6. Mathematics II Paper (Math.312), 2066

(Analytical Geometry & Vector Analysis)

Time : 3 hrs. Full Marks: 75

Attempt ALL the questions.

Group "A" 5×7=35

1. When does the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represent a parabola, an ellipse and hyperbola ? What conic does the equation $14x^2-4xy+11y^2-44x-58y+71=0$ represent ? Find the centre of the conic. [3+4]

OR

Define the tangent to a conic, $S=ax^2+2hxy+by^2+2gx+2fy+c=0$ at (x_1,y_1) obtain the condition that the line $lx+my+n=0$ may be a tangent to the conic $S=0$ at (x_1,y_1). [2+5]

2. Define the auxiliary circle and eccentric angle of a point with respect of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. [1+1+5]

Find the foci, directrices, eccentricity, the ends of latus rectum and length of latus rectum of the ellipse $9x^2+25y^2=225$.

3. **Define skew lines and line of shortest distance. Find the shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$. Find also the equation of shortest distance.** [1+1+5]

4. **Define a great and small circle of a sphere. Find the equation of a sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.** [1+1+5]

Define tangent line and tangent plane at a point of the sphere. Find the equation to the spheres which pass through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touch the plane $4x+3y=15$. [1+1+5]

5. **Define scalar triple product and prove geometrically that the scalar triple product represents the volume of the parallelopeped. Also verify that in the scalar triple product position of dot and cross can be interchanged.** [1+3+3]

Group "B" 10×4=40

6. **What are the equation $(x-a)^2+(y-b)^2=c^2$ become when it is transformed to parallel axes through the point $(a-c,b)$?** [4]

OR

Find the polar coordinates of the points (3,4,5) and (-2,1,2). [2+2]

7. **Show that the line $x\cos\alpha+y\sin\alpha=p$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $p^2=a^2\cos^2\alpha+b^2\sin\alpha$.**

OR [4]

Show that the tangent at the extremity of any diameter of an ellipse is parallel to the chords which it bisect.

8. State the condition under which the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent an ellipse. Find the centre of the conic section $2x^2 - 5xy - 3y^2 - x - 4y + 6 = 0$ and its equation when transformed to the centre. [1+3]

9. Find the equation of the plane through the line $\frac{x-a}{l} = \frac{y-\beta}{m} = \frac{z-y}{n}$ parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$. [4]

10. Show that the equation to a right circular cone whose vertex is 0, axes OX and semivertical angle α is $y^2 + z^2 = x^2 \tan^2\alpha$.

OR [4]

Define reciprocal cone. Prove that the cone $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.

11. Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = y = z$. [4]

12. Obtain the condition that the plane $lz + my + nz = p$ may touch the central conicoid $ax^2 + by^2 + cz^2 = 1$. [4]

13. Prove the following :

a. $(\vec{a} \times \vec{b})^2 = a^2 b^2 - (\vec{a}.\vec{b})^2$

b. If $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \times \vec{b} = +\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. [4]

14. If $\vec{r} = a\cos t\,\vec{i} + a\sin t\,\vec{j} + at\tan\alpha\,\vec{k}$ find $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|$ and $\left[\vec{r}\,\frac{d\vec{r}}{dt}\,\frac{d^2\vec{r}}{dt^2}\right]$. [4]

15. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$.

OR [4]

Prove that curl $(grad\phi) = 0$.

Mathematics II Paper (Math.312), 2067

(Analytical Geometry & Vector Analysis)

Bachelor Level / I Year / Sc. & Tech. + Hum. **Full Marks: 75**

Time: 3 hrs.

Attempt ALL the questions

Group 'A' $5 \times 7 = 35$

1. What is conic section ? When it becomes hyperbola ? Obtain the length of the axes, eccentricity, coordinates of foci, equation of directrix and length of latus rectum of the hypoerbola $6x_2 - 25y_2 = 400$ [1+1+5]

2. What are the conditions under which the second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent (i) a hyperbola (ii) an ellipse (iii) a parabola ? What conic does the equation

$12x^2 - 23xy + 10y^2 - 25x + 26y - 14 = 0$ represent ?

If possible, find the centre and its equation referred to the circle. [1+1+3+2]

Or

Define pole and polar with respect to conic. Obtain the equation of polar or any point (x', y') w.r.t. to the conic represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [2+5]

3. Find the equation straight line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2).

Find the point where the line joining (2, 1, 3) and (4, –2, 5) cuts the plane $2x + y - z - 3) = 0$. [3+4]

4. Define reciprocal cone. Prove that the equation $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ represent a cone which touches the coordinate planes and that equation of the receiprocal cone is $fyz + gzx + hxy = 0$. [1+6]

OR

Define a cone. Obtain the equation of cone with vertex (α, β, γ) and base the parabola $z^2 = 4ax$, $y = 0$. [1+6]

5. Define vector triple product of any non zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and give its geometrical meaning. Find an expression for $\vec{a} \times (\vec{b} \times \vec{c})$. [1+2+4]

Group "B" 10×4=40

6. What does the equation $2x^2 + y^2 - 4x + 4y = 0$ become, when it is transferred to parallel axis through the point (1, –2) ? [4]

OR

Find the distance of the point (1, 2, 3) from the coordinate axes. Also find its distance from the origin. [3+1]

7. If e and e' the eccentricity of hyperbola and its conjugate prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$. [4]

8. What is a focal chord of a conic ? In any conic prove that the sum of the receiprocal of the segments of any focal chord is constant.

9. Find the equation of plane through (2, –3, 1) normal to the line joining (3, 4, –1) and (2, –1, 5) [4]

10. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinates axes at A, B, C.

Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is

$$yz\left(\frac{b}{c}+\frac{c}{b}\right) + zx\left(\frac{c}{a}+\frac{a}{c}\right) + xy\left(\frac{a}{b}+\frac{b}{a}\right) = 0$$ [4]

OR

Show that the equation to a right circular cone whose vertex is O, axis OX and semi-vertical angle 'α' is $y^2 + z^2 = x^2 \tan^2\alpha$. [4]

11. Find the equation of the sphere which passes through the origin and the points (0, 1, –1), (–1, 2, 0) and (1, 2, 3). [4]

OR

Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.

12. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ through the point (a, b, g). Prove that the perpendiculars to them form the origin generates the cone $(\alpha x + \beta y + \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2$. [4]

13. Show that : $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2\,[\vec{a}\ \vec{b}\ \vec{c}]$ [4]

14. Prove that : $[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2$. [4]

15. The necessary and sufficient condition for the vector function of a scalar variable to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$. [4]

OR

If $\vec{a}$ is a constant vector then prove that

(i) $\vec{a} \cdot \nabla\left(\frac{1}{r}\right) = -\frac{\vec{a} \cdot \vec{r}}{r^3}$

(ii) grad $(\vec{r} \cdot \vec{a}) = \vec{a}$. [2+2]

Mathematics II Paper (Math.312), 2068

(Analytical Geometry & Vector Analysis)

Bachelor Level/I Year/ Sc. & Tech.+ Hum. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions

Group "A" 5×7=35

1. What type of the conic section is the hyperbola ? Define its foci and eccentricity and directrix.

Determine the centre, coordinates of foci, the eccentricity, length of the latus rectum and the equation of the directrices of the hyperbola.

$$5x^2 - 6y^2 = 30.$$ [1+2+4]

2. What conic does the equation

$3x^2 - 8xy - 3y2 + 10x - 13y + 8 = 0$ represent ? If possible, find the centre and its equation referred to the centre. [2+5]

Or

Define pole and polar with respect to a conic. Determine the equation of the polar with respect to the conic represented by

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$ [2+5]

3. Define the skew lines and the line of shortest distance. Find the shortest distance between the lines

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4} \text{ and } \frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}.$$

Also find the equation of the shortest distance. [1+4+2]

4. What do you mean by a great circle and a small circle of the sphere ? Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$; $x - 2y + 2z = 5$ as a great circle, determine its centre and radius. [1+1+5]

Or

Find the tangent line and tangent plane at a point of a sphere. Show that the plane $2x - y + 3z = 14$ touches the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$. Find the point of contact. [2+3+2]

5. Define reciprocal system of vectors. If $\vec{a}'$, $\vec{b}'$, & $\vec{c}'$, be reciprocal system to three non coplanar vectors $\vec{a}$, $\vec{b}$, & $\vec{c}$, then prove the followings :

(i) $\vec{a} . \vec{b}' = \vec{b} . \vec{b}' = \vec{c} . \vec{c}' = 1$

(ii) $\vec{a} . \vec{b}' = \vec{b} . \vec{c}' = \vec{c} . \vec{a}' = 0$

(iii) $[\vec{a}' \vec{b}' \vec{c}'] [\vec{a} \vec{b} \vec{c}] = 1$ [+1+1+4]

Group "B"

10×4=40

6. If the axes be turned through an angle $\tan^{-1}(2)$ what does the equation $4xy - 3x^2 = a^2$ becomes ? [4]

7. Find the locus of the point of intersection of the tangents to the ellipse which meet at right angles. What is the nature of the locus ? [3+1]

OR

Show the line $x \cos\alpha + y \sin\alpha = p$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $p^2 = a^2 \cos^2\alpha + b^2 \sin^2\alpha$. [4]

8. Find the equation of the polar with respect to the conic represented by the equation

$$ax^2 + 2hxy + by^2 \ 2gx + 2fy + c = 0$$ [4]

OR

Prove that the equation $\frac{\ell}{r} = 1 - e\cos\theta$ and $\frac{\ell}{r} = e\cos\theta - 1$ represent the same conic.

9. Find the equation of the plane through (–1, 1, –1) and (6, 2, 1) normal to the plane $2x + y + z = 5$. [4]

10. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and

$$\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$$ [4]

11. Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having it generators parallel to the line $x = y = z$. [4]

12. Obtain the condition that the plane $\ell x + my + nz = p$ may touch the central conicoid $ax^2 + by^2 + cz^2 = 1$. [4]

Or

Find the equation of the planes which contain the line given by $5x + 6y - 18 = 0$ and $3y - z = 0$ and touch the ellipsoid $5x^2 + 3y^2 = 36$.

13. If $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$ verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c}$.

14. Show that the necessary and sufficient condition for the vector function $\vec{a}$ of a scalar variable t to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

OR [4]

A particle moves along the curve $x = 4 \cos t$, $y = 4 \sin t$ $z = 6t$. Find the magnitude of acceleration at time $t = \pi$

15. Define curl of a vector function. If ϕ be a scalar function prove that curl (grad ϕ) = 0. [1+3]

OLD COURSE

Attempt ALL the questions

Group "A"

$5 \times 7 = 35$

1. What type of locus is the ellipse ? In what respect does ellipse differ from hyperbola ? Find the centre, coordinates of foci, the eccentricity, latus rectum and the equation of the directrices of the ellipse $9x^2 + 16y^2 = 144$. [1+1+6]
2. Define a conic. Obtain the polar equation of a conic having given its eccentricity e and latus rectum equal to 2ℓ and focus being taken as pole. Also find the equation of its directrix. [1+4+2]
3. What does the shortest distance between two lines mean ? Find the magnitude and the equation of the line of shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$ [1+3+3]

OR

What are coplanar lines ? Find the condition that the two lines in symmetrical form are coplanar. Also show that the lines

$$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}; \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$ are coplanar.

4. Define a cone. Determine the equation of the cone with vertex (α, β, γ) and base $y^2 = 4ax$, $z = 0$. [1 + 6]

OR

Define the generator of a cone. Find the condition that the cone has three mutually perpendicular generators. [1+6]

5. Define the scalar triple product of three non-zero vectors and interpret it geometrically. Prove that the scalar triple product of three noonzero vectors is zero when two of the vectors are equal and parallel. [1+1+5]

Group "B"

6. What does the equation $(x-h)^2 + (y-k)^2 = r^2$ become when it is transferred to parallel axes through the point $(h, k, -r)$? [4]
7. Find the equation to the common tangent of the circle $x^2 + y^2 = 4ax$ and the parabola $y^2 = 4ax$. [4]

OR

Show that line $\ell x + my = n$ is a normal to the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $\frac{a^2}{\ell^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$ [4]

8. Prove that the equation $\frac{\ell}{r}=1-e\cos\theta$ and $\frac{\ell}{r}=e\cos\theta-1$ represent the same conic. [4]

9. Find the equation of the plane through (2, –3, 1) normal to the line joining (3, 4 –1) and (2, –1, 5). [5]

10. Prove that $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represent a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$ [4]

OR

Define reciprocal cone.

Prove that the cone $ayz+bzx+cxy=0$ and $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$ are reciprocal. [1+3]

11. What are coplanar lines ?

Prove that the lines $\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2}$ and

$3x+2y+z-2=0=x-3y+2z-13$ are coplanar. [1 + 3]

12. Obtain the condition that the line $\ell x+my+nz=p$ may touch the central conicoid $ax^2+by^2+cz^2=1$. [4]

OR

Prove the equations of two planes which contain the line given by $5x+6y-18=0$ and $3y-z=0$ and touch the ellipsoid $5x^2+3y^2+z^2=36$.

13. Prove that following :

(a) $(\vec{a}\times\vec{b})^2=a^2b^2-(\vec{a}\cdot\vec{b})^2$

(b) If $\vec{a}+\vec{b}+\vec{c}=0$, then $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ [2+2]

14. Prove that the necessary and sufficient condition for the vector function of a scalar variable to have a constant magnitude is

$\vec{a}\cdot\frac{d\vec{a}}{dt}=0.$ [4]

OR

If $\vec{r}_1$ and $\vec{r}_2$ are the directional vector functions then prove that $\frac{d}{dt}(\vec{r}_1\times\vec{r}_2)=\vec{r}_1\times\frac{d\vec{r}_2}{dt}+\frac{d\vec{r}_1}{dt}\times\vec{r}_2.$ [4]

15. Show that div $(\hat{r})=\frac{2}{r}$, where

$\vec{x}'=x\vec{i}+y\vec{j}+2\vec{k}.$ [4]

Mathematics II Paper (Math.312), 2069

(Analytical Geometry & Vector Analysis)

Bachelor Level/I Year/Sc. & Tech. + Hum. — Full Marks : 100

(For: Regular Examinee only) — Time :3hrs.

Attempt ALL the questions.

GROUP 'A' $5\times7=35$

1. Define conic section. Find the centre, foci, eccentricity, latus rectum and length of axes of the ellipse $x^2+4y^2-4x+24y+24=0$ [1+6]

2. Define general equation of second degree and show that general equation of second degree in x & y represent a conic section. [1+6]

   Or

   Find the centre of the conic $9x^2-4xy+6y^2-14x-8y+1=0$ show that this conic is an ellipse. Also find its semi-axes and eccentricity. [7]

3. Defin. shortest distance between the lines. Obtain the equation of the line of shortest distance between the lines. [1+6]

4. Define a cone. Determine the equation of the cone with vertex (α, β, γ) and base $y^2=4ax$, $z=0$. [1+6]

   Or

   Define the generator of a cone. Find the condition that the cone has three mutually perpendicular generator. [1+6]

5. Define reciprocal system of vectors. If $\vec{a}'$, $\vec{b}'$, $\vec{c}'$ be reciprocal system to three non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ then prove the followings :

   (i) $\vec{a}.\vec{a}'=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'=1$

   (ii) $\vec{a}.\vec{b}'=\vec{b}.\vec{c}'=\vec{c}.\vec{a}'=0$

   (iii) $[\vec{a}'\ \vec{b}'\ \vec{c}']\ [\vec{a}\ \vec{b}\ \vec{c}]=1$ $[1+1^{1/2}+1^{1/2}+3]$

GROUP 'B' $10\times4=40$

6. What does the equation $(x-a)^2+(y-b)^2=c^2$ become when it is transferred to parallel axes through the point $(a, b-c)$? [4]

7. Define normal to the ellipse. Prove that the straight line $\ell x+my+n=0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $\frac{a^2}{\ell^2}+\frac{b^2}{m^2}=\frac{(a^2-b^2)^2}{n^2}$

   Or

   Show that the tangent at the extremity of any diameter of an ellipse is parallel to the chords which it bisect. [4]

8. Find the centre of the conic section $9x^2-4xy+6y^2-14x-8y+1=0$. [4]

9. Find the point where the line joining (2, 1, 3) (4, –2, 5) cuts the plane $2x+y-z-3=0$. [4]

   Or

   Find the equation of line through the point (2, 3, 1) and parallel to the planes $2x+3y+4z=5$ and $3x+4y+5z=6$. [4]

10. Find the shortest distance between the lines

$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$ [4]

11. A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right) + zx\left(\frac{c}{a}+\frac{a}{c}\right) + xy\left(\frac{a}{b}+\frac{b}{a}\right) = 0.$$ [4]

12. Planes through OX and OY include an angle α. Show that their line of intersection lies on the cone $z^2(x^2 + y^2 + z^2) = x^2y^2\tan^2\alpha$.

Or

Prove that the cone $ayz + bzx + cxy = 0$ and $\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$ are reciprocal.

13. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors prove that $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c} = 3$. [4]

14. if $\vec{r}_1 = 2t^2\vec{i} + 3(t-1)\vec{j} + 4t^2\vec{k}$ and $\vec{r}_2 = (t-1)\vec{i} + t^2\vec{j} + (t-2)\vec{k}$, show that $\int_0^2 (\vec{r}_1.\vec{r}_2)\,dt = \frac{4}{3}$. [4]

15. Prove curl (grand ϕ) = 0. [4]

Or

If $\vec{f} = x^2z\vec{i} - 2y^3z^2\vec{j} + xy^2z\vec{k}$ find div $\vec{f}$ and curl $\vec{f}$ at (1, –1, 1). [4]

Mathematics II Paper (Math.312), 2070

(Analytical Geometry & Vector Analysis)

Bachelor Level/I Year/ Sc. & Tech. + Hum. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

GROUP "A"

5×7=35

1. What type of conic section is the hyperbola? Find the coordinates of centre, foci, equation of directrix, eccentricity and latus rectum of the hyperbola $4x^2 - 9y^2 + 8x + 18y - 41 = 0$. [1+6]

OR

Define conjugate hyperbola. Give an example to show that a hyperbola and its conjugate have the same asymptotes. Find the equation to the hyperbola, whose asymptotes are the straight lines $x + 2y + 3 = 0$ and $3x + 4y + 5 = 0$ and which passes through the point (1, –1) [1+1+5]

2. Define tangent and normal to a curve. Find the condition that any straight line $x + my + n = 0$ may touch the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [2+5]

3. What are skew lines and line of shortest distance?

Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z+2}{2}$.

Also find the equation of the shortest distance. [1+4+2]

4. Find the equation of the tangent plane at (α, β, γ) to the conicoid $ax^2 + by^2 + cz^2 = 1$ and hence write down the equation of the tangent plane at a point (α, β, γ) of the ellipsoid. [5+2]

OR

What is conicoid? Give the condition under which it represents an ellipsoid. Show that six normal's can be drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ from any given point. [1+1+5]

5. Define derivative of a vector function of a scalar variable. Prove that the necessary and sufficient condition for the vector function $\vec{a}$ of a scalar variable to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.

A particle moves along the curve $x = 2 \sin 3t$, $y = 2\cos 3t$, $z = 8t$. Find the magnitude of the velocity at $t = \pi/3$. [1+4+2]

GROUP "B" 10×4=40

6. Find the equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$ referred to parallel axes through (–1, 2). [4]

7. Find the equation to the common tangents of the circle $x^2 + y^2 = 4ax$ and the parabola $y^2 = 4ax$. [4]

OR

Show that the line $x + my = n$ is normal to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $\frac{a^2}{\ell^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$

8. Prove that the equation $\frac{\ell}{r} = 1 + e \cos \theta$ and $\frac{\ell}{r} = -1 + e \cos \theta$ represent the same conic. [4]

9. Find, the equation of the plane through (2, –3, 1) normal to the line joining (3, 4, –1) and (2, –1, 5). [4]

OR

Find the equation of plane through (–1, 1, –1) and (6, 2, 1) normal to the plane $2x + y + z = 5$.

10. Show that the equation to a right circular cone whose vertex is 0, axis 0X and semivertical angle 'α' is $y^2 + z^2 = x^2 \tan^2\alpha$. [4]

11. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touching the plane $z = 0$. [4]

12. If 2r be the distance between two parallel tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ prove that the line through the origin perpendicular to the planes lies on cone $(a^2 - r^2)x^2 + (b^2 - r^2)y^2 + (c^2 - r^2)z^2 = 0$. [4]

OR

Find the equation of the tangent plane at (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

13. Show that : $[\vec{\ell} + \vec{m} \;\; \vec{m} + \vec{n} \;\; \vec{n} + \vec{\ell}] = 2\,[\vec{\ell} \;\; \vec{m} \;\; \vec{n}]$. [4]

14. If $\vec{r} = a \cos t\vec{i} + a \sin t\vec{j} + a \tan \alpha t\vec{k}$, find

$$\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right| \text{ and } \left[\vec{r} \;\; \frac{d\vec{r}}{dt} \;\; \frac{d^2\vec{r}}{dt^2}\right]$$ [4]

15. Define divergence and curl of a vector function if $\phi = \log(x^2 + y^2 + z^2)$, find curl (grad ϕ). [4]

OR

Define curl of a vector function. If ϕ be a scalar function prove that curl (grad ϕ) = 0.

OLD COURSE

Attempt ALL the questions.

Group "A" 5×7=35

1. Define conic section. When does it become ellipse? Obtain the length of axes, the eccentricity, the coordinates of foci, the length of latus rectum, and the equation of directrices of the ellipse $9x^2 + 25y^2 = 225$. [1+1+5]

2. State the conditions under which the general equation of second degree may represent (1) a parabola (2) an ellipse (3) a hyperbola. What conic does the equation $2x^2 - 72xy + 23y^2 - 4x - 28y - 48 = 0$ represent?
If possible, find the centre and its equation referred to the centre. [1+1+3+2]

OR

Define normal to a curve. Obtain the equation of the normal at any point (x' y') of the conic represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [1+6]

3. Find the equation of straight line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2). Find K so that the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ may be perpendicular to each other. [3+4]

4. What are coplanar lines? Find the condition that the two lines in symmetrical form are coplanar. Also, show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$; $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. [1+3+3]

5. What are reciprocal system of vectors? Show that the scalar product of any vector of one system with a vector of the other system which does not correspond to it is zero. Find a set of vectors which form a reciprocal system to the set of vectors $-\vec{i} + \vec{j} + \vec{k}, \vec{i} - \vec{j} + \vec{k}, \vec{i} + \vec{j} - \vec{k}$. [1+2+4]

OR

Define scalar product of three non-zero vectors. Interpret it geometrically. Show that the position of dot and cross can be interchanged without changing its value. [1+3+3]

Group "B" 10×4=40

6. Reduce the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ by translating the axes into an equation with linear term missing. [4]

7. Define focal chord of a conic. In any conic, prove that the sum of the reciprocals of two perpendicular focal chord is constant. [1+3]

OR

Obtain the polar equation of the conic section in the form $r = \frac{1}{1 + e\cos\theta}$ [4]

8. Prove that the point $x = \frac{a(1-t^2)}{1+t^2}$ and $y = b\left(\frac{2t}{1+t^2}\right)$ is a point of an ellipse, where t is a parameter. [4]

9. Obtain the equation of plane through the intersection of the planes $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and the origin. [4]

OR

Obtain the angle between the two planes represented by $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$.

10. Find the equation of the sphere have the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle. [4]

11. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = a^2$ with vertex at the point (α, β, γ). [4]

OR

Find the equation of enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the lines $x = y = z$.

12. If the normal at any point P of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ meets the principal planes at G_1, G_2, G_3 respectively show that $PG_1 : PG_2 : PG_3 = a^2 : b^2 : c^2$. [4]

13. Prove that $[\vec{\ell}\ \vec{m}\ \vec{n}]\,[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} \vec{\ell}.\vec{a} & \vec{\ell}.\vec{b} & \vec{\ell}.\vec{c} \\ \vec{m}.\vec{a} & \vec{m}.\vec{b} & \vec{m}.\vec{c} \\ \vec{n}.\vec{a} & \vec{n}.\vec{b} & \vec{n}.\vec{c} \end{vmatrix}$ [4]

14. Show that any vector $\vec{r}$ may be expressed as $\vec{r} = (\vec{r}\,.\,\vec{a}')\,\vec{a} + (\vec{r}\,.\,\vec{b}')\,\vec{b} + (\vec{r}\,.\,\vec{c}')\,\vec{c}$ where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors. [4]

15. Define gradient of a scalar function and divergence of a vector function. Prove that $\text{Div}\,(\phi\,\vec{a}) = \phi\,\text{div}\,\vec{a} + \vec{a}\,.\,(\text{grad}\,\phi)$, where ϕ is a scalar function of x, y, z.

OR [4]

Define divergence and curl of a vector function. If $\phi = \log(x^2 + y^2 + z^2)$, find curl (grad ϕ)

Mathematics II Paper (Math.102), 2070 (New course)

(Analytical Geometry & Vector Analysis)

Four Year Bachelor Level/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

Group "A" 5×7=35

1. Find the centre, eccentricity, length of axes, length of latus rectum, foci of the ellipse $8(x-1)^2 + 6(y-1)^2 - 1 = 0$. [1+1+2+2+1]

2. Find the condition that any straight line $\ell x + my + n = 0$ may touch the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Show that $x + 4y = 8$ touches the conic $x^2 + 4xy + 3y^2 - 5x - 6y + 3 = 0$. [4+3]

OR

What conic does the equation $12x^2 - 23xy + 10y2 - 25x + 26y - 14 = 0$ represent? If possible, find the equation referred to the conic. [2+5]

3. Show that the equation of the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1$; $x = 0$ and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1$; $y = 0$ is $\frac{x}{a} - \frac{y}{b} = 1$ and if 2d is the shortest distance between the line, show that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$. [3.5 + 3.5]

Or

Find the shortest distance between the line $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z-2}{5}$ and $\frac{x-2}{3} = \frac{y+3}{-2} = \frac{z-4}{1}$. Also find the equation of the line of shortest distance. [3.5+3.5]

4. Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = 2y = z$.

5. Define curl of a vector function. Prove that if ϕ is a scalar field, then curl $(\phi \vec{a}) = \phi \operatorname{div} \vec{a} + \vec{a} \cdot (\operatorname{grad} \phi)$ [1+6]

GROUP "B" 10×4=40

6. If the axes be turned through an angle $\tan^{-1}(3)$, what does the equation $3xy - 4y^2 = a^2$ becomes. [4]

7. Show that line $\ell x + my = n$ is tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2\ell^2 - b^2m^2 = n^2$. [4]

OR

Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ and equation of the conjugate hyperbola. [2+2]

8. Prove that in any conic, the semi-latus rectum is a harmonic mean between the segment of any focal chord. [4]

9. Find the point where the line joining (1, 2, 3), (3, –1, 4) cuts the plane $3x + 2y + z - 2 = 0$. [4]

10. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 8y - 2z - 4 = 0$, $x + 2y + z = 2a$s the great circle. [4]

OR

A plane passes through a fixed point (f, g, h) and cuts the axes at A, B, C. Prove that the locus of the sphere OABC if $\frac{f}{x} + \frac{g}{y} + \frac{h}{z} = 2$ [4]

11. Show that the angle between lines given by $x + y + z = 0$ and $fyz + gzx + hxy = 0$ is $\frac{\pi}{2}$ if $f + g + h = 0$. [4]

OR

Find the equation of the cone with vertex at the origin and passes through the curve intersection of $x^2 + y^2 + 3Z^2 - 1$ and $x + y + 2z = 3$. [4]

12. Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid $3x^2 + dy^2 + 9z^2 + 17 = 0$ and find the point of contact. [4]

13. How many different type of product among three vectors $\vec{a}, \vec{b}, \vec{c}$ can be made ? Prove that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$. [2+2]

14. If $\vec{r} = \vec{a} \cos wt + \vec{b} \sin wt$, prove that $\vec{r} \times \frac{d\vec{r}}{dt} = w\,(\vec{a} \times \vec{b})$ [4]

15. Define divergent of a vector function.
If $F = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$, find div (curl F). [4]

OR

Define gradient of a scalar field. If $f = ayz^2 + bzx^2 + cxy^2$, find curl (grad f)

Mathematics II Paper (Math.102), 2071

(Analytical Geometry & Vector Analysis)

Bachelor Level (4 Yrs.)/I Year/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

GROUP "A" 5×7=35

1. Find the nature of the conic represented by the equation
$36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$ and trace it. [5+2]

OR

Find the centre of the conic $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$ and what conic does the equation represent? [3½ + 3½]

2. Find the centre, eccentricity, foci and directrices of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$. [2+1+2+2]

3. Define cylinder. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and passing through the curve $x^2 + 2y^2 = 1$, $z = 0$. [1+6]

4. Define sphere. Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point (1, 1 – 1) and passes through the origin. [1+6]

5. Define curl of a vector function, prove that
Div. $(\vec{a} \times \vec{b}) = \vec{b} \cdot (\text{curl } \vec{a}) - \vec{a} \cdot (\text{curl } \vec{b})$. [1+6]

OR

If $\vec{f} = x^2z\vec{i} - 2y^3z^2\vec{j} + xy^2z\vec{k}$, find div $\vec{f}$ and curl $\vec{f}$ at (1, –1, 1). [3½ +3½]

GROUP "B" 10×4=40

6. Find the distance of the point (2, –1, 3) from the coordinate axes. Also find its distance from the origin. [3+1]

7. Show that straight line $\ell x + my = n$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ if $a^2\ell^2 + b^2m^2 = n^2$. [4]

8. In any conic, prove that the sum of the reciprocals of two perpendicular focal chords is constant. [4]

OR

Prove that the equation $\frac{\ell}{r} = 1 - e\cos\theta$ and $\frac{\ell}{r} = e\cos\theta + 1$ represent the same conic. [4]

9. Define plane. Find the equation of the plane through the intersection of the plane $2x + 3y + 10z = 8$, $2x - 3y + 7z = 2$ and normal to the plane $3x + 2y + 4z = 5$. [1+3]

10. When will two given lines be coplanar? Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also find their point of intersection. [1+2+1]

OR

Find the shortest distance between the lines

$\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+3}{4}$ and $\frac{x+3}{4} = \frac{y-2}{5} = \frac{z-1}{3}$ [4]

11. Define cone. Show that a second degree homogeneous equation in x, y, z represents a cone whose vertex is at the origin. [1+3]

OR

Find the equation of the cone with vertex at the origin and passes through the curve of intersection of $x + y + 2z = 1$ and $x - y + 2z = 5$. [4]

12. Find the equation of the tangent plane at (α, β, γ) to the conicoid $ax^2 + by^2 + cz^2 = 1$. [4]

13. Define vector triple product, of any non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$. Find an expression for $\vec{a} \times (\vec{b} \times \vec{c})$. [1+3]

14. What do you mean by constant vector? Evaluate $\frac{d}{dt}\left(\frac{\vec{r} - \vec{a}}{r^2 + a^2}\right)$ [1 +3]

15. Prove that the curl of the linear velocity of a rigid body equals twice the angular velocity of the body.

OR [4]

If $\vec{V} = e^{xyz}(\vec{i} + \vec{j} + \vec{k})$, then find curl $\vec{V}$.

Mathematics II Paper (Math.102), 2072

(Analytical Geometry & Vector Analysis)

Bachelor Level (4yrs. prog.) I Year/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

Group "A" 5×7 = 35

1. Explain the auxiliary circle and the eccentric angle of a point in an ellipse. Find the point at which the line $\ell x + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$. Also, find the condition for the line to be a normal to the ellipse.

[3+2+2]

2. Define the tangent and normal to a conic. Find the equation of tangent at my point (x_1, y_1) of the conic.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0.$$ [2+5]

OR

Write the condition for the second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent different types of conics. If the centre of the hyperbola.

$f(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is at (h, k) then prove that the pair of asymptotes are given by $f(x, y) = f(h, k)$ [3.5+3.5]

3. Obtain the expression for the angle between a line with direction ratios ℓ, m, n and a plane $ax + by + cz + d = 0$. Find the points at Which the line $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ cuts surface $11x^2 - 5y^2 + z^2 = 0$. [4+3]

4. Write the equation of a sphere in diameter form. Find the equation of a sphere that cuts each positive coordinate axes at a unit distance and the radius as small as possible. [1+6]

OR

What is a plane section of a sphere? Find the centre and the radius of the circle

$$x^2 + y^2 + z^2 + 12x - 12y - 16z + 111 = 0 = 2x + 2y + z - 17.$$ [2+5]

5. Prove that in a scalar triple product, the position of the dot and the cross can be interchanged. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, find the angles which $\vec{a}$ and $\vec{c}$ if $\vec{b}$ and $\vec{c}$ are not parallel. [2.5+2.5+2.5]

Group "B"

$10 \times 4 = 40$

6. Find the transformed equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$, if the origin is shifted at (–1, 2) but the direction of axes are not changed. [4]

7. Find the condition for the line $x \cos\alpha + y \sin\alpha = p$ will be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. [4]

8. If PSP' and QSQ' are two perpendicular focal chords of a conic $\frac{l}{r} = 1 + e\cos q$, prove that $\frac{1}{pp'} + \frac{1}{QQ'}$ is constant, where S is the focus. [4]

OR

Find the equation of tangent at point whose vectional angle is α for the conic $\frac{l}{r} = 1 + e\cos q$.

9. Find the equation of the plane passing through the point (1, –2, 3) and perpendicular to the line passing through (3, 4 – 5) and (1, 2, 3). [4]

10. Find the point of intersection of the lines

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} \text{ and } x - 4 = \frac{y+3}{-4} = \frac{z+1}{7}.$$ [4]

11. If the section of a cone with vertex at L and guiding curve, the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1; z=0$ by the xy – plane is a rectangular hyperbola, find the locus of L.

[4]

OR

Find the equation of the right cylinder that passes through $y^2 = 4ax$, $z = 0$ and whose generators are parallel to the line $x = y = z$.

12. Find the equation of the tangent plane at the point (f, g, h) to the conicoid $ax^2 + by^2 + cz^2 = 1$.

[4]

OR

If $3x + 12y - 6z = 17$ is a tangent plane to the conicoid $3x^2 - 6y^2 + 9z+2 + 17 = 0$, find the point of contact.

13. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, then show that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are also non-coplanar. [4]

14. For the space curve $x = 3t$, $y = 3t^2$, $z = 2t^3$, prove that

$$\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right] = 216.$$ [4]

15. Find the unit vector normal to the surface $z = x^2 + y^2$ at the point (–1, –2, 5). **[4]**

OR

For any space vector $\vec{v}$ prove that

grad (div $\vec{v}$) = curl (curl $\vec{v}$) + $\sum \frac{\partial^2 v^{TM}}{\partial x^2}$ **[4]**

7. Statistics I Paper (Stat.311), 2066

(Descriptive Statistics & Introduction to Probability)

Time : 3 hour **Full Marks : 100**

Attempt ALL the questions.

Group "A"

1. Compulsory Question. **[5×3=15]**

Attempt any FIVE questions.

a) Give the difference between ordinal scale and nominal scale.

b) Discuss what you know about the classification and categorization. Write down the formula for estimating the number of classes required.

c) For a number of 51 observations, the arithmetic mean and standard deviation are 58.5 and 11 respectively. It was found after the calculations were made that the one of the observations recorded as 15 was incorrect. Find the standard deviation of the 50 observations if this incorrect observation is omitted.

d) Write down the normal equation in fitting the modely $Y = a + bx + cx^2$.

e) If X and Y sare independent variates, prove that they are uncorrelated, that is $r_{xy} = 0$. Show by an example that the converse of theorem is not necessarily true.