5. Mathematics I Paper (Math. 311), 2066
(Calculus)

Time : 3 hrs. Full Marks: 75

Attempt ALL the questions.

Group "A" $5 \times 7 = 35$

1. Define and deduce the expressions for the polar subtangent and polar subnormal at any point P(r, θ) of a curve $r = f(\theta)$. Find the angle between the curves $r^2 = a^2 \cos 2\theta$ and $r^2 = b^2 \sin 2\theta$. [1+2+4]

2. State Taylor's series extended to infinity. Let Rn denote the remainder after n terms of the series. Prove that $\lim_{n \to \infty} Rn = 0$ is both necessary and sufficient condition that the function $f(x+h), |h| < \delta$ can be expanded in an infinite series. Hence show that

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots\ldots\ldots \text{ to } \infty \text{ for all x.}$$ [1+2+4]

OR

State Leibnitz theorem.

If $y = \tan^{-1} x$. prove $(1+x^2)y_1 = 1$

and hence show that

$$(1+x^2)y_{n+1} + 2nxyn + n(n-1)y_{n-1} = 0$$ [1+2+4]

3. Define Beta and Gamma functions.

Prove that : $\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\tau\left(\frac{m+1}{2}\right)\tau\left(\frac{n+2}{2}\right)}{2\tau\left(\frac{m+n+2}{2}\right)}$ and hence show that

$\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{x}$. [2+3+2]

4. How do you define the maximum and minimum values of a function of two variables ?

Find the minimum values of $x^2 + y^2 + z^2$ when $x + y + z = 3a^2$. [2+5]

OR

State and establish Eulers theorem for a homogeneous function of degree n. Use this theorem to show that

$x\frac{2u}{2x} + y\frac{2u}{2y} = \tan u$ if $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$. [1+2+4]

5. State the condition of exactness of a first order differential equation. Verify that the equation $(2xy + y^2)dx + (x^2 2xy - y)dy = 0$ is exact and hence find its general solution.

Group "B" 10×4=40

6. What is the angle between the curve $r = \psi_1(\theta_2), r = \phi(\theta)$? Show that the curves ax $ax^2 + by^2 = 1$ and $a^1x^2 + b^1y^2 = 1$ cut orthogonally if $\frac{1}{a} - \frac{1}{a^{-1}} = \frac{1}{b} - \frac{1}{b^1}$. [1+3]

OR

Show that the tangent drawn at the extremities of any chord of the cardiode $r = (1 + \cos\theta)$ which passes through the pole are perpendicular to each other. [4]

7. What do you mean by indeterminate form ?

Evaluate : $\lim_{x \to \pi/2} (\sin x)^{\tan x}$. [1+3]

8. Evaluate : $\int_0^{\pi/2}\int_{\pi/2}^{\pi} e^x \cos(y - x) dy dx$

OR [4]

Show that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy \neq \int_0^1 dy \int_0^1 \frac{(x-y)}{(x+y)^3} dx$ [4]

9. Show that $\int_0^1 \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = \pi/2 \log(1/2)$. [4]

10. Let the circle $x^2 + y^2 = a^2$ revolves round the x-axis, show that the volume of the whole sphere generated is $\frac{4}{3}\pi a^3$ [4]

11. Obtain the reduction formula for $\int \operatorname{cosec}^n x\,dx$ and find $\int \operatorname{cosec}^5 x\,dx$.

OR [4]

12. Find the complete solution and the singular solution of the equation $y = px + p - p^2$. [4]

13. Find the complimentary function and particular integral of the differential equation.

$$\frac{d^2y}{dx^2} + a^2y = \sec ax$$ [1+3]

Solve: $(D^2 + 3D + 2)y = e^{2x}$, given that $y = 0$, $\frac{dy}{dx} = 0$ when $x = 0$. [4]

14. Solve: $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$. [4]

15. Define Clairaut equation and solve

$$y = px + \sqrt{a^2p^2 + b^2}.$$ [4]

Mathematics I Paper (Math. 311), 2067
(Calculus)

Bachelor Level /Science & Tech. / I Year — Full Marks: 100

Time: 3 hrs.

Attempt ALL the questions.

Group "A" — **5×7=35**

1. Define the length of perpendicular from the pole on the tangent to a curve. Also define pedal equation and obtain its expression for the curve $r^2 = a^2 \cos 2\theta$. [3+1+3]

OR

What the pedal equation of a curve is ? Deduce its equation from Cartesian equation.

Find geometrically the pedal equation of the ellipse with respect to focus. [1+3+3]

2. State Rolle's theorem and give its geometrical meaning. Verify that the function $y = f(x) = x^2 - 4x + 3$ satisfies the Rolle's theorem in the interval $1 \le x \le 3$ and hence find the number c such that $f'(c) = 1$. [1+2+3+1]

3. What do you mean by the term 'integration'? Explain it. Of f (x) is continuous in the interval (a, b), b > a, show that the integral $\int_a^b f(x)\,dx$ geometrically represents the area of the space enclosed by the curve y = f (x), the ordinate x =

a, x = b and the x-axis. [2+5]

OR

State fundamental theorem of integral calculus.

Evaluate : $\int_a^b \frac{1}{x}\, dx$ as the limit of a sum. [1+6]

4. State and establish Euler's theorem for homogeneous function of degree n. Use this theorem to show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u \text{ if } u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$$ [1+2+4]

5. Define a differential equation of the second order. What do you mean by complimentary function and the particular integral ?

Solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$. [2+5]

Group "B"

10×4=40

6. Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant. [4]

7. State L Hospital's rule.

Show that $\lim_{x \to 0} (\cos x)^{\cot^2 x} = e^{-\frac{1}{2}}$ [1+3]

8. Evaluate : $\int_1^2 dy \int_3^4 \frac{dx}{(x + y)^2}$ [4]

OR

Show that $\int_0^1 dx \int_0^1 \frac{x - y}{(x + y)^3}\, dy \neq \int_0^1 dy \int_0^1 \frac{x - y}{(x + y)^3}\, dx$ [4]

9. Define Beta function. Show that for m > –1, n > –1.

$$\int_a^b (x - a)^m (b - x)^n\, dx = (b - a)^{m+n+1} \beta(m + 1, n + 1).$$ [1 + 3]

OR

Prove that $\int_0^\infty \sqrt{y}\, e^{-y^2}\, dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\, dy = \frac{\pi}{2\sqrt{2}}$ [4]

10. Find the area of a loop of the curve r = a sin 3θ. [4]

11. Show that $\int_0^x \frac{x \tan x}{\sec x + \cos x}\, dx = \frac{x^2}{4}$ [4]

12. If $y = (\sin^{-1} x)^2$, prove that [4]

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

and hence show that

$(1-x)^2 y_{n+2} - (2n+1)\,xy_{n+1} - x^2 y_n = 0$ [4]

13. What do you mean by linear differential equation of first order ? Solve: $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$ [1+3]

14. State Clairaut's equation:
Find the general solution of $y = 2px + y^2p^3$. [4]

OR

Find the general and singular solution of [1+3]
$y = px + \sqrt{a^2p^2 + b^2}$

15. Find the particular integral $\frac{1}{f(D^2)}$ sin ax,
where $f(D^2) = D^2 + a^2$ [4]

Mathematics I Paper (Math. 311), 2068
(Calculus)

Bachelor Level/I Year/ Sc. & Tech.+ Hum. **Full Marks: 75**
Time: 3 hrs.

Attempt ALL the questions

Group "A" 5×7=35

1. **What the pedal equation of a curve is? Deduce its equation from Cartesian equation.**
Find geometrically the pedal equation of the ellipse with respect to the focus. [1+3+3]

2. **State the criteria for a function of two variables to have maximum and minimum values. Find the maximum and minimum values of the function $x^3 + y^3 - 3axy$.** [2+5]

Or

State and establish Ruler's theorem for a function of two variables. Use this theorem to prove that
$$x\frac{2u}{2x} + y\frac{2u}{2y} + \frac{1}{2}\cot u = 0, \text{ where } u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$ [1+3+3]

3. **What do you mean by indeterminate form ? State various forms of indeterminacy.**
Evaluate : $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$ [1+2+4]

Or

What do you mean by the curvature and radius of curvature of a curve? Show that the circle is a curve of uniform curvature, and its radius of curvature at every point is constant. [2+1+4]

4. State and prove LaGrange's Mean value theorem and give its geometrical meaning.

Find the value of θ in the mean value theorem

$f(x + h) = f(x) + hf'(x + \theta h)$ if $f(x) = 1/x$ [1+3+3]

5. Show that the necessary and sufficient condition for the differential equation of the form $Mdx + Ndy = 0$, where M and N are functions of x and y to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Verify that the equation $(2x^3 + 4y)\,dx + (4x + y - 1)\,dy$ is exact and obtain its general solution. [3+1+3]

Group "B"

6. Define an asymptote to a curve. Find asymptote to a curve ;

$x(x-y)^2 - 3(x^2-y^2) + 8y = 0$ [1+3]

7. If $y = a^{\sin^{-1}x}$ then prove that

(a) $(1-x^2)\,y_1^2 = xy_1 + a^2y$

(b) $(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} - (n^2+a^2)\,y_n = 0$ [1+3]

OR

State Rolle's theorem. Verify the theorem for the function

$f(x) = (x-2)(x-3)(x-4)$ in [2, 4]. [1+3]

8. Show that : $\displaystyle\int_0^{\pi/2} \frac{\sqrt{\sin x}\,dx}{\sqrt{\sin x} + \sqrt{\cos x}} = \frac{\pi}{4}$ [4]

9. If f (x, y) is homogeneous function of x & y of degree n, prove that

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n\,f(x, y)$ [4]

OR

State Euler's theorem on homogeneous function of two independent variable. Verify it for $u = x^n \tan^{-1}(y/x)$. [1+3]

10. Define a Gama function

Prove that : $\Gamma(m)\,\Gamma(1-m) = \frac{\pi}{\sin m\pi}$ $(0 < m < 1)$. [1+3]

11. Find the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base. [4]

12. Prove, by evaluating the repeated integrals, that

$$\int_0^1 dx \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dy \neq \int_0^1 dy \int_0^1 \frac{x^2-y^2}{(x^2+y^2)^2}\,dx.$$ [4]

13. Find the solution of the equation $(x+1)\,p - xp^2 + 2 = 0$ [4]

OR

State Clairaut's equation and show that how it provides complete solution and singular solution.

14. Solve $(D^2+1)\,y = \sin 2x$, when $x = 0$, $y = 0$ and $y_1 = 0$. [4]

15. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ [4]

OLD COURSE

Attempt ALL the questions

Group "A" $5\times7=35$

1. What do you mean by curvature and radius of curvature of a curve ? Show that the circle is a curve of uniform curvature, and its radius of curvature a every point is constant. [2+1+4]

2. State and prove Rolle's theorem. Write its geometrical interpretation. Verify Rolle's theorem for

$$f(x) = \frac{\sin x}{e^x},\ x = \in [0, x].$$ [4+1+2]

OR

State Leibnitz's theorem.

If $y = (\sin^{-1}x)^2$, prove that [1+3+3]

$$(1-x)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2 = 0$$

and hence show that $(1-x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - x^2 y_n = 0$

3. Define Beta and Gamma functions.

Prove that $\int_0^{x/2} \sin^m x\ \cos^n x\ dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$ and hence show

that $\int_0^{\infty} e^{-x^2} = \frac{1}{2}\sqrt{x}$ [2+3+2]

4. How do you define the maximum and minimum value of a function of two variables ?

Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a^2$. [2+5]

OR

State and establish Euler's theorem for homogeneous function of degree n. Use this theorem to show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$ if $\sin u = \frac{x^2 + y^2}{x+y}$ [1+2+4]

5. What type of first order differential equation is called Clairauts equation ? Explain the method of solving such equation. Find the complete primitive and the singular solution of $y = px + p^n$. [1+2+4]

Group "B" $10\times4=40$

6. Define pedal equation.

Find the pedal equation of the curve $r^2 = a^2\cos 2\theta$. [1+3]

OR

Write the expression for polar substangent and polar subnormal at any point of a curve $r = f(\theta)$.

Show that for the curve $r = e^{\theta}$, the polar substangent is equal to the polar subnormal. [1+3]

7. State L-Hospital's rule and use it to evaluate $\lim_{x \to 0} \frac{(e^x - 1)\tan^2 x}{x^3}$ [4]

8. Define homogeneous function with examples of degree 0, $\frac{1}{2}$ and 2. check the homogeneity for the function.

   $f(x, y) = ax^2 + 2hxy + by^2$ [2+2]

9. Show that : $\int_0^x \frac{x \tan x}{\sec x + \cos x}\, dx = \frac{x^2}{4}$ [4]

10. Find the area of a loop of the curve

    $a^2y^2 = a^2x^2 - x^2$ [4]

    OR

    Find the volume of the reel formed by the revolution of the cycloid

    $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$.

11. Define centre of curvature of a curve. Find the centre of curvature at any point (x, y) on the parabola $y^2 = 4ax$. [1+3]

    Or

    Show that the radius of curvature at any point of the equiangular spiral $r = e^{\theta\cot\alpha}$ is $r \operatorname{cosec} \alpha$ and subtend a right angle at the pole. [2+2]

12. Find the particular solution of the equation

    $y(1 - x^2)\frac{dy}{dx} + (1 - y^2) = 0$ given that $y = 1$ when $x = 0$.

13. Solve : $y - xp = a(y^2 + p)$ [4]

14. Define linear differential equation with constant coefficient. Solve :

    $(D^2 + a^2)\, y = \sin ax$. [4]

    OR

    Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + x$.

15. Solve : $\frac{dy}{dx} - \frac{\tan y}{1 + x} = (1 + x)\, e^x \sec y$

Mathematics I Paper (Math. 311), 2069
(Calculus)

Bachelor Level/I Year /Sc. & Tech. + Hum. Full Marks : 100

(For: Regular Examinee only) Time :3hrs.

Attempt ALL the questions.

GROUP 'A' 5×7=35

1. State MaClaurin's series in finite form. What is the condition under which this series can be extended to infinity? Hence or otherwise show that

   $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \ldots\ldots\ldots\ldots$ [1+2+4]

2. What do you mean by the curvature of a curve? Hence define the radius of curvature.

Show that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the radius of curvature at any extremity of the major axis is equal to half the latus rectum. [1+1+5]

OR

Describe the curve tracing techniques for a given Cartesian equation. Trace the curve of the function $x^2y^2-x^2+a^2=0$ [4+3]

3. Define Beta and Gamma function. Prove that.

$$\int_0^{\frac{\pi}{2}} \sin^p x \cos^q (x)\, dx = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\frac{(q+1)}{2}}{2\Gamma\left(\frac{p+q+2}{2}\right)},\ p, q > -1$$

and hence show that $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$

4. What do you mean by the Lagrange's undetermined multipliers? Use Lagrange's multiplier to find the minimum value of $x^2+y^2+z^2$ subject to the constrain

$x+y+z=3a$.

Or

State and establish Euler's theorem for a function of two variables. Use this theorem to show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{20}u,\ \text{where } u=\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{5}}+y^{\frac{1}{5}}}.$$ [1+3+3]

5. State Clairaut's equation and show that how Claraut's equation provided complete solution and singular solution. Find the complete and singular solutions of the differential equation $y=px+p-p^2$. [1+3+3]

GROUP 'B' 10×4=40

6. What do you mean by the pedal equation of a curve? Find the pedal equation of the curve $r=a(1+\cos\theta)$. [1+3]

Or

Define the polar subtangent and polar subnormal at any point of a curve $r=f(\theta)$.

Show that for the curve $r=e^{\theta}$, the polar subtangent is equal to the polar subnormal. [2+2]

7. If $y=a\cos(\log x)+b\sin(\log x)$, then show that $x^2y_{n+2}+(2n+1)\,x\,y_{n+1}+(n^2+1)\,y_n=0$. [4]

8. State L' Hospital's rule. Evaluate $\lim_{x\to\frac{\pi}{4}} (\tan x)^{\tan 2x}$ [1+3]

9. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ and use it, prove

$$\int_0^{\pi} \frac{x \sin x \, dx}{1+\cos^2 x} = \frac{\pi^2}{4}.$$

10. Obtain a reduction formula for $\int \sin^n x \, dx$ and find $\int \sin^4 x \, dx$. [4]

Or

Define Gamma function.

Use it, prove that $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}} = \frac{\pi}{3}$. [1 + 3]

11. Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. [4]

Or

Prove that the volume of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis is $\frac{4\pi ab^2}{3}$. [4]

12. Evaluate $\int_0^{\frac{\pi}{2}} dy \int_0^{\frac{\pi}{2}} \cos(x+y)\, dx$. [4]

13. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$. [4]

14. Solve $(D^2 - 3D + 2)\, y = e^x$ where $D = \frac{d}{dx}$ [4]

Or

Solve : $(D^2 + 4)\, y = \sin 2x$. [4]

15. Solve : $(D^2 + 1)\, y = \cos 2x$, when $x = 0$, $y = 0$ and $y_1 = 0$. [4]

Mathematics I Paper (Math. 311), 2070
(Calculus)

Bachelor Level/I Year/ Sc. & Tech.+ Hum. Full Marks: 75

Time: 3 hrs

Attempt ALL the questions.

Group "A" 5×7=35

1. Find angle of intersection of two curves whose polar equations are $r = f(\theta)$ and $r = d(\theta)$. What happens if the two curves touch? Find the angle between the curves $r = 2\sin\theta$ and $r = 2\cos\theta$. [2+1+4]

2. State Cauchy's mean value theorem and state when it reduces to Lagrange's mean value theorem, verify Lagrange's mean value theorem for the function $f(x) = x(x-1)(x-2)$ in the interval [0, 1/2].

   Also, show that the function f(x) is concave downward at $x = \frac{1}{5}$. [1+½+4+1½]

3. What do you mean by the asymptotes to a curve? Obtain the condition that a line $y = mx + c (m \neq 0)$ is a asymptote to the curve $f(x, y) = 0$.

   Find the asymptote of $x^3 - y^3 = 3ax^2$. [1+3+3]

OR

What do you mean by the curvature of a curve at a point? Show that the circle is a curve of uniform curvature. Find the radius of curvature of the curve $r^m = a^m \cos m\theta$. [1+3+3]

4. Show that the area of the region bounded by a curve $y = f(x)$, the axis of x, and two ordinates $x = a$ and $x = b$ is $\int_a^b f(x)\,dx$ and hence find the area bounded by the parabola $y^2 = 4ax$ and its latus rectum. [3+4]

5. Show that the necessary and sufficient condition for the differential equation of the form $Mdx + Ndy = 0$, where M and N are functions of x and y to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Verify that the equation $(x^2 + xy^2)\,dx + (x^2y + y^2)\,dy = 0$ is exact and obtain its general solution. [3+1+3]

OR

State the homogenous equation of the first order and first degree. Is the equation $\frac{dy}{dx} = \frac{y - x + 1}{y + x + 5}$ homogeneous ? If not make it homogeneous and solve it. [1/2+1/2+2+4]

Group "B" 10×4=40

6. If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2y_{n+2} + (2n+1)\,xy_{n+1} + (n^2+1)\,y_n = 0$. 4

7. Find the extreme value of the function $x^2 + y^2$ under the condition $x + 4y = 2$. [4]

OR

If $u = \log\frac{x^4 + y^4}{x + y}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{34}{e^4}$. [4]

8. Show that $\int_0^{\pi/2} \log\sin x\,dx = \int_0^{\pi/2} \log\cos x\,dx = \frac{\pi}{2}\log\frac{1}{2}$. [4]

9. Prove that $\int_0^{\infty} e^{-ax}\cos bx\,dx = \frac{a}{a^2 + b^2}$, $a > 0$. [4]

OR

Obtain a reduction formula for $\int \sec^n x\,dx$ and hence find $\int \sec^6 x\,dx$.

10. Find the area bounded by the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ and its base. [4]

OR

Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.

11. Evaluate $\int_0^{4a} dx \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy$ [4]

12. Solve $(D^2 + 2D + 1)y = e^x + e^{-x}$, where $D = \frac{d}{dx}$ [4]

13. Find the complete primitive and singular solution of $y = px + ap(1 - p)$ where $p = \frac{dy}{dx}$ [4]

14. Find the equation of the curve for which the sum of the reciprocals of the radius vector and the polar subtangent is constant. [4]

OR

What do you mean by an initial condition for a differential equation?

Solve: $y(1 - x^2)\frac{dy}{dx} + x(1 - y^2) = 0$, given that $y = 1$ when $x = 0$. [1+3]

15. State L' Hospital's rule and use it to evaluate $\lim_{x \to 0} (\cos x)^{\cot^2 x}$ [1+3]

OLD COURSE

Attempt ALL the questions.

Group "A" 5×7=35

1. What is the pedal equation of a curve? Deduce is equation from Cartesian equation.
   Find geometrically the pedal equation of the ellipse with respect to focus. [1+3+3]

2. State Rolle's Theorem and interpret it geometrically. If $f'(x) = 0$ for all values of x in an interval, then show that f(x) is constant in the interval. [1+2+4]

OR

State Cauchy mean value theorem and state when it reduces to Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x) = x(x - 1)(x - 2)$ in the interval [0, ½].

Also, show that the function f(x) is concave downward at $x = \frac{1}{5}$ [1+ ½+4+1 ½]

3. What do you mean by the indeterminate form? State various forms of indeterminancy. When do you apply L'Hospital's rule? Evaluate $\lim_{x \to 0} (\sec x)^{x^{1/2}}$ [1+2+1+3]

4. Define Beta and Gamma functions. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ and use Gamma function to prove that $\int_0^1 \frac{dx}{(1 - x^6)^{1/6}} = \frac{\pi}{3}$ [1+3+3]

5. Define a differential equation of the second order. What do you men by complementary function and the particular integral?

   Solve : $\frac{d^2y}{dx^2} + a^2y = \sec ax$. [1+2+4]

OR

Define auxiliary equation of the differential equation of the second order. If the auxiliary equation has two equal roots say α, may its general solution be $y = c e^{ax}$? If not why? And deduce the correct general solution.

Solve: $(D^2 + 2D + 5)y = 0$, where $D = \frac{d}{dx}$ [1+1+3+2]

Group "B" 10×4=40

6. What are polar subtangent and polar subnormal? Show that for the curve $r\theta = a$, the polar subtangent is constant and for the curve $r = a\theta$, the polar subnormal is constant. [1+3]

7. Define centre of a curvature of a curve. Find the centre of curvature at any point (x, y) on the parabola $y^2 = 4ax$. [1+3]

OR

Show that the chord of curvature parall l to y - axis for the curve $y = c \cosh \frac{x}{c}$ is double of the ordinate. [4]

8. Trace the curve $y = x^3 - 12x - 6$. [4]

9. Show that $\int_0^{\pi} \frac{x \sin x \, dx}{1 + \cos^2 x} = \frac{\pi^2}{4}$ [4]

10. Find the perimeter of the cardioid $r = a(1 + \cos\theta)$ and show that the arc of the upper half is bisected by $\theta = \frac{\pi}{3}$. [4]

OR

Find the area of a loop of the curve $a^2y^2 = a^2x^2 - x^4$.

11. Examine where the equation $x\, dx + y\, dy + (x^2 + y^2)\, dy = 0$ is exact or not and hence solve it. [1+3]

12. State Euler's theorem on homogeneous function of two independent variables. Verify it for $u = (x^2 + y^2)^{1/3}$. [1+3]

OR

Find $\frac{dy}{dx}$ of $(\tan x)^y + (y)^{\tan x} = 0$. [4]

13. Find the general and singular solution of $y = px + p(1 - p)$. [4]

14. Define linear differential equation of the first order.

Solve: $\frac{dy}{dx} + y \tan x = \sec x$. [1+3]

15. Solve: $(D^2 + D - 2)y = e^{2x}$, given that when $x = 0$, $y = 0$ and $y_1 = 0$. [4]

OR

Solve: $(D^2 + 16)y = \cos 4x$.

Mathematics I Paper (Math. 101), 2070
(Calculus), (New course)

Four Year Bachelor Level/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

Group "A" 5×7=35

1. Define and deduce the expressions for the polar subtangent and polar subnormal at any point $p(r, \theta)$ of a curve $r = f(\theta)$.

Show that the portion of the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ Intercepted between the axes is of constant length. [1+2+4]

2. State the various mean value theorems. In what respect the Rolle's theorem and Lagrange's mean value theorem are differ by geometrically? Give the different generalisation of Lagrange's mean value theorems. [3+2+2]

OR

State Leibnitz theorem.

If $y = \sin^{-1}x$, show that

(1) $(1 - x_2)\, y_2' - xy_1 = 0$

(2) $(1 - x^2)\, y_{n+2} - (2n + xy_{n+1} - n^2 y_n = 0$

(3) $(y_{n+2})_0 = n^2 (y_n)_0$. Find also the value of $(y_n)_0$. [7]

3. Define Beta and Gamma functions.

Is the property of Beta function commutative"? How?

Prove that : $\sqrt{\pi}\,\Gamma(2n) = 2^{2n-1}\,\Gamma(n)\,\Gamma\left(n + \frac{1}{2}\right)$ [2+1+4]

4. Define the term quadrature. Write the expression for the - areas in Cartesian coordinates. What happen if area is divided into one or more points?

Find the area of the loop of the folium $x^3 + y^3 - 3axy = 0$. [3+4]

OR

Define solid-of revolution.

Find the volume of the solid generated by the revolution of the cardiod $r = a(1 - \cos\theta)$ about the initial line. What will be the volume of the solid formed by the curve $r = f(\theta)$ and the line $\theta = \theta_1$ and $\theta = \theta_2$ is rotated about the line $\theta = 0$? [1+5+1]

5. What do you mean by exact differential equation? Is the equation $xdy - ydx = 0$ is exact? If not how can it be made exact?

Verity the equation $(2xy + y^2)\,dx + (x^2 + 2xy - y)\,dy = 0$ is exact and hence find its general solution. [3+4]

Group "B"

10×4=40

6. What do you mean by pedal equation of a curve? Find the pedal equation of the curve $r^2 = a^2\cos 2\theta$. [1+3]

OR

Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon coordinate axes is constant. [4]

7. What do you mean by n^{th} derivative of a function? Obtain the n^{th} to derivative of $e^{ax}\cos bx$. [1+3]

8. How can you define indeterminate forms?

Write its various forms. Find $\lim_{x\to 0} \sin x . \log x^2$. [4]

9. Show that: $\displaystyle\int_0^{\pi} \frac{x}{(a^2\sin^2x + b^2\cos^2x)^2}\,dx = \frac{\pi^2(a^2 + b^2)}{4a^3b^3}$ [4]

10. Trace the curve $r = a(1 - \cos\theta)$. [4]

OR

Show that the chord of curvature through the pole of the curve $r^m = a^m \cos m\theta$ is $\frac{2r}{m+1}$.

11. State and establish Euler's theorem for homogeneous function of degree n. Show that $f(x, y) = ax^2 + 2hxy + by^2$ is homogeneous of degree 2. [4]

12. Evaluate : $\int_0^{\pi/2} \int_{\pi/2}^{\pi} e^x \cos(y - x)\, dy\, dx$. [4]

OR

Evaluate : $\int_0^{\pi/2} \int_0^{\pi} \sin(x + y)\, dx\, dy$.

13. What do you mean by linear differential equation of first order ?

Solve : $(x + y + 1)\frac{dy}{dx} = 1$. [1 + 3]

14. State Clairaut's equation.

Find the general solution of $(px - y)(py + x) = h^2p$. [4]

OR

Solve : $xy^2(p^2 + 2) = 2py^3 + x^3$

15. Show that $\frac{1}{f(D)} e^{ax} = \frac{1}{\phi(a)} xe^{ax}$, when $f(a) = 0$ [4]

Mathematics I Paper (Math. 101), 2071
(Calculus)

Bachelor Level (4 Yrs.)/ I Year/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

Group "A" 5×7=35

1. Obtain the derivatives of arc-length in Cartesian form $y = f(x)$. Also define pedal equation and obtain the expression for $x^{2/3} + y^{2/3} = a^{2/3}$. [3+1+3]

2. Define the term curvature and Radius of curvature. What are the methods to obtain the Radius of curvature at the origin? Find the radius of curvature at the point (p, r) of the curve $r^{m+1} = a^m p$. [3+4]

OR

Obtain the radius of curvature for the pedal curve $p = f(r)$.

Show that the chord of curvature parallel to y - axis for the curve $y = a \log\left(\sec\left(\frac{x}{a}\right)\right)$ is constant. [4+3]

3. Define the term solid of revolution and surface of solid of revolution. Show that the volume of the solid formed by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the line $x = 2a$ is $4\pi^2a^2b$ [2+5]

4. Define improper integral with examples.

Show that $\int_0^\infty e^{-x} x^n \, dx = n!$ [2+5]

OR

State and prove Walli's formula. [7]

5. Define linear equations with constant coefficients.
Obtain the general solution of $(D^2 + P_1 D + P) y = 0.$ [1+6]

Group "B"

10×4=40

6. State L-Hospital rule.

Show that $\lim_{x \to 0} \frac{x \cos x - \log(1+x)}{x^2} = \frac{1}{2}.$ [1 + 3]

7. If $y^{1/m} + y^{-1/m} = 2x$, show that
$(x^2 - 1) y_{n+2} + (2n+1) x y_{n+1} + (n^2 - m^2) y_n = 0$ [4]

8. Evaluate: $\int_0^{\pi/2} dy \int_0^{\pi/2} \cos(x+y) \, dx$

OR [4]

Evaluate : $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$

9. Find the extreme value for the function $x^2 + y^2$ under the condition $x + 4y = 2$. [4]

10. Prove: $\int_0^{2a} f(x) \, dx = 2 \int_0^{a} f(x) \, dx$, when $f(2a - x) = f(x)$

$= 0$, when $f(2a - x) = -f(x)$

and hence obtain the value of $\int_0^\pi \sin^n x \, dx$ and $\int_0^\pi \cos^4 x \, dx.$ [3+1]

OR

Show that $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx = \frac{\pi}{2} (\pi - 2).$ [4]

11. Find the area of a loop of the curve $r = a \sin 3\theta$. [4]

12. Define the Cartesian subtangent and subnormal at any point of a curve. Show that for the curve $by^2 = (x + a)^3$, the square of the subtangent varies as the subnormal. [1+3]

OR

Find the angle between the curves $r = a\sin 2\theta$ and $r = a \cos 2\theta$. [4]

13. Verify that $y = e^{m\sin^{-1}x}$ is a solution of

$(1 - x^2)\frac{d^2}{d}$ [illegible] $- m^2 y = 0.$ [4]

14. Find the general solution of

$$x^2(y - px) = yp^2.$$ [4]

15. Solve: $(D^2 + 2D + 1)\,y = x\cos x$.

OR [4]

Show that $\frac{1}{f(D)}\,e^{ax} = \frac{1}{f(a)}\,e^{ax}$ provided $a \neq 0,\ f(a) \neq 0$.

Mathematics I Paper (Math. 101), 2072

(Calculus)

Bachelor Level (4yrs. prog.) I Year/Science & Tech. Full Marks: 75

Time: 3 hrs.

Attempt ALL the questions.

Group "A" $5\times7 = 35$

1. Define the length of perpendicular from the pole on the tangent to a curve. Also define pedal equation and obtain its expression for the curve $r^2 = a^2\cos 2\theta$. [3+1+3]

OR

Define polar subtangent and polar subnormal and deduce expressions at any point $P(r, \theta)$ of a curve $r = f(\theta)$.

Find the angle of intersection of the curves $r^2 = a^2\cos 2\theta$ and $r^2 = b^2\sin 2\theta$. [1+2+4]

2. State Rolle's theorem and give its geometrical meaning. Verify that the function $y = f(x) = x^2 - 4x + 3$ satisfy the Rolle's Theorem in the interval $1 \le x \le 3$ and hence find the number c such that $f'(c) = -1$. [1+2+3+1]

OR

State Leibnitz's theorem. If $y = (\sin^{-1}x)^2$

Prove that (i) $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

(ii) $(1 - x^2)\,y_{n+2} - (2n + 1)\,xy_{n+1} - n^2y_n = 0$. [1+3+3]

3. State and establish Euler's theorem for homogeneous function of degree n. Use this theorem to show that.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u \text{ if } u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$$

4. Define second Eulerian integral or Gamma function and prove that

(i) $\Gamma 1 = 1,\ \Gamma(n + 1) = n\,\Gamma n$

If $I_n = \int \sin^n x\,dx$ then prove that

$$I_n = \frac{-\sin^{n-1}x\cos x}{n} + \left(\frac{n-1}{n}\right) I_{n-2}.$$ [1+2+4]

5. Explain the method of solving linear differential equation

$$\frac{dy}{dx} + py = Q$$

Where P and Q are function of x or constants.

Hence solve $\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x)\, e^x. \sec y.$ [4+3]

Group "B" 10×4 = 40

6. Prove that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant.

OR [4]

Show that for the curve $by^2 = (x + a)^3$, the square of subtangent varies as the subnormal.

7. Evaluate: $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right].$

8. Evaluate: $\int_1^2 \int_0^x \frac{1}{(x^2 + y^2)}\, dx\, dy.$

9. Evaluate by using Gamma function

$\int_0^a x^3 (a^2 - x^2)^{\frac{5}{2}}\, dx.$

10. The area bounded by the parabola $y^2 = 4px$ and the line $x = a$ revolves about the x –axis. Find the volume generated. [4]

11. Show that $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x}\, dx = \frac{\pi^2}{4}.$ [4]

12. Solve: $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x.$ [4]

13. Solve the differential equation $\frac{d^2y}{dx^2} + y = \cos x.$ [4]

OR

Solve: $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - 4y = x^2.$ [4]

14. Find the general and singular solution of [4]

$y = px + \sqrt{a^2p^2 + b^2}$

OR

Find the value of y which satisfies the equation

$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, given that $x = 0,\ y = 3,\ \frac{dy}{dx} = 0.$

15. Find the asymptote of the Folium of Descartes [4]

$x^3 + y^3 = 3axy.$

OR

Show that for the curve $y = a \log \sec \left(\frac{x}{a}\right)$, the chord of curvature parallel to y –axis is of constant length. [4]