30. Show that the points A, B and C with position vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-7\vec{j} + 10\vec{k}$ respectively are collinear. [Q.N. 10(a), 2058]

31. Show that the vectors $2\vec{i} + 3\vec{j} - 8\vec{k}$ and $2\vec{i} + 4\vec{j} + 2\vec{k}$ are orthogonal. [Q.N. 3(b), 2057]

32. If the position vector of M and N are $3\vec{i} + \vec{j} - 3\vec{k}$ and $4\vec{i} - 2\vec{j} + \vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines. [Q.N. 10(a), 2057]

$\left(\text{Ans: } \vec{MN} = \vec{i} - 3\vec{j} + 4\vec{k}, \left(\frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)\right)$

33. A B C D E F is a regular hexagon. Express $\vec{AC}$ and $\vec{AD}$ in terms of $\vec{AB}$ and $\vec{BC}$.

$\left(\text{Ans: } \vec{AC} = \vec{AB} + \vec{BC}, \vec{AD} = 2\vec{BC}\right)$ [Q.N. 4(a), 2057]

6.2 Product of Vectors

1. Find the angle between the vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$. **(Ans: 90°)** 2[Q.N.3(c), 2072'C']
2. Define Vector product of two Vectors. Prove by Vector method: $\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6[Q.N.10, 2072'C']
3. If $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$, prove that $|\vec{a}| = |\vec{b}|$. 2[Q.N.3(c), 2072'D']
4. Define Vector product of two Vectors. Prove by Vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 6[Q.N.10, 2072'D']
5. If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and its direction cosines. 2[Q.N.2(c), 2072'E']

$\left(\text{Ans: } 4\vec{i} - 5\vec{j} + 11\vec{k}, \left(\frac{4}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}\right)\right)$

6. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$. **(Ans: 20.5 sq. unit)** 2[Q.N.3(c), 2072'E']
7. Define scalar product of two vectors. Give the geometrical interpretation of the scalar product of two vectors. In any triangle prove vectorically that $a^2 = b^2 + c^2 - 2bc\cos A$. [Q.N.10, 2072'E']
8. For what value of m is the pair of vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} - 7\vec{j} + m\vec{k}$ orthogonal ? **(Ans: m = 3)** 2 [Q.N. 3(c), Set 'C' 2071]
9. Define vector product of two vectors. Prove by vector method that $\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6 [Q.N. 10, Set 'C' 2071]
10. Find a unit vector perpendicular to each of the vectors $3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$. 2 [Q.N. 3(c), Set 'D' 2071]

$\left(\text{Ans: } \frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)$

11. Define scalar product of two vectors. Prove by vector method that: $\cos(A + B) = \cos A\cos B - \sin A\sin B$. 6 [Q.N. 10, Set 'D' 2071]

12. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, prove that $\vec{a}$ is perpendicular to $\vec{b}$ 2 [Q.N. 3(c), 2070 'C']

13. Define vector product of two vectors. Using vector method, prove that:
$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ 6[Q.N. 10, 2070 'C']

14. Find the since of the angle between the two vectors
$2\vec{i}-\vec{j}+\vec{k}$ and $3\vec{i}+4\vec{j}-\vec{k}$, 2 [Q.N. 3(c), 2070 'D']
$\left(\text{Ans: }\sqrt{\frac{155}{156}}\right)$

15. Define scalar product of two vectors. 6[Q.N. 10, 2070 'D']
Prove by vector method that : cos (A – B) = cosA cosB + sinA sinB

16. If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=0$, prove that $|\vec{a}|=|\vec{b}|$. 2 [Q.N. 3(c), Supp. 2069]

17. Define vector product of two vectors and geometrically interpret it. Also show that $\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$ where $\vec{a}$ and $\vec{b}$ are any two non zero vectors.
6 [Q.N. 10, Supp. 2069]

18. Find the area of the parallelogram determined by the vectors $\vec{i}+2\vec{j}+3\vec{k}$ and $-3\vec{i}-2\vec{j}+\vec{k}$ [Q.N. 3(c), Set 'A' 2069]
(Ans: $6\sqrt{5}$ sq uints)

19. Define scalar product of two vectors. Prove by the method of vectors that:
cos (A–B) = cosA cosB + sinA sinB. 6 [Q.N. 10, Set 'A' 2069]

20. If $\vec{a}=\vec{i}+\vec{j}-2\vec{k}$ and $\vec{b}=2\vec{i}-\vec{j}-\vec{k}$ are any two vectors, find the cosine of the angle between the two vectors. [Q.N. 3(c), Set 'B' 2069]
$\left(\text{Ans: }\frac{1}{2}\right)$

21. Define vector product of two vectors. Interpret the vector product of two vectors geometrically. Prove by vector method that:
$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ [Q.N. 10, Set 'B' 2069]

22. Find the area of the triangle determined by the vectors
$3\vec{i}+4\vec{j}$ and $-5\vec{i}+7\vec{j}$ [Q.N. 3(b), 2068]
[Ans: 20.5 sq. units]

23. Using vector method prove that: $c^2=a^2+b^2-2ab\cos C$. [Q.N. 11(b), 2068]

24. Given $\vec{a}=(3,1,2)$ and $\vec{b}=(2,-2,4)$, find the projection of $\vec{a}$ on $\vec{b}$.
(Ans: $\sqrt{6}$) [Q.N. 3(b), 2067]

25. Prove by vector method : Cos (A+B) = Cos A Cos B – Sin A Sin B. [Q.N. 11(a), 2067]

26. For what value of m are the vectors $\vec{i}-2\vec{j}+4\vec{k}$ and $2\vec{i}+7\vec{j}+m\vec{k}$ orthogonal?
(Ans: 3) [Q.N. 3(b), 2066]

27. Use vector method to prove that, in any triangle ABC, a = b cos C + c cos B.
[Q.N. 11(a), 2066]

28. Find the value of r if the vectors $3\vec{i}-\vec{j}-2\vec{k}$ and $2\vec{i}-2\vec{j}+r\vec{k}$ are orthogonal.
(Ans: 4) [Q.N.4(a), 2065]

29. By using vectors, prove that in any ΔABC,
$\frac{\text{Sin A}}{a}=\frac{\text{Sin B}}{b}=\frac{\text{Sin C}}{c}$ [Q.N.11(a), 2065]

30. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$.
(Ans: 41 sq. units) [Q.N. 3(b), 2064]

31. Using vector method, prove in any triangle, that :
$b^2 = c^2 + a^2 - 2ca\ CosB$ [Q.N. 11(a), 2064]

32. If $\vec{i}$, $\vec{j}$, $\vec{k}$ are three mutually perpendicular unit vectors and $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - 3\vec{j} - \vec{k}$, find the cosine of the angle between the two vectors. [Q.N. 4(a), 2063]
$\left(Ans.: \cos^{-1}\sqrt{\frac{7}{12}}\right)$

33. Using vector method, prove in any triangle that :
$a = b\ CosC + c\ CosB$ [Q.N. 11(a), 2063]

34. Find the area of the parallelogram determined by the vectors
$\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$ [Q.N. 3(b), 2062]
(Ans.: $6\sqrt{5}$ sq. unit)

35. Prove vectorically that : $Cos\ (A-B) = CosA\ Cos\ B + SinA\ SinB$ [Q.N. 11(a), 2062]

36. Find a unit vector perpendicular to $2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{i} + \vec{j} - 2\vec{k}$.
$\left(Ans: \frac{-5}{\sqrt{35}}\vec{i} + \frac{3}{\sqrt{35}}\vec{j} - \frac{1}{\sqrt{35}}\vec{k}\right)$ [Q.N. 3(b), 2061]

37. If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them. Show that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$ [Q.N. 4(a), 2061]

38. Prove, in any triangle, by vector method that :
$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$ [Q.N. 11(a), 2061]

39. Find the cosine of the angle between the vectors
$2\vec{i} + \vec{j} + \vec{k}$ and $4\vec{i} + 3\vec{j} + 5\vec{k}$ [Q.N. 3(b), 2060]
$\left(Ans: \frac{8}{5\sqrt{3}}\right)$

40. Prove by vector method: $\sin(A - B) = \sin A \cos B - \cos A \sin B$. [Q.N. 11(a), 2060]

41. Find the angle between two vectors $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$.
(Ans: 60°) [Q.N. 4(a), 2059]

42. Prove by vector method: $\sin(A+B) = \sin A \cos B + \cos A \sin B$. [Q.N. 11(a), 2059]

43. Show that the area of the parallelogram determined by:
$\vec{i} + \vec{j} - 3\vec{k}$ and $-\vec{i} - 2\vec{j} - 3\vec{k}$ is $\sqrt{118}$ sq. units [Q.N. 3(b), 2058]

44. Prove by vector method.
$\cos(A-B) = \cos A \cos B + \sin A \sin B$. [Q.N. 11(a), 2058]

45. Prove by vector method
$Cos\ (A - B) = CosA\ CosB + SinA\ SinB$ [Q.N. 11(a), 2057]

Unit 7: Derivative and its Application

7.1 Continuity and differentiability

1. Show that $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$ [Q.N. 2(b), 2057]

7.2 Differential Coefficients by definition (by first principle)

1. Find, from first principles, the derivative of xlnx. [Q.N.11(Or), 2072'C']
(Ans: 1 + ℓnx)
2. Find from first principles the derivative of ln $\cos^{-1}x$. [Q.N.11(Or), 2072'D']
$\left(\text{Ans: } \frac{-1}{\sqrt{1-x^2\cos^{-1}x}}\right)$
3. Solve : $x\frac{dy}{dx} + y - 1 = 0$. 2[Q.N.4(a), 2072'E']
(Ans: x(y – 1) = e)
4. Find from first principles the derivative of $\sin x^2$. [Q.N.11(Or), 2072'E']
(Ans: $2x\cos x^2$)
5. Find from first principle, the derivative of $\tan^{-1}x$. 6 [Q.N. 11(OR), Set 'C' 2071]
$\left(\text{Ans: } \frac{dy}{dx} = \frac{1}{1+x^2}\right)$
6. Find from first principle, the derivative of: $\ln(\sin\frac{x}{a})$.
$\left(\text{Ans: } \frac{d}{dx}\left(\log\sin\frac{x}{a}\right) = \frac{1}{a}\cot\frac{x}{a}\right)$ 6 [Q.N. 11(OR), Set 'D' 2071]
7. Find from first principles, the derivative of log (tanx). [Q.N. 11(OR), 2070 'C']
$\left(\text{Ans: } \frac{1}{\sin x.\cos x}\right)$
8. Find from first principles, that derivative of x^x. [Q.N. 11(OR), 2070 'D']
(Ans: x^x (1 + logx))
9. Find from first principles the derivative of $\log\cos^{-1}x$. [Q.N. 11(OR), Supp. 2069]
$\left(\text{Ans: } \frac{-1}{\cos^{-1}x\sqrt{1-x}}\right)$
10. Find from first principle, the derivative of sin(logx) [Q.N. 11(OR), Set 'A' 2069]
$\left(\text{Ans: } \frac{1}{x}\cos(\log x)\right)$
11. Find from first principle, the derivative of $e^{\sin x}$. [Q.N. 11(OR), Set 'B' 2069]
(Ans: $\cos x.\ e^{\sin x}$)
12. Find from first principle, the derivative of : $e^{\sin x}$ [Q.N. 10(b), 2068]
(Ans: $\cos x\ e^{\sin x}$)
13. Find from first principles, the derivative of : $\sin^{-1}x$. [Q.N. 10(b), 2067]
$\left(\text{Ans: } \frac{1}{\sqrt{1-x^2}}\right)$
14. Find the derivative of Sin (log x) from first principles. [Q.N. 10(b), 2066]
$\left(\text{Ans: } \frac{1}{x}\cos(\log x)\right)$
15. Find from first principles, the derivative of a^x. [Q.N.10(b), 2065]
(Ans: $a^x\log a$)
16. Find from first principles, the derivative of log sinx. [Q.N. 10(b), 2064]
(Ans: cotx)

17. Find from first principles, the derivative of *log tanx* [Q.N. 10(b), 2063]

$\left(Ans.: \frac{1}{\sin x \cdot \cos x}\right)$

18. Find from first principles, the derivative of $\tan^{-1}x$. [Q.N. 10(b), 2062]

$\left(Ans.: \frac{1}{1+x^2}\right)$

19. Find from first principles, the derivative of $e^{\sqrt{x}}$. [Q.N. 10(b), 2061]

$\left(Ans: \frac{1}{2\sqrt{x}} e^{\sqrt{x}}\right)$

20. Find from first principles, the derivative of $e^{\sin x}$ [Q.N. 10(b), 2060]

$\left(Ans: \cos x.e^{\sin x}\right)$

21. Find from first principles, the derivative of $e^{\tan x}$. [Q.N. 10(b), 2058]

$\left(Ans: \sec^2 x\, e^{\tan x}\right)$

7.3 Derivative of hyperbolic function

1. Find the derivative of: $\left(\text{Cosh}\frac{x}{a}\right)^{\log x}$ [Q.N. 2(b), 2068]

$\left[Ans: \left(\cosh\frac{x}{a}\right)^{\log x}\left(\frac{1}{a^2}\tanh\frac{x}{a}\log x + \frac{1}{x}\log\cosh\frac{x}{a}\right)\right]$

2. Find the derivative of $x^{\cosh\frac{x}{a}}$. [Q.N. 2(b), 2067]

$\left[Ans: x^{\cos h\frac{x}{a}}\left[\frac{\cos h\frac{x}{a}}{x} + \frac{\log x \sin h\frac{x}{a}}{a}\right]\right]$

3. Find the derivative of $2\tanh^{-1}\left(\tan\frac{1}{2}x\right)$ [Q.N. 2(b), 2066]

(Ans: secx)

4. Find the derivative of Arc tan Sin hx. [Q.N.2(b), 2065]

$\left(Ans: \frac{\cosh x}{1-\sinh^2 x}\right)$

5. Find the derivative of : $\left(\sin h\frac{x}{a}\right)^{x^2}$ [Q.N. 2(b), 2064]

$\left(Ans: \left(\sin\frac{x}{a}\right)^{x^2}\left[\frac{x^2}{a}\cot\frac{x}{a} + 2x\log\sinh\frac{x}{a}\right]\right)$

6. Find the derivative of $x^{Cosh^2\frac{x}{a}}$. [Q.N. 2(b), 2063]

$\left(Ans.: x^{\cosh^2\frac{x}{a}}\left(\frac{1}{x}.\cosh^2\frac{x}{a} + \frac{1}{a}.\log x.\sin\frac{2x}{a}\right)\right)$

7. Find the derivative of $\log(\sinh^x/_a)$ [Q.N. 2(b), 2062]

$\left(Ans.: \frac{1}{a}\coth\frac{x}{a}\right)$

8. Find the derivative of : $x^{\cos h\, x/a}$ [Q.N. 2(b), 2061]

$\left(\text{Ans: } x^{\cosh\frac{x}{a}}\left[\frac{\cosh x/a}{x}+\frac{\log x \sinh x/a}{a}\right]\right)$

9. Find the derivative of $2\tan^{-1}\left(\tanh\frac{x}{2}\right)$ [Q.N. 2(b), 2060]

(Ans: Sechx)

10. Find the derivative of $e^{\cosh^{-1}x}$. [Q.N. 2(a), 2059]

$\left(\textit{Ans: }\frac{1}{\sqrt{x^2-1}}e^{\cosh^{-1}x}\right)$

11. Find, from definition, the derivative of $e^{\tan x}$. [Q.N. 10(b), 2059]

$\left(\textit{Ans: } sec^2x e^{\tan x}\right)$

12. Find the derivative of x^{Coshx} [Q.N. 2(b), 2058]

$\left(\textit{Ans: } x^{\cosh x}\left(\frac{\cosh x}{x}+\log x \sinh x\right)\right)$

13. Find the derivative of $\left(\text{Sinh}\frac{x}{a}+\text{Cosh}\frac{x}{a}\right)^{nx}$ [Q.N. 10(b), 2057]

$\left(\text{Ans: } n\left(\sinh\frac{x}{a}+\cosh\frac{x}{a}\right)^{nx}\left[\frac{x}{a}+\log\left(\sinh\frac{x}{a}+\cosh\frac{x}{a}\right)\right]\right)$

7.4 Tangent and Normal

1. Find the points on the circle $x^2 + y^2 = 16$ at which the tangents are parallel to X-axis. 2[Q.N.3(a), 2072'E']
(Ans: (0, 4), (0, – 4))
2. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent is parallel to the x-axis. 2
(Ans: (0, 1), (2, – 3)) [Q.N. 3(a), Set 'C' 2071]
3. Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4)
(Ans: 4x – y – 4 = 0) 2 [Q.N. 3(a), 2070 'D']
4. At what angle does the curve $y(1+x) = x$ cut the x-axis ? [Q.N. 5(c), 2068]

$\left(\textit{Ans: }\frac{\pi}{4}\right)$

5. Find the angle of intersection between the curves $y = x^2$ and $6y = 7 - x^3$ at (1, 1). [Q.N. 2(c), 2067]

$\left(\textit{Ans: } \tan^{-1}\left(\pm\frac{3}{2}\right)\right)$

6. Find the angle of intersection of the curves $y^2 = x^3$ and $y = 2x$ at the point (0, 0).

$\left(\textit{Ans: } \tan^{-1}(\pm 2) \text{ at } (0, 0)\ \tan^{-1}\left(\pm\frac{1}{7}\right) \text{ at } (4, 8)\right)$ [Q.N. 2(c), 2066]

7. Find where the tangent is parallel to the x-axis for the curve $y = x^3 - 3x^2 - 9x + 15$. [Q.N.2(c), 2065]
(Ans: (–1, 20) and (3, –12))
8. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to x-axis.
(Ans: (0, 1), (2, –3)) [Q.N. 2(c), 2064]
9. Find the angle of intersection of the curves [Q.N. 2(c), 2062]
$4y = x^2 + 12$ and $y^2 = 8x$ at (2,4)
(Ans.: 0°)
10. Find the slope and inclination with the x-axis of the tangent of $y = -3x - x^4$ at $x = -1$.

$\left(\textit{Ans: 1 and } \frac{\pi}{4}\right)$ [Q.N. 2(c), 2061]

11. Find the points on the curve $4y = x^4 - 8x^2$ where the tangents are paralleled to the x-axis. [Q.N. 5(c), 2060]
(Ans: (0, 0), (–2, –4) and (2, –4)

12. Find the slope and inclination with x -axis of the tangent of: $x^2 + y^2 = 36$ at (0,6). [Q.N. 5(c), 2058]
(Ans: 0 and 0°)

13. Find the slope and inclination with X -axis of the tangent of the curve $2y = 2 - x^2$ at $x = 1$. [Q.N. 5(c), 2057]
$\left(\text{Ans: } -1, \frac{3\pi}{4}\right)$

7.5 L Hospital's rule, Roll's Theorem & Mean Value Theorem

1. Evaluate, using L' Hospitals rule: $\lim_{x \to 0} \frac{\tan ax}{\tan bx}$. 2[Q.N.3(a), 2072'C']
$\left(\text{Ans: } \frac{a}{b}\right)$

2. State Rolle's theorem. Verify Rolle's theorem for the functions $f(x) = 2x^3 - 3x + 1$ in $\left[\frac{1}{2}, 1\right]$. 6[Q.N.11, 2072'C']

3. Using L' Hospitals rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$. 2[Q.N.3(b), 2072'D']
$\left(\text{Ans: } \frac{1}{2}\right)$

4. State Mean Value theorem. Verify the mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$, $x \in [2, 4]$. 6[Q.N.11, 2072'D']
(Ans: $c = \sqrt{6}$)

5. State Rolle's theorem. What is the geometrical interpretation of Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$, $x \in [-1, 1]$. 6[Q.N.11, 2072'E']
(Ans: c = 0)

6. State mean value theorem. Interpret it geometrically. Verify the mean value theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in $[1, 4]$. 6 [Q.N. 11, Set 'C' 2071]

7. Using L Hospital's rule, evaluate:
$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$. 2 [Q.N. 3(a), Set 'D' 2071]
[Ans: 1]

8. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = x(x-3)^2$ for $x \in [0, 3]$. 6 [Q.N. 11, Set 'D' 2071]

9. Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{\tan x - x}{x - \sin x}$ 2 [Q.N. 3(a), 2070 'C']
[Ans: 2]

10. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = x(x - 3)^2$ for $x \in [0, 3]$. 6 [Q.N. 11, 2070 'C']
[Ans: C = 1]

11. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x(x - 1)^2$ in $[0, 2]$. 6 [Q.N. 11, 2070 'D']
$\left(\text{Ans: } c = \frac{4}{3}\right)$

12. Evaluate, using L'Hospital rule $\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$ [Q.N. 3(a), Supp. 2069]2
[Ans: 1]

13. Using mean value theorem, find a point on the parabola $y = (x-3)^2$ where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

$\left(\text{Ans: } \frac{7}{2}, \frac{1}{4}\right)$ 6 [Q.N. 11, Supp. 2069]

14. Using L Hospital's rule, evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x^2}$ [Q.N. 3(a), Set 'A' 2069]

$\left(\text{Ans: } \frac{1}{2}\right)$

15. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$. 6 [Q.N. 11, Set 'A' 2069]

16. Using L Hospital's rule, evaluate:

$\lim_{x \to 0} \frac{x - \sin x}{x^3}$. [Q.N. 3(a), Set 'B' 2069]

$\left(\text{Ans: } \frac{1}{6}\right)$

17. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1,4]$. [Q.N. 11, Set 'B' 2069]

Unit 8: Antiderivatives

1. Evaluate : $\int \frac{dx}{1 - 2\cos x}$ 2[Q.N.3(b), 2072'C']

$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \frac{\sqrt{3}\tan\frac{x}{2} - 1}{\sqrt{3}\tan\frac{x}{2} + 1} + C\right)$

2. Evaluate : $\int \frac{dx}{(x-2)^2 (x-3)^3}$ 4[Q.N.7(a), 2072'C']

$\left(\text{Ans: } -\frac{1}{2}\left(\frac{x-2}{x-3}\right)^2 + 3\left(\frac{x-2}{x-3}\right) - \log\frac{x-2}{x-3} - \frac{x-3}{x-2} + C\right)$

3. Compute the integral $\int \frac{\coth x\, dx}{\sinh x - 9\,\text{cosech}x}$ 2[Q.N.3(a), 2072'D']

$\left(\text{Ans: } \frac{1}{6}\log\frac{\sinh x - 3}{\cosh x + 3} + c\right)$

4. Evaluate : $\int \frac{dx}{(x-1)^2 (x-2)^3}$ 4[Q.N.7(a), 2072'D']

$\left(\text{Ans: } -\frac{1}{2}\left(\frac{x-1}{x-2}\right)^2 + 3\left(\frac{x-1}{x-2}\right) - 3\log\left(\frac{x-1}{x-2}\right) - \frac{x-2}{x-1} + c\right)$

5. Evaluate: $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}} (\beta > \alpha)$ 2[Q.N.3(b), 2072'E']

(Ans: $2\log(\sqrt{x-\alpha} + \sqrt{x-\beta}) + c$)

6. Evaluate : $\int \frac{dx}{2+3\cos x}$ 4[Q.N.7(a), 2072'E']

$$\left(\text{Ans: } \frac{1}{\sqrt{5}} \frac{\log \sqrt{5} + \tan \frac{x}{2}}{\sqrt{5} - \tan \frac{x}{2}} + C\right)$$

7. Evaluate : $\int \frac{6x+1}{x^2+9}\, dx$. 2 [Q.N. 3(b), Set 'C' 2071]

$$\left(\text{Ans: } 3 \log (x^2+9) + \frac{1}{3}\tan^{-1}\frac{x}{3} + C\right)$$

8. Evaluate: $\int \frac{dx}{1-2\cos x}$. 4 [Q.N. 7(a), Set 'C' 2071]

$$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \left(\frac{\sqrt{3}\tan\frac{x}{2} - 1}{\sqrt{3}\tan\frac{x}{2} + 1}\right) + C\right)$$

9. Evaluate : $\int \frac{dx}{\sqrt{2ax+x^2}}$. 2 [Q.N. 3(b), Set 'D' 2071]

[Ans: $\log (x + a + \sqrt{x^2 + 2ax}) + C$

10. Evaluate: $\int \frac{dx}{1-3\sin x}$ 4 [Q.N. 7(a), Set 'D' 2071]

$$\left(\text{Ans: } \left(\frac{1}{2\sqrt{2}} \log \frac{\tan\frac{x}{2} - 3 - 2\sqrt{2}}{\tan\frac{x}{2} - 3 + 2\sqrt{2}} + C\right)\right)$$

11. Evaluate : $\int \frac{2x+3}{4x^2+1}\, dx$. 2 [Q.N. 3(b), 2070 'C']

$$\left(\text{Ans: } \frac{1}{4}\log (4x^2+1) + \frac{3}{2}\tan^{-1} 2x + C\right)$$

12. Evaluate: $\int \frac{dx}{2+\cos x}$ 4 [Q.N. 7(a), 2070 'C']

$$\left(\text{Ans: } \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan^{-1}\frac{x}{2}}{\sqrt{3}}\right) + C\right)$$

13. Evaluate : $\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, $(\beta > \alpha)$ 2 [Q.N. 3(b), 2070 'D']

(Ans: $2\log (\sqrt{x-\alpha} + \sqrt{x-\beta}) + C$)

14. Evaluate: $\int \frac{dx}{1+2\sin x}$ 4 [Q.N. 7(a), 2070 'D']

$$\left(\text{Ans: } \frac{1}{\sqrt{3}} \log \frac{\tan\frac{x}{2} + 2 - \sqrt{3}}{\tan\frac{x}{2} + 2 + \sqrt{3}} + C\right)$$

15. Evaluate : $\int \frac{dx}{1+\sin x+\cos x}$. 2 [Q.N. 3(b), Supp. 2069]

$\left(\text{Ans: } \log\left(1+\tan\frac{x}{2}\right)+c\right)$

16. Evaluate: $\int \frac{x^2}{x^4-2x^2-15}\,dx$. 4 [Q.N. 7(a), Supp. 2069]

$\left(\text{Ans; } \frac{\sqrt{3}}{8}\tan^{-1}\frac{x}{3}+\frac{\sqrt{5}}{16}\log\frac{x-\sqrt{5}}{x+\sqrt{5}}+C\right)$

17. Evaluate : $\int \frac{dx}{\sqrt{2ax-x^2}}$. [Q.N. 3(b), Set 'A' 2069]

$\left(\text{Ans: } \sin^{-1}\frac{x-a}{a}+c\right)$

18. Evaluate: $\int \frac{dx}{3\sin x-4\cos x}$. [Q.N. 7(a), Set 'A' 2069]

$\left(\text{Ans: } \frac{1}{5}\frac{\tan\frac{x}{2}-\frac{1}{2}}{\tan\frac{x}{2}+2}+c\right)$

19. Evaluate : $\int \frac{dx}{e^x+e^{-x}}$. [Q.N. 3(b), Set 'B' 2069]

(Ans: $\tan^{-1} e^x + c$)

20. Evaluate: $\int \frac{dx}{2+3\cos x}$. [Q.N. 7(a), Set 'B' 2069]

$\left(\text{Ans: } \frac{1}{\sqrt{5}}\log\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}+C\right)$

21. Evaluate: $\int \frac{dx}{\sqrt{2ax-x^2}}$ [Q.N. 3(a), 2068]

$\left(\text{Ans: } \sin^{-1}\frac{x-a}{a}+C\right)$

22. Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$ [Q.N. 11(b), 2068]

$\left(\text{Ans: } \log\left(1+\tan\frac{x}{2}\right)+C\right)$

23. Evaluate : $\int \frac{1}{x^2} e^{\frac{-1}{x}}\,dx$ [Q.N. 3(a), 2067]

(Ans: $e^{-\frac{1}{x}} + C$)

24. Evaluate : $\int \frac{dx}{1+2\sin x}$ [Q.N. 11(b), 2067]

$\left(\text{Ans: } \frac{1}{\sqrt{3}}\log\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}+C\right)$

25. Integrate: $\int \frac{dx}{\sqrt{2ax + x^2}}$ [Q.N. 3(a), 2066]

(Ans: $\log(x + a) + \sqrt{x^2 + 2ax} + c$)

26. Integrate: $\int \frac{x^2}{(x + 2)(x + 3)^2} dx$ [Q.N. 11(b), 2066]

$\left(Ans: 4\log(x + 2) - 3\log(x + 3) + \frac{9}{x + 3} + c\right)$

27. Evaluate : $\int \frac{dx}{e^x + e^{-x}}$ [Q.N.3(a), 2065]

(Ans: $\tan^{-1}(e^x) + c$)

28. Integrate : $\int \frac{dx}{3 + 4\cos hx}$ [Q.N.11(b), 2065]

$\left(Ans: \frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{\tan h\frac{x}{2}}{\sqrt{7}}\right) + c\right)$

29. Evaluate : $\int \frac{dx}{x^2 - 16}$ [Q.N. 3(a), 2064]

$\left(Ans: \frac{1}{8} \log \frac{x - 4}{x + 4} + C\right)$

30. Integrate : $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx$ [Q.N. 11(b), 2064]

$\left(Ans: \log\{(\sin x + \cos x) + \sqrt{\sin 2x}\} + c\right)$

31. Integrate : $\int \frac{dx}{1 + \sin x + \cos x}$ [Q.N. 11(b), 2063]

$\left(Ans.: \log\left(1 + \tan\frac{x}{2}\right) + C\right)$

32. Integrate : $\int \frac{dx}{\sqrt{2ax - x^2}}$ [Q.N. 3(a), 2062]

$\left(Ans.: \sin^{-1}\frac{x - a}{a} + C\right)$

33. Integrate : $\int \frac{dx}{\sin x + \cos x}$ [Q.N. 11(b), 2062]

$\left(Ans.: \frac{1}{\sqrt{2}} \log\left[\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right] + C\right)$

34. Evaluate : $\int \sqrt{\frac{1 + x}{1 - x}} dx$ [Q.N. 3(a), 2061]

$\left(Ans: -\sqrt{1 - x^2} + \sin^{-1} x + c\right)$

35. Find the value of : $\int \frac{dx}{3\sin x - 4\cos x}$ [Q.N. 11(b), 2061]

$\left(\text{Ans: } \frac{1}{5} \log \frac{\tan\frac{x}{2} - \frac{1}{2}}{\tan\frac{x}{2} + 2} + C \right)$

36. Evaluate : $\int \frac{dx}{\sqrt{a^2 - x^2}}$ [Q.N. 3(a), 2060]

$\left(\text{Ans: } \sin^{-1}\frac{x}{a} + C \right)$

37. Find the value of :

$\int \frac{\sin x \cdot \cos x}{(\sin x + \cos x)^2} dx$ [Q.N. 11(b), 2060]

$\left(\text{Ans: } -\frac{1}{2}\frac{1}{(\tan x + 1)} + C \right)$

38. $\int \frac{dx}{a + b\cos x}$ a<b . [Q.N. 11(b), 2059]

$\left(\text{Ans: } \frac{1}{\sqrt{b^2 - a^2}} \log \left(\frac{\sqrt{b+a} + \sqrt{b-a}\tan\frac{x}{2}}{\sqrt{b+a} - \sqrt{b-a}\tan\frac{x}{2}} \right) + C \right)$

39. Prove that: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c.$ [Q.N. 3(a), 2058]

40. Integrate: $\int \sqrt{\frac{1+x}{1-x}} dx.$ [Q.N. 11(b), 2058]

$\left(\text{Ans: } -\sqrt{1 - x^2} + \sin^{-1}x + c \right)$

41. Prove: $\int \text{Cosec}\, x\, dx = \log \left| \tan\frac{x}{2} \right| + c$ [Q.N. 3(a), 2057]

42. Integrate: $\int \frac{dx}{a + b\cos x}$ when a> b. [Q.N. 11(b), 2057]

$\left(\text{Ans: } \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \tan\frac{x}{2} \right) + C \right)$

Unit 9: Differential Equations and their Applications

1. Solve : $\frac{dy}{dx} + \frac{y}{x} = 1.$ 2[Q.N.4(a), 2072'C']

$\left(\text{Ans: } xy = \frac{x^2}{2} + C \right)$

2. Solve : $\frac{dy}{dx} = y\tan x - 2\sin x$ 4[Q.N.7(b), 2072'C']

$\left(\text{Ans: } y\cos x = \frac{\cos 2x}{2} + C\right)$

3. Solve : $xy\frac{dy}{dx} - y^2 = x^2$ 4[Q.N.7(b)(Or), 2072'C']

(Ans: $y^2 = 2x^2(\log x + C)$)

4. Solve : $\frac{dy}{dx} + \frac{1+\cos 2y}{1-\cos 2y} = 0$. 2[Q.N.4(a), 2072'D']

(Ans: $\tan y - y + x = c$)

5. Reduce the equation $\frac{dy}{dx} + \frac{y}{x} = y^2$ in liner form hence solve it. 4[Q.N.7(b), 2072'D']

$\left(\text{Ans: } \frac{1}{xy} = -\log x + c\right)$

6. Solve : $\frac{dy}{dx} = \frac{y+1}{x+y+1}$ [Q.N.7(b)(Or), 2072'D']

(Ans: $y + 1 = ce^{\frac{x}{y+1}}$)

7. Solve: $\frac{dy}{dx} = \frac{y}{x} - \sin^2\frac{y}{x}$ 4[Q.N.7(b), 2072'E']

$\left(\text{Ans: } \cot\left(\frac{y}{x}\right) = \log x + c\right)$

8. Solve : $\sin x\frac{dy}{dx} + \cos xy = x\sin x$. [Q.N.7(b)(Or), 2072'E']

(Ans: $y\sin x = -x\cos x + \sin x + c$)

9. Solve: $e^{x-y}.dx + e^{y-x}dy = 0$. 2 [Q.N. 4(a), Set 'C' 2071]

(Ans: $e^{2x} + e^{2y} = C$)

10. Solve: $\frac{dy}{dx} = \frac{x^2+y^2}{2x^2}$. 4 [Q.N. 7(b), Set 'C' 2071]

(Ans: $2x = (x - y)\log(cx)$)

11. Solve: $\sin x\frac{dy}{dx} + (\cos x).y = \sin x.\cos x$. 4 [Q.N. 7(b)(OR), Set 'C' 2071]

$\left(\text{Ans: } y\sin x + \frac{1}{4}\cos 2x = c\right)$

12. Solve: $\frac{dy}{dx} + 4x = 2e^{2x}$. 2 [Q.N. 4(a), Set 'D' 2071]

[Ans: $y = e^{2x} - 2x^2 + C$

13. Solve: $xy\frac{dy}{dx} = x^2 + y^2$. 4 [Q.N. 7(b), Set 'D' 2071]

[Ans: $y^2 = 2x^2(\log x + C)$]

14. Solve: $\frac{dy}{dx} + \frac{2x}{1+x^2}.y = \frac{1}{(1+x^2)^2}$. 4 [Q.N. 7(b)(OR), Set 'D' 2071]

[Ans: $y(1 + x^2) = \tan^{-1}x + C$]

15. Solve: $\frac{dy}{dx} = \frac{e^x + 1}{y}$ 2 [Q.N. 4(a), 2070 'C']

[Ans: $y^2 = 2e^x + 2x + C$]

16. Solve: $\cos^2 x\frac{dy}{dx} + y = 1$. [Q.N. 7(b), 2070 'C']

[Ans: $y = 1 + Ce^{-\tan x}$]

17. Solve: $\frac{dy}{dx} = \frac{y}{x} - \text{Sin}^2\frac{y}{x}$. [Q.N. 7(b)(OR), 2070 'C']

$\left(\text{Ans: Cot}\frac{y}{x} = \log x + C\right)$

18. Solve: $x^2dy - y^2dx = 0$ 2 [Q.N. 4(a), 2070 'D']

(Ans: $y - x = Axy$)

19. Solve: $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$. 4 [Q.N. 7(b), 2070 'D']

$\left(\text{Ans: } (1 + x^2)\, y = \frac{4}{3}x^3 + C\right)$

20. Solve: $(x^2 + y^2)dy = xydx$ [Q.N. 7(b)(OR), 2070 'D']

(Ans: $x^2 = 2y^2 \log(yc)$)

21. Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$. 2 [Q.N. 4(a), Supp. 2069]

[Ans: $y - x = C(1 + xy)$]

22. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Supp. 2069]

[Ans: $y \sin x = x + C$]

23. Solve: $\frac{dy}{dx} = \frac{y-x+1}{y-x+5}$ [Q.N. 7(b)(OR), Supp. 2069]

[Ans: $x^2 + y^2 - 2xy + 10y - 2x = C$]

24. Solve: $\frac{dy}{dx} = e^{x-y} + x^3 \cdot e^{-y}$ [Q.N. 4(a), Set 'A' 2069]

$\left(\textit{Ans: } e^y = e^x + \frac{x^4}{4} + c\right)$

25. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Set 'A' 2069]

(Ans: $y \sin x = x + c$)

26. Solve: $xy\frac{dy}{dx} = x^2 + y^2$ [Q.N. 7(b)(OR), Set 'A' 2069]

(Ans: $y^2 = 2x^2(\log x + c)$)

27. Solve: $e^{x-y}\,dx + e^{y-x}.\,dy = 0$ [Q.N. 4(a), Set 'B' 2069]

(Ans: $e^{2x} + e^{2y} = C$)

28. Solve: $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ [Q.N. 7(b), Set 'B' 2069]

$\left(\textit{Ans: } (1 + x^2)\, y = \frac{4}{3}x^3 + C\right)$

29. Solve: $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ [Q.N. 7(b)(OR), Set 'B' 2069]

$\left(\textit{Ans: } \sin\left(\frac{y}{x}\right) = Cx\right)$

30. Solve: $x^2dy - y^2dx = 0$ [Q.N. 6(c), 2068]

(Ans: $y - x = c\,y$, c is a constant.)

31. Solve: $\sin x\frac{dy}{dx}\cos x.y = x \sin x$ [Q.N. 11(b)(Or), 2068]

(Ans: $y + x\cot x - c\,\text{cosec}\ x = 1$; c is constant.)

32. Solve : $xdy + (x+y)\,dx = 0$ [Q.N. 6(c), 2067]

(Ans: $2xy + x^2 = c^2$)

33. Solve : $(1-x^2)\frac{dy}{dx} = 1 + xy$ [Q.N. 11(b) (Or)), 2067]

(Ans: $\sqrt{1-x^2}\ y = \sin^{-1} x + C$)

34. Solve the differential equation $(x + 2y - 3)\,dy - (2x - y + 1)\,dx = 0$. [Q.N. 6(c), 2066]

(Ans: $xy + y^2 - x^2 - 3y - x = c$)

35. Solve the differential equation: $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ [Q.N. 11(b)Or, 2066]

$\left(\textit{Ans: } y = \frac{1}{2}e^{\tan^{-1}x} + ce^{-\tan^{-1}x}\right)$

36. Solve: $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$. [Q.N.6(c), 2065]

(Ans: $2x^3 + 2y^3 + 3x^2y^2 = c$)

37. Solve: $\frac{dy}{dx} + y \cot x = x$. [Q.N.11(b, or), 2065]

(Ans: $y \sin x = -x \cos x + \sin x + c$)

38. Solve : $x\,dy + (x + y)\,dx = 0$ [Q.N. 6(c), 2064]

(Ans: $y = x \log (cx)$)

39. Solve : $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$ [Q.N. 11(b)Or, 2064]

$\left(\textit{Ans: } (y-x)^2 = cxy^2\right)$

40. Solve : $e^{x-y}\,dx + e^{y-x}\,dy = 0$ [Q.N. 6(c), 2063]

(Ans.: $e^{2x} + e^{2y} = C$)

41. Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$. [Q.N. 11(b) Or, 2063]

$\left(\textit{Ans.: } \sin\frac{y}{x} = cx\right)$

42. Solve : $x^2dy - y^2dx = 0$ [Q.N. 6(c), 2062]

(Ans.: $y - x = Axy$)

43. Solve : $\frac{dy}{dx} + \frac{1}{x} \cdot y = x^2$ [Q.N. 11(b)Or, 2062]

$\left(\textit{Ans.: } xy = \frac{x^4}{4} + C\right)$

44. Solve : $x^2\,dy - y^2\,dx = 0$ [Q.N. 6(c), 2061]

(Ans: $y - x = Axy$)

45. Solve: $(x^2 - y^2)\frac{dy}{dx} = xy$ [Q.N. 11(b)Or, 2061]

(Ans: $2y^2\ \log yC + x^2 = 0$)

46. Solve : $\sqrt{1-x^2}\ dy + \sqrt{1-y^2}\ dx = 0$ [Q.N. 6(c), 2060]

(Ans: $y = \tan^{-1} x + C$)

47. Solve: $\tan x\frac{dy}{dx} + y = \sec x$ [Q.N. 11(b)Or, 2060]

(Ans: $y \sin x = x + C$)

48. Solve: $x\,dy - y\,dx = 0$. [Q.N. 2(b), 2059]

(Ans: $y = cx$)

49. Solve: $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ [Q.N. 11(b) Or, 2059]

$\left(\textit{Ans: } y^2 + x^2 = cx\right)$

50. Solve: $\frac{dy}{dx} = \frac{x^3+1}{y^3+1}$ [Q.N. 6(c), 2058]

$\left(Ans: \frac{y^4}{4} + y = \frac{x^4}{4} + x + c\right)$

51. Solve : $\cos^2 x \frac{dy}{dx} + y = 1$. [Q.N. 11(b)Or, 2058]

$\left(Ans: y = 1 + ce^{-\tan x}\right)$

52. Solve: $\frac{dy}{dx} = \frac{x^2+x+1}{y^2+y+1}$ [Q.N. 6(c), 2057]

$\left(Ans: \frac{y^3}{3} + \frac{y^2}{2} + y = \frac{x^3}{3} + \frac{x^2}{2} + x + A\right)$

53. Solve: $\tan x \frac{dy}{dx} + y = \sec x$. [Q.N. 11(b)Or, 2057]

(Ans: ysinx = x + c)

Unit 10: Dispersion, Correlation and Regression

10.1 Measures of dispersions

1. The information about the daily temperature of two cities X and y are as follows:

	X	Y
Average temp. (°F)	84	92
Variance of temp.	16	25

Determine which city has greater consistency in climate. 2[Q.N.4(b), 2072'C']
(Ans: City X)

2. In the distribution of data 20, 25, 30, 36, 32, 43 ; find standard deviation.
(Ans: 7.39) 2[Q.N.4(b), 2072'D']

3. For a group of 50 items; circle $\Sigma x^2 = 600$, $\Sigma x = 150$ and $m_0 = 1.75$, find the Pearsonian coefficient of skewness. 2[Q.N.4(b), 2072'E']
(Ans: 0.72)

4. Calculate the coefficient of Skewness based on mean, mode and standard deviation from the following data: 4 [Q.N. 8(a), Set 'C' 2071]

Wages (in Rs.)	100	110	120	130	140
No. of persons	2	6	10	8	4

(Ans: 0.18)

5. If n = 10, $\Sigma x = 120$, $\Sigma x^2 = 1530$, find the standard deviation and the coefficients of variation. 2 [Q.N. 4(b), Set 'D' 2071]
[Ans: r = 3, C.V. = 25%]

6. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$ find the skewness based on mean, mode and standard deviation. 4[Q.N. 8(a), 2070 'C']
[Ans: –0.22]

7. Consider the following distribution.

	Distribution A	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation:	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness? 4[Q.N. 8(a), 2070 'D']
(Ans: Yes, same)

8. The information about the daily temperature of two cities A and B are as follows:

	City A	City B
Average Temperature (F)	84	92
Variance of Temperature	16	25.

Determine which city has greater consistency in climate.
[Ans: A] 2 **[Q.N. 4(b), Supp. 2069]**

9. Determine the standard deviation and the coefficient of variation from the following distribution. 4 **[Q.N. 8(a), Set 'A' 2069]**

Profit (in Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	8	13	16	8	5

(Ans: Rs. 11.88, 52.11%)

10. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, $N = 10$ and $M_0 = 12.45$, find the skewness based on mean, mode and standard deviation. **[Q.N. 8(a), Set 'B' 2069]**
(Ans: – 0.22)

11. Weights of a group of individuals are given below. Find out the mean and the standard deviation. **[Q.N. 12(a), 2068]**

Weight (in kg.)	0-10	10-20	20-30	30-40	40-50
Frquency	12	33	30	15	10

(Ans: 27, 11.45)

12. Consider the following distribution:

	distribution A	distribution B
Arithmetic mean	100	90
Medium	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness ? **[Q.N. 12(b), 2068]**
(Ans: Yes, same)

13. In a frequency distribution of a set of data C.V. = 5%, $\sigma = 2$ and Karl Pearson coefficient of skewness = 0.5; find the mean of the data. **[Q.N. 4(b), 2067]**
(Ans: 40)

14. Define standard deviation. Also prove that the root mean square deviation is not less than the standard deviation. **[Q.N. 12(b), 2067]**

15. Find the standard deviation from the following data:

x:	10	11	12	13	14
f:	3	12	18	12	2

(Ans: 0.967) **[Q.N. 4(b), 2066]**

16. Calculate the coefficient of skewness from the following frequency distribution: **[Q.N.12(b), 2066]**

Investment	10-20	20-30	30-40	40-50	50-60	60-70	70-80
	12	18	20	15	10	3	2

(Ans: 0.227)

17. Calculate Karl Pearson's coefficient of skewness of the data: **[Q.N.12(a), 2065]**

Marks	above 0	above 10	above 20	above 30	above 40
Frequency	150	140	100	80	80

(Ans: 0.89)

18. The information about the wages distribution of the firms A and B are given below :

	Firm A	Firm B
No. of workers	500	600
Average monthly wages	Rs. 586	Rs. 575
Variable of wages distribution	81	100

In which firm is the wages distribution uniform ? **[Q.N. 4(b), 2064]**
(Ans: firm A)

19. For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24{,}270$ and mode = 43.7, find the Pearson's coefficient of skewness. **[Q.N. 12(a), 2064]**
(Ans: 0.076)

20. Consider the following distribution.

	Distribution a	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the skewness ?
(Ans.: Same) **[Q.N. 4(b), 2063]**

21. Find the standard deviation of the following data :
10, 15, 20, 25, 30, 35, 40 **[Q.N. 4(b), 2062]**
(Ans.: 10)

22. Following are the marks obtained by two students A and B in 10 tests of 100 marks each.

Test:	1	2	3	4	5	6	7	8	9	10
Marks of A :	44	80	76	48	52	72	68	56	60	54
Marks of B :	48	75	54	60	63	69	72	51	57	66

Who is intelligent ? **[Q.N. 12(a), 2062]**
(Ans.: B)

23. Find the mean deviation from mean of the following data :
6, 8, 10, 13 and 5. **[Q.N. 4(b), 2061]**
(Ans: 2.72)

24. Find out the mean and Standard Deviation from the following data :

Variable:	5-10	10-15	15-20	20-25	25-30	30-35
Frequency:	2	9	29	54	11	5

(Ans: 23.95 and 4.87) **[Q.N. 12(a), 2061]**

25. The median, mode and coefficient of skewness for a certain distribution are respectively 17.4, 15.3 and 0.35. Calculate mean and C.V. **[Q.N. 12(b), 2061]**
(Ans: 18.45 and 48.78%)

26. Find the mean deviation from median of the numbers 5, 7, 10, 12 and 6.
(Ans: 1.57) **[Q.N. 4(b), 2060]**

27. Find the mean and S.D. from the following table wages (Rs.) :

Wages:	10–20	10–30	10–40	10–50	10–60
No. of workers :	15	33	63	83	100

(Ans: Mean = 35.06 and S.d. = 12.45) **[Q.N. 12(a), 2060]**

28. Prove that in a discrete distribution the standard deviation is not less than the mean deviation from the mean. **[Q.N. 12(b), 2060]**

29. The coefficient of variation and mean of a certain frequency distribution are 50.2% and 22.8 respectively. Find the s.d. **[Q.N. 4(c), 2059]**
(Ans: 11.4456)

30. Find the standard deviation of the following data:
100, 150, 200, 250, 300. **[Q.N. 4(b), 2058]**
(Ans: 70.7)

31. Write the expressions for the first four central moment and for β_2. Hence show that for a discrete distribution, $\beta_2 \geq 1$. **[Q.N. 12(b), 2057]**

$$\left(\textit{Ans: } \mu_1 = 0, \mu_2 = \frac{\Sigma x^2}{N}, \mu_3 = \frac{\Sigma x^3}{N}, \mu_4 = \frac{\Sigma x^4}{N}, \beta_2 = \frac{\mu_4}{\mu_2} \right)$$

10.2 Correlation

1. Define correlation. Find Karl Pearson's coefficient of correlation of the marks of the following distribution. 4[Q.N.8(a), 2072'D']

X	20	30	40	50	60
Y	50	46	30	24	8

(Ans: – 0.937)

2. If n = 15, σ_x = 3.2, σ_y = 3.4 and $\Sigma(X - \bar{X})(Y - \bar{Y}) = 122$, find the correlation coefficient between the two variables. 2 **[Q.N. 4(b), Set 'C' 2071]**
(Ans: 0.75)

3. Calculate Karl Pearson's correlation coefficient between the two variables height (in cms) and weight (in kg) from the data gives below: 4 **[Q.N. 8(a), Set 'D' 2071]**
[Ans: 0.51]

Height	160	162	165	161	163
Weight	63	62	64	60	61

4. If $\Sigma (X - \bar{X})^2 = 40$, $\Sigma(Y - \bar{Y})^2 = 63$ and $\Sigma(X - \bar{X})(Y - \bar{Y}) = 35$, find the correlation coefficient between the two variables. 2 **[Q.N. 4(b), 2070 'C']**
[Ans: 0.697]

5. If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. 2 **[Q.N. 4(b), 2070 'D']**
(Ans: 0.59)

6. Find Karl Pearsons' coefficient of correlation of the marks of the following two group of students 4 **[Q.N. 8(a), Supp. 2069]**
[Ans: –0.937]

X	20	30	40	50	60
Y	50	46	30	24	8

7. Calculate the correlation coefficient between two variables from the following data: $\Sigma x^2 = 114$, $\Sigma y^2 = 422$, and $\Sigma xy = 174$ **[Q.N. 4(b), 2068]**
[Ans: 0.793)

8. If the covariance between the variable x and y is 18 and the variances of x and y are 16 and 81 respectively, find the coefficient of correlation between them.
(Ans: 0.5) **[Q.N.4(b), 2065]**

9. From the following table, calculate the coefficient of correlation by Karl Pearson's method. **[Q.N. 12(b), 2063]**

X :	6	2	10	4	8
Y :	9	11	–	8	7

Arithmetic means of X and Y series are 6 and 8 respectively.
(Ans.: – 0.92)

10. Prove that the correlation coefficient between two variables lies between –1 and +1. **[Q.N. 12(b), 2062]**

11. Calculate Karl Pearson's coefficient of correlation from the following data:

x:	12	9	8	10	11	13	7
y:	14	8	6	9	11	12	3

(Ans: 0.95) **[Q.N. 12(b), 2059]**

12. Prove that the coefficient of correlation lies between-1 and 1. **[Q.N. 12(b), 2058]**

13. Calculate r_{xy} if $\Sigma x^2 = 114$; $\Sigma y^2 = 442$; $\Sigma xy = 174$. **[Q.N. 4(b), 2057]**
(Ans: 0.793)

10.3 Regression

1. Define regression and lines of regression. Find the correlation coefficients between the two variables when $b_{xy} = 1.8$ and $b_{yx} = 0.35$. 4[Q.N.8(a), 2072'C']
(Ans: 0.79)
2. The regression coefficients of x on y and y on x are 0.84 and 0.32 respectively. if the arithmetic means of x and y series are 42 and 26 respectively, find two equations of lines of regression. 4[Q.N.8(a), 2072'E']
(Ans: y = 0.32x + 12.56, x + 0.84y + 20.16)
3. Find the regression equation of y on x when:
$\Sigma x = 15, \Sigma y = 25, \Sigma x^2 = 55, \Sigma y^2 = 140, \Sigma xy = 78, n = 5.$
(Ans: y = 0.3x + 4.1) [Q.N. 4(b), Set 'A' 2069]
4. The regression coefficient of y on x is 0.32. If the arithmetic means of x and y series are 42 and 36 respectively, find the regression equation of y on x.
(Ans: 0.32x – y – 6 = 0) [Q.N. 4(b), Set 'B' 2069]

Unit 11: Probability

11.1 Probability

1. In rolling a pair of dice, determine the probability of obtaining a sum of 10.
$\left(\text{Ans: } \frac{1}{12}\right)$ 2[Q.N.4(c), 2072'C']
2. In a draw of a card from well shuffled deck of 52 cards what is the probability that it is a king or a queen? 2[Q.N.4(c), 2072'D']
$\left(\text{Ans: } \frac{2}{13}\right)$
3. Two dice are rolled once. What is the probability of getting a total of 8 or 7 ?
$\left(\text{Ans: } \frac{11}{36}\right)$ 2[Q.N.4(c), 2072'E']
4. The chance that A can solve the problem is $\frac{3}{5}$ and the chance that B can solve the problem is $\frac{2}{3}$. Find the probability that the problem is solved. 2[Q.N. 4(c), Set 'C' 2071]
$\left(\text{Ans: } \frac{13}{15}\right)$
5. Two coins are tossed simultaneously. Find the sample space. Find the probability that both are heads. 2 [Q.N. 4(c), Set 'D' 2071]
[Ans: S ={HH, HT, TH, TT}, $P(HH) = \frac{1}{4}$]
6. A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that one is boy and one girl? 2 [Q.N. 4(c), 2070 'C']
$\left(\text{Ans: } \frac{16}{33}\right)$
7. A card is drawn from a well–shuffled back of 52 cards. What is the probability that it is a king or a Diamond? 2 [Q.N. 4(c), 2070 'D']
$\left(\text{Ans: } \frac{4}{13}\right)$