

- b. For what value of k will the point $(2, -1)$ lie on the curve $kx^2 - 2y^2 - 2x + 3y - 3 = 0$ [3]
 c. Find the equation of a straight line passing through the point of intersection of the line $3x + y = 6$ and $x - y = 2$ and the point $(-1, 3)$. [3]

26. 2062 Q.No. 5

Ans: (b) 3 (c) $x + y = 2$

- a. Show that the points $A(1, 2)$, $B(3, 4)$ and $C(-3, -2)$ are collinear. [3]
 b. Find the equation of the locus of a point, which moves such that its distance from $(-1, 3)$ is always 8. [3]
 c. Find the slope and the equation of a straight line passing through the points $(8, 5)$ and $(-3, -6)$. [3]

27. 2061 Q.No. 5

Ans: (b) $x^2 + y^2 + 2x - 6y - 54 = 0$ (c) 1, $x - y = 3$

- a. Show that the points $(-2, -1)$, $(1, 0)$, $(4, 3)$ and $(1, 2)$ are the vertices of a parallelogram. [3]
 b. If $A(a, 0)$ and $B(-a, 0)$ and two fixed points, find a locus of a point P which moves so that $PA^2 + PB^2 = AB^2$. [2]
 c. Find the equation of a straight line passing through the points $(2, -3)$ with slope $-\frac{2}{3}$. [3]

Ans: (b) $x^2 + y^2 = a^2$ (c) $2x + 3y = -5$

28. 2060 Q.No. 5

- a. Show that the triangle formed by joining the co-ordinates of the two points $(4, 3)$ and $(5, 0)$ with the origin is an isosceles triangle. [3]
 b. Find the equation of the locus of a point if its distance from the y -axis is double the distance from the point $(2, 2)$. [3]
 c. Find the equation of the line which passes through the origin and the point of intersection of $2x - 3y + 1 = 0$ and $x + 2y = 3$. [3]

Ans: (b) $3x^2 + 4y^2 - 16x - 16y + 32 = 0$ (c) $x - y = 0$

29. 2059 Q.No. 5

- a. Find the co-ordinates of the points on the x -axis which are at a distance of 5 units from the point $(4, 6)$. [3]
 b. Obtain the equation of the locus of a point, which moves so that its distance from $(4, 3)$ is half the distance from x -axis. [3]
 c. Find the equation of the straight line passing through the points $(4, 7)$ and $(2, y)$ if its slope is $-\frac{3}{2}$. [3]

Ans: (a) Does not exist such points on the x -axis
 (b) $4x^2 + 3y^2 - 32x - 24y + 100 = 0$ (c) $3x + 2y = 26$

30. 2058 Q.No. 5

- a. Find the co-ordinate of the middle point joining the two points $(5, 6)$ and $(3, 4)$. Also find the distance between them and the slopes of line joining them. [3]
 b. Find the co-ordinates of the point dividing the line joining the points $(5, -2)$ and $(9, 6)$ in the ratio 3:1 (i) internally (ii) externally. [3]
 c. Find the equation of a straight line passing through the points $(-1, -1)$ and $(8, 11)$. [3]

Ans: (a) $(4, 5)$, $2\sqrt{2}$ and 1 (b) (i) $(8, 4)$ (ii) $(11, 10)$ (c) $4x - 3y = -1$

31. 2057 Q.No. 5

- a. A point divides internally the line joining the points $(3, 5)$ and $(7, 15)$ in the ratio of 3:4. Find the co-ordinates of that point. [3]
 b. Find the equation of the straight line through the point $(2, 3)$ and making equal intercepts on the axes. [3]
 c. Find the point of intersection of the straight lines $x - 3y + 4 = 0$ and $7x + 4y + 8 = 0$. [3]

Ans: (a) $(\frac{33}{7}, \frac{65}{7})$ (b) $x + y = 5$ (c) $(\frac{-8}{5}, \frac{4}{5})$

UNIT 7: LOGARITHMS

1. 2073 Set C Q.No. 5a

Using log table, simplify: $\frac{(3.678)^4}{\sqrt[3]{42.75}}$

[3]

Ans: 52.38

2. 2073 Set D Q.No. 5a

Using 4 figure log-table, evaluate: $\frac{46.23 \times 92.75}{\sqrt[3]{0.0029}}$

[3]

Ans: 30070

3. 2072 Supp Q.No. 5a

Using 4 figure log table, evaluate: $\frac{(3.678)^4}{\sqrt[3]{42.75}}$

[3]

Ans: 52.38

4. 2072 Set C Q.No. 5a

Evaluate $\frac{18.29 \times \sqrt{0.741}}{(4.05)^2}$ by using log-table

[3]

Ans: 0.9596

5. 2072 Set D Q.No. 5a

Find the value of $\sqrt{0.7654} \times 328 + (3.421 \times \sqrt[3]{0.7358})$ using log table.

[3]

Ans: 92.94

6. 2072 Set E Q.No. 5a

Using four figure log table, Evaluate $\sqrt{\frac{67.58}{9.765}} + (3.57)^2$.

[3]

Ans: 15.381

7. 2071 Supp Q.No. 5a

Using log table, find the value of $(42 \times \sqrt{0.338}) \div (8.87)^2$.

[3]

Ans: 0.3103

8. 2071 Set C Q.No. 5a

Using 4 figure log table, evaluate: $3.142 \sqrt{\frac{98.1}{32.2}}$

[3]

Ans: 5.484

9. 2071 Set D Q.No. 5a

Evaluate using the four figure log table: $\frac{\sqrt{0.7654}}{3.421}$

[3]

Ans: 0.2546

10. 2070 Supp Q.No. 5a

Using log table, find the value of $\frac{(39.4)^2}{\sqrt[5]{(374)^4}}$

[3]

Ans: 13.57

11. 2070 Set C Q.No. 5 a

Using 4 figure log table, evaluate $\sqrt[3]{\frac{9620}{108 \times (62.4)^3}}$

[3]

Ans: 0.07157

12. 2070 Set D Q.No. 5 a

Using log table find the value of $\sqrt[3]{\frac{1}{0.3852 \times 3.256}}$

[3]

Ans: 0.9272

13. 2069 Supp Q.No. 5 a

Using 4 figure log table, evaluate $\sqrt[7]{\frac{1}{0.8176 \times 36.21}}$ [3]

Ans: 0.6163

14. 2069 Q.No. 5a

Evaluate using four figure log table $\frac{(3.678)^4}{\sqrt[3]{42.75}}$ [3]

Ans: 52.38

15. 2068 Q.No. 5a

Using 4 figure log table, evaluate: $\left(\frac{8.12}{62.9 \times 0.28}\right)^{1/6}$ [3]

Ans: 0.8790

16. 2067 Q.No. 5 a

Evaluate using the four figure log table: $\sqrt[3]{\frac{12.7 \times (0.86)^4}{(0.625)^3}}$ [3]

Ans: 3.053

17. 2067 Supp Q.No. 5a

Using log table, find the value of $\frac{23.1 \times 2.56}{\sqrt[3]{52.89}}$ [3]

Ans: 15.75

18. 2066 Q.No. 5 a

With the help of log table, find the value of: $\sqrt[3]{\frac{12.7 \times (0.86)^4}{(0.625)^3}}$ [3]

Ans: 3.053

19. 2066 Old Q.No. 4 a

Using log table, evaluate $3.142 \times \sqrt{\frac{98.1}{32.2}}$ [3]

Ans: 5.484

20. 2066 Supp Q.No. 5 a

Using logarithms, find the value of $\frac{23.1 \times 2.56}{\sqrt[3]{52.89}}$ [3]

Ans: 15.75

21. 2065 Q.No. 5 a

Using log table, find the value of: $\sqrt[3]{\frac{59.26 \times (1.414)^2}{0.022 \times 365}}$ [3]

Ans: 2.452

22. 2065 Old Q.No. 4 a

Evaluate by using log table: $\frac{\sqrt{18.5} \times (19.3)^{-1/3}}{(65.4)^{3/4} \times (15.9)^{1/5}}$ [3]

Ans: 0.0401

23. 2064 Q.No. 5 a

Evaluate the following, using log tables: $\frac{\sqrt[3]{42.75}}{(3.678)^2}$ [3]

Ans: 0.2583

24. 2064 Q.No. 4 a Old

Using log tables, evaluate: $\frac{2731 \times (0.0354)^3}{\sqrt[5]{0.224}}$ [3]

Ans: 0.1635

25. 2063 Q.No. 4 a

Using logarithm, evaluate $\sqrt[3]{\frac{1}{2.35 \times 6.472}}$

[3]

Ans: 0.4036

26. 2062 Q.No. 4 a

Using logarithm, compute: $\sqrt[3]{\frac{(1.414)^2}{3.65}}$

[3]

Ans: 0.8181

27. 2061 Q.No. 4 a

Using logarithms, find the value of: $\frac{23.1 \times 2.56}{\sqrt[3]{52.89}}$

[3]

Ans: 15.75

28. 2060 Q.No. 4 a

Using logarithms, find the value of y where $y = \frac{\sqrt[3]{12.7(0.84)^4}}{0.625}$

[3]

Ans: 1.859

29. 2059 Q.No. 4 a

Find the value by using log table: $\frac{\sqrt[7]{4396.93}}{\sqrt{0.7654}}$

[3]

Ans: 3.793

30. 2058 Q.No. 4 a

Evaluate the following using four figure log table: $y = \frac{(50.38)^4}{(0.9648)^{10}}$

[3]

Ans: 9208000

31. 2057 Q.No. 4 a

Find the value using log table $\frac{\sqrt[3]{(60.45)^2}}{\sqrt[5]{0.08952}}$

[3]

Ans: 24.96

UNIT 8: FUNCTIONS, LIMIT AND CONTINUITY**A. FUNCTIONS**

1. 2071 Set D Q.No. 5b

If $f(x) = x - \frac{1}{x}$ show that: $f(a) = f\left(\frac{-1}{a}\right)$

[3]

2. 2058 Q.No. 6 b

If fixed cost of 20 articles is 500 and variable cost for each article is 40. Find the total cost function? Also find the total cost of 90 articles.

[3]

Ans: $C(x) = 40x - 300$ and 3300**B. LIMIT**

1. 2073 Set C Q.No. 5b

Evaluate: $\lim_{x \rightarrow \infty} \frac{x^2 + 9}{x^2 - 9}$

[3]

Ans: 1

2. 2073 Set D Q.No. 5b

Evaluate: $\lim_{x \rightarrow \infty} \frac{4x^2 + x - 1}{3x^2 + 2x + 1}$

[3]

Ans: $\frac{4}{3}$

3. 2072 Supp Q.No. 5b

Evaluate: $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 5x + 6}$

[3]

Ans: 27

4. 2072 Set C Q.No. 5b

Evaluate: $\lim_{x \rightarrow 1} \frac{x - \sqrt{2 - x^2}}{2x - \sqrt{2 + 2x^2}}$

[3]

Ans: 2

5. 2072 Set D Q.No. 5b

Evaluate: $\lim_{x \rightarrow \infty} \frac{2\sqrt{1+x} - \sqrt{2x-1}}{x}$

[3]

Ans: 0

6. 2072 Set E Q.No. 5b

Evaluate: $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}}$

[3]

Ans: 1

7. 2071 Supp Q.No. 5b

Evaluate: $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$

[3]

Ans: 0

8. 2071 Set C Q.No. 5b

Evaluate: $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 4}$

[3]

Ans: 0

9. 2070 Supp Q.No. 5b

Evaluate: $\lim_{x \rightarrow 1} \frac{x^2 - 5x + 4}{x^2 + 6x - 7}$

[3]

Ans: Ans: -3/8

10. 2070 Set C Q.No. 5b

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

[3]

Ans: 1/4

11. 2070 Set D Q.No. 5b

Evaluate: $\lim_{x \rightarrow a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)}$

[3]

Ans: $-\frac{1}{4\sqrt{2a}}$

12. 2069 Supp Q.No. 5b

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{x}$

[3]

Ans: 1

13. 2069 Q.No. 5b

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{5x}$

[3]

Ans: $\frac{1}{10\sqrt{2}}$

14. 2068 Q.No. 5b

Evaluate: $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9}{x^3 - 3x^2} \right)$

[3]

Ans: $\frac{2}{3}$

15. 2067 Q.No. 5 b

Find the limit of: $\lim_{x \rightarrow 5} \frac{x-5}{\sqrt{4x+5}-5}$ [3]

Ans: 5/2

16. 2067 Supp Q.No. 5b

Evaluate: $\lim_{x \rightarrow 0} \left(\frac{7x}{\sqrt{3x+4}-2} \right)$ [3]

Ans: $\frac{28}{3}$

17. 2066 Q.No. 5 b

Evaluate, if exist: $\lim_{x \rightarrow 0} \frac{7x}{\sqrt{3x+4}-2}$ [3]

Ans: $\frac{28}{3}$

18. 2066 Old Q.No. 6 a

Prove that: $\lim_{x \rightarrow 1} \frac{\sqrt{x-1} + \sqrt{x-1}}{\sqrt{x^2-1}} = \frac{1}{\sqrt{2}}$ [3]

19. 2066 Supp Q.No. 5 b

Evaluate: $\lim_{x \rightarrow 3} \frac{x^3 + 2x^2 - 14x - 3}{x^2 - x - 6}$ [3]

Ans: 5

20. 2065 Q.No. 5 b

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+3} - \sqrt{3-x^2}}{x^2}$ [3]

Ans: $1/\sqrt{3}$

21. 2065 Old Q.No. 6 a

Prove that: $\lim_{x \rightarrow 1} \frac{x - \sqrt{2-x^2}}{2x - \sqrt{2+2x^2}} = 2$ [3]

22. 2064 Q.No. 5 b

Find the limit of: $\lim_{x \rightarrow 0} \frac{7x}{\sqrt{3x+4}-2}$ [3]

Ans: 28/3

23. 2064 Q.No. 6 a Old

Evaluate: $\lim_{x \rightarrow 5} \left(\frac{1}{x-5} - \frac{25}{x^3-5x^2} \right)$ [3]

Ans: 2/5

24. 2063 Q.No. 6 a

Evaluate: $\lim_{x \rightarrow 3} \frac{x^3-27}{x^2-5x+6}$ [3]

Ans: 27

25. 2062 Q.No. 6 a

Evaluate: $\lim_{x \rightarrow \infty} \frac{6x^2+5x-8}{8x^2+9x+3}$ [3]

Ans: $\frac{3}{4}$

26. 2061 Q.No. 6 a

Find the limit of: $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$ [3]

Ans: 0

27. 2060 Q.No. 6 a

Find the limit of: $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$ [3]

Ans: 5/2

28. 2059 Q.No. 6 a

Evaluate: $\lim_{x \rightarrow 0} \frac{\sqrt{3+x^2} - \sqrt{3-x^2}}{x^2}$ [3]

Ans: $\frac{1}{\sqrt{3}}$

29. 2058 Q.No. 6 a

Find the limit of: $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} - \frac{9}{x^3-3x^2} \right)$ [3]

Ans: 2/3

30. 2057 Q.No. 6 a

Find the limit of $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$ [3]

Ans: 1/4

C. CONTINUITY

1. 2073 Set C Q.No. 6a

Examine the continuity of the function $f(x) = \begin{cases} \frac{4x^2-16}{2x-4} & ; x \neq 2 \\ 8 & ; x = 2 \end{cases}$; at $x = 2$ [3]

Ans: Continuous

2. 2073 Set D Q.No. 6a

Examine the continuity or the discontinuity of the function [3]

$f(x) = \frac{x^2-4}{x+2}$ at $x = 2$

Ans: Continuous

3. 2072 Supp Q.No. 6a

Examine the continuity of the function: $f(x) = 4x^2 + 2x - 9$ at $x = \frac{1}{2}$. [3]

Ans: Continuous

4. 2072 Set C Q.No. 6a

A function $f(x)$ is defined as follows: [3]

$f(x) = \begin{cases} \frac{x^2-2x}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$

Find the value of k so that the function $f(x)$ is continuous at $x = 2$. [3]

Ans: 2

5. 2072 Set D Q.No. 6a

Examine the continuity of: $f(x) = \begin{cases} \frac{x^2-3x+2}{x-2} & \text{for } x \neq 2 \\ 1 & \text{for } x = 2 \end{cases}$ at $x = 2$ [3]

Ans: Continuous

6. 2072 Set E Q.No. 6a

Test the continuity of the function. $f(x) = \begin{cases} \frac{x^2-3x}{x-3} & \text{at } x \neq 3 \\ 3 & \text{at } x = 3 \end{cases}$ at $x = 3$. [3]

Ans: Continuous

7. 2071 Supp Q.No. 6a

Test the continuity of the function $f(x) = \begin{cases} \frac{(x-2)(x-1)}{x-1} & \text{at } x \neq 1 \\ -1 & \text{at } x = 1 \end{cases}$ at $x = 1$ [3]

Ans: Continuous

8. 2071 Set C Q.No. 6a

Examine the continuity of the function $f(x) = \frac{x+1}{2x+5}$ at $x = 2$ [3]

Ans: Continuous

9. 2071 Set D Q.No. 6a

Examine the continuity or the discontinuity of the function: $f(x) = \frac{3x+2}{2x-1}$ at $x = 1$. [3]

Ans: Continuous

10. 2070 Supp Q.No. 6a

Determine whether the function $f(x) = \begin{cases} 2x+3 & \text{when } 0 \leq x \leq 2 \\ x+7 & \text{when } x > 2 \end{cases}$ is continuous or discontinuous at $x = 2$. [3]

Ans: Discontinuous

11. 2070 Set C Q.No. 6a

Examine the continuity or discontinuity of the function. $f(x) = \frac{x^2-4}{x-2}$ at $x = 2$. [3]

Ans: Discontinuous at $x = 2$

12. 2070 Set D Q.No. 6a

A function $f(x)$ is defined as follows. $f(x) = \begin{cases} \frac{x^2-2x}{x-2} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$ [3]

Find the value of k so that $f(x)$ is continuous at $x = 3$.

Ans: 3

13. 2069 Supp Q.No. 6a

Find the continuity or discontinuity of the function $f(x) = \frac{x^2-4}{x+2}$ at $x = 2$. [3]

Ans: Continuous

14. 2069 Q.No. 6a

Test the continuity of the function $f(x) = \begin{cases} \frac{x^2-4x}{x-4} & \text{when } x \neq 4 \\ 4 & \text{when } x = 4 \end{cases}$ at $x = 4$. [3]

Ans: Continuous

15. 2068 Q.No. 6a

Examine the continuity or discontinuity of the function $f(x) = 3x^2 - 8x + 6$ at $x = 1$ [3]

Ans: Continuous

16. 2067 Q.No. 6a

Examine the continuity or discontinuity of the function $f(x) = \frac{x^2-9}{x-3}$ at $x = 3$. [3]

Ans: Discontinuous

17. 2067 Supp Q.No. 6a

Examine the continuity or discontinuity or $f(x) = \frac{x^2-16}{x-4}$ at $x = 4$. [3]

Ans: Discontinuity

18. 2066 Q.No. 6a

Examine the continuity of the function: $f(x) = \begin{cases} \frac{x^2-4x}{x-4} & x \neq 1 \\ 3 & x = 1 \end{cases}$ at $x = 1$ [3]

Ans: Discontinuous

19. 2066 Old Q.No. 6b

Show that the function $f(x)$ given by

$f(x) = \begin{cases} kx-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ is continuous at the point $x = 2$ if $k = 1$ [3]

20. 2066 Supp Q.No. 6 a

Discuss the continuity of the function $f(x)$ at $x = 1$ where

$$\begin{aligned} f(x) &= -x \text{ when } x < 0 \\ &= x \text{ when } 0 < x < 1 \\ &= 2 - x \text{ when } x \geq 1 \end{aligned}$$

[3]
Ans: $f(x)$ is continuous at $x = 1$

21. 2065 Q.No. 6 a

If $f(x) = \frac{x^2 - 1}{x - 1}$, discuss the continuity of $f(x)$ at $x = 1$

[3]
Ans: Discontinuous

22. 2065 Old Q.No. 6 b

A function $f(x)$ is defined as follows: $f(x) = \begin{cases} \frac{x^2 - 4x}{x - 4}, & \text{when } x \neq 4 \\ = k, & \text{when } x = 4 \end{cases}$

Find the value of k so that the function $f(x)$ is continuous at $x = 4$.

[3]
Ans: 4

23. 2064 Q.No. 6 a

Examine the continuity of the function: $f(x) = \frac{2x + 3}{8x - 3}$ at $x = 1$

[3]
Ans: Continuous

24. 2064 Q.No. 6 b Old

Examine the continuity of the following function at the point $x = 5$.

$$f(x) = \begin{cases} \frac{x^2 - 25}{x - 5}, & \text{when } x \neq 5 \\ = 10, & \text{when } x = 5 \end{cases}$$

[3]
Ans: Continuous

25. 2063 Q.No. 6 b

Examine the continuity or the discontinuity of the function, $f(x) = \frac{3x + 2}{2x - 1}$ at $x = 1$

[3]
Ans: Continuous

26. 2062 Q.No. 6 b

Test for continuity the function, $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & \text{when } x \neq 3 \\ 6, & \text{when } x = 3 \end{cases}$

[3]
Ans: Continuous

27. 2061 Q.No. 6 b

Examine the continuity or the discontinuity of the function: $f(x) = \frac{x^2 - 4}{x + 2}$ at $x = 2$

[3]
Ans: Continuous

28. 2060 Q.No. 6 b

Find the point of discontinuity of the function $\frac{4x^2 - 16}{2x - 4}$, if any. Justify why the function is not continuous at that point.

[3]
Ans: $x = 2$

29. 2059 Q.No. 6 b

Examine the continuity and discontinuity of the function. $f(x) = \begin{cases} \frac{x^2 - 7x}{x - 7}, & x \neq 7 \\ = 5, & x = 7 \end{cases}$ at $x = 7$

[3]
Ans: Discontinuous

30. 2057 Q.No. 6 b

Examine the continuity and discontinuity of the function

$$f(x) = 3x^2 + 2x - 1 \text{ at } x = 2$$

[3]
Ans: Continuous

UNIT 9: DIFFERENTIATION

A. DIFFERENTIATION BY FIRST PRINCIPLES OR DEFINITION

1. 2073 Set C Q.No. 12

Find from first principles, the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

2. 2073 Set D Q.No. 12

Find from first principle, the derivative of $x^2 - 2x$.

$$\text{Ans: } 2(x - 1) \quad [5]$$

3. 2072 Supp Q.No. 12

Find from first principles, the derivative of $x^2 - 2x$.

$$\text{Ans: } 2(x - 1) \quad [5]$$

4. 2072 Set C Q.No. 12

Find, from the first principles, the derivative of $\frac{3x+5}{\sqrt{x}}$.

$$\text{Ans: } \frac{3x-5}{2x^{3/2}} \quad [5]$$

5. 2072 Set D Q.No. 12

Find, from the first principles, the derivative of $\frac{1}{1-x}$.

$$\text{Ans: } \frac{1}{(1-x)^2} \quad [5]$$

6. 2072 Set E Q.No. 12

Find the derivate from first principles, $y = \frac{1}{x}$.

$$\text{Ans: } -\frac{1}{x^2} \quad [5]$$

7. 2071 Supp Q.No. 12

Find from first principles the derivative of $\sqrt{x+1}$.

$$\text{Ans: } \frac{1}{2\sqrt{x+1}} \quad [5]$$

8. 2071 Set C Q.No. 12

Find from first principles, the derivative of $x(x+1)$.

$$\text{Ans: } 2x + 1 \quad [5]$$

9. 2071 Set D Q.No. 12

Find from first principle the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

10. 2070 Supp Q.No. 12

Find from first principles the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

11. 2070 Set C Q.No. 12

Find from first principles, the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

12. 2070 Set D Q.No. 12

Find the derivative of \sqrt{x} from the first principles.

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

13. 2069 Supp Q.No. 12

Find from first principles the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

14. 2069 Q.No. 12

Find from first principles the derivative of \sqrt{x} .

$$\text{Ans: } \frac{1}{2\sqrt{x}} \quad [5]$$

15. 2068 Q.No. 12

Find from first principles, the derivative of $\frac{1}{x+3}$.

$$\text{Ans: } -\frac{1}{(x+3)^2} \quad [5]$$

16. 2067 Q.No. 12

Find from the first principle the derivative of $\frac{1}{2x+3}$.

$$\text{Ans: } \frac{-2}{(2x+3)^2} \quad [5]$$

17. 2067 Supp Q.No. 12

Find from first principles the derivative of $x + \frac{1}{x}$.

$$\text{Ans: } 1 - \frac{1}{x^2} \quad [5]$$

18. 2066 Q.No. 12

Find from first principle, the differential coefficients of: $2x^2 + 3x + 1$.

$$\text{Ans: } 4x + 3 \quad [5]$$

19. 2066 Supp Q.No. 12

Find, from the principles the differential coefficient of $\frac{1}{\sqrt{2x+3}}$.

$$\text{Ans: } \frac{dy}{dx} = \frac{-1}{(2x+3)^{3/2}} \quad [5]$$

B. DIFFERENTIATION

1. 2073 Set C Q.No. 6b

Find the derivative of $x^2 + x$.

$$\text{Ans: } 2x + 1 \quad [3]$$

2. 2073 Set D Q.No. 6b

Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt[3]{x^2 - 2x + 1}}$.

$$\text{Ans: } \frac{-2}{3\sqrt[3]{(x-1)^5}} \quad [3]$$

3. 2072 Supp Q.No. 6b

Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt{x+a} - \sqrt{x}}$.

$$\text{Ans: } \frac{1}{2a} \left(\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x}} \right) \quad [3]$$

4. 2072 Set C Q.No. 6b

If $x = \frac{2a}{t}$ and $y = \frac{a}{t^2}$, prove that $\frac{dy}{dx} = \frac{1}{t}$.

[3]

5. 2072 Set D Q.No. 6b

Find the differential coefficient of $e^{2x} (1 + \ln x)$.

$$\text{Ans: } e^{2x} \left\{ \frac{1}{x} + 2(1 + \ln x) \right\} \quad [3]$$

6. 2072 Set E Q.No. 6b

Find $\frac{dy}{dx}$ when $x = 2$ at, $y = at^2$.

[3]

Ans: t

7. 2071 Supp Q.No. 6b

Find the derivative of $(x+1)(x^2+1)$.

$$\text{Ans: } 3x^2 + 2x + 1 \quad [3]$$

8. 2071 Set C Q.No. 6b

Find $\frac{dy}{dx}$ if $y = (x^2 + 3x)(5x^3 - 6x)$

$$\text{Ans: } 25x^4 + 60x^3 - 18x^2 - 36x \quad [3]$$

9. 2071 Set D Q.No. 6b

Find the derivate of: $\frac{1}{\sqrt{3x+1} - \sqrt{3x-2}}$

$$\text{Ans: } \frac{1}{2} \left(\frac{1}{\sqrt{3x+1}} + \frac{1}{\sqrt{3x-2}} \right) \quad [3]$$

10. 2070 Supp Q.No. 6b

Find $\frac{dy}{dx}$ when $x^2 + y^2 = 3y$

$$\text{Ans: } \frac{2x}{3-2y} \quad [3]$$

11. 2070 Set C Q.No. 6 b

Find $\frac{dy}{dx}$ if $y = z^3 + 2z + 1, x = z^2 + 2$.

$$\text{Ans: } \frac{3z^2 + 2}{2z} \quad [3]$$

12. 2070 Set D Q.No. 6 b

Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.

$$\text{Ans: } \frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax^2} \quad [3]$$

13. 2069 Supp Q.No. 6 b

Find $\frac{dy}{dx}$ if $y = \frac{x^2}{x-1}$.

$$\text{Ans: } \frac{x^2 - 2x}{(x-1)^2} \quad [3]$$

14. 2069 Q.No. 6b

Find the derivative of: $e^x \log x$.

$$\text{Ans: } \frac{e^x}{x} (1 + x \log x) \quad [3]$$

15. 2068 Q.No. 6b

Find $\frac{dy}{dx}$ if: $y = (x^2 + 5x)(3x^2 - x)$

$$\text{Ans: } 2x(6x^2 + 21x - 5) \quad [3]$$

16. 2067 Q.No. 6 b

Find the derivative of $\frac{1}{\sqrt{x+a} - \sqrt{x-b}}$

$$\text{Ans: } \frac{1}{2(a+b)} \left[\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x-b}} \right] \quad [3]$$

17. 2067 Supp Q.No. 6b

Find the derivative of $\sqrt{\frac{1+x}{1-x}}$

$$\text{Ans: } \frac{1}{(1-x)^2} \sqrt{\frac{1-x}{1+x}} \quad [3]$$

18. 2066 Q.No. 6 b

Find the derivative of: $e^{2x} \log x$.

$$\text{Ans: } \frac{e^{2x}}{x} (1 + 2x \log x) \quad [3]$$

19. 2066 Old Q.No. 7 a

Find $\frac{dy}{dx}$ when (i) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ (ii) $x = \frac{3at}{1+t^2}$, $y = \frac{3at^2}{1+t^2}$ [6]

Ans: (i) $-\left(\frac{y}{x}\right)^{\frac{1}{3}}$ (ii) $\frac{2t}{1-t^2}$

20. 2066 Supp Q.No. 6 b

Find $\frac{dy}{dx}$ when $ax^2 + 2hxy + by^2 = 0$ [3]

Ans: $-\frac{(ax + hy)}{hx + by}$

21. 2065 Q.No. 6 b

Find the derivative of: $y = \frac{1}{\sqrt[3]{x^2 - 2x + 1}}$ [3]

Ans: $\frac{-2}{3\sqrt[3]{(x-1)^3}}$

22. 2065 Q.No. 12

What do you mean by the derivatives of a function? [5]

Find $\frac{dy}{dx}$ of $y = \sqrt{\frac{1-x}{1+x}}$

Ans: $\frac{-1}{(1+x)\sqrt{(1-x^2)}}$

23. 2065 Old Q.No. 7 a

Find $\frac{dy}{dx}$ when (i) $y = \frac{1}{\sqrt{2x-3} - \sqrt{2x-5}}$ (ii) $x = \frac{3at}{1+t^2}$, $y = \frac{3at^2}{1+t^2}$ [6]

Ans: (i) $\frac{1}{2} \left[\frac{1}{\sqrt{2x-3}} + \frac{1}{\sqrt{2x-5}} \right]$ (ii) $\frac{2t}{1-t^2}$

24. 2064 Q.No. 6 b

Find the derivative of: $\frac{1}{\sqrt{x+a} - \sqrt{x-b}}$ [3]

Ans: $\frac{1}{2(a+b)} \left[\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x-b}} \right]$

25. 2064 Q.No. 12

Find $\frac{dy}{dx}$ of (i) $y = x^2 \log x$ (ii) $x = t^2 - 1$, $y = t^4 - 1$ [5]

Ans: (i) $x + 2x \log x$ (ii) $2t^2$

26. 2064 Q.No. 7 a Old

Find $\frac{dy}{dx}$ when (i) $y = \frac{1}{x + \sqrt{a^2 + x^2}}$ (ii) $x = at^3$, $y = 3at$ [6]

Ans: (i) $\frac{1}{a^2} \left(\frac{x}{\sqrt{a^2 + x^2}} - 1 \right)$ (ii) $\frac{1}{t^2}$

27. 2063 Q.No. 7 a

Find $\frac{dy}{dx}$ of the following.

(i) $y = \frac{1}{\sqrt{x+a} - \sqrt{x-b}}$ (ii) $x = 3at^2$, $y = at^3$ [6]

Ans: (i) $\frac{1}{2(a+b)} \left[\frac{1}{\sqrt{x+a}} + \frac{1}{\sqrt{x-b}} \right]$ (ii) $\frac{1}{2}$

28. 2062 Q.No. 7 a

Find $\frac{dy}{dx}$ (i) $y = \frac{x^2}{x-1}$ (ii) $y = z^3 + 2z + 1$, $x = z^2 + 2$ [6]

Ans: (i) $\frac{x^2 - 2x}{(x-1)^2}$ (ii) $\frac{3z^2 + 2}{2z}$

29. 2061 Q.No. 7 a

Find $\frac{dy}{dx}$ at (i) $y = \sqrt{ax^2 + bx + c}$ (ii) $x = t^2 - 1, y = t^4 - 1$ [6]

Ans: (i) $\frac{2ax + b}{2\sqrt{ax^2 + bx + c}}$; (ii) $2t(x + 1)$

30. 2060 Q.No. 7 a

Geometrically, interpret $\frac{dy}{dx}$. Also find $\frac{dy}{dx}$ when $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$. [6]

Ans: $\frac{t^2 + 1}{t^2 - 1}$

31. 2058 Q.No. 7 b

Differentiate the following by first principle: $y = 2x^2 + 3x + 1$. [6]

Ans: $4x + 3$

UNIT 10: APPLICATION OF DERIVATIVES

1. 2073 Set C Q.No. 13

The demand equation for a certain commodity is $P = \frac{1}{12}Q^2 - 10Q + 300$. [5]

Find the value of Q and the corresponding price P that maximizes the revenue.

Ans: $Q = 20$ and $P = \frac{400}{3}$

2. 2073 Set D Q.No. 13

The demand equation for a certain commodity is $P = \frac{1}{3}Q^2 - 10Q + 75$. [5]

Find the value of Q and the corresponding value of P that maximize the revenue.

Ans: $Q = 5, P = \frac{100}{3}$

3. 2072 Supp Q.No. 13

Given the demand function $P = 20 - Q$ and the total cost function $C = Q^2 + 8Q + 2$, determine the output Q and total profit under profit maximization. [5]

Ans: $Q = 3, \text{Profit} = 16$

4. 2072 Set C Q.No. 13

A company produces Q units of output at a total cost of $C = \frac{1}{4}Q^2 + 3Q + 100$. Verify that the minimum average cost is equal to the marginal cost at the level minimizing the average cost. [5]

5. 2072 Set D Q.No. 13

Find the minimum average cost if the cost function is given by $C = 36Q - 10Q^2 + 2Q^3$. Find also the marginal cost at the point at which the average cost is minimum. [5]

Ans: 23.5, 23.5

6. 2072 Set E Q.No. 13

The demand equation for a certain commodity is $P = \frac{Q^2}{3} - 10Q + 75$. Find the value of Q and the corresponding value of P that maximizes the revenue. [5]

Ans: $Q = 5$ and $P = \frac{100}{3}$

7. 2071 Supp Q.No. 13

A plant produces Q metric tonnes of steel per week at a total cost of Rs. $(\frac{1}{3}Q^3 - Q^2 - 452Q + 50)$. If the market price is fixed at Rs. 500 per metric tonnes, show that the plant should produce 8 metric tonnes per week to make the profit maximum. [5]

8. 2071 Set C Q.No. 13

The demand equation for a certain commodity is $p = \frac{1}{12}Q^2 - 10Q + 300$. ($0 \leq Q \leq 60$)

Find the value of Q and the corresponding price P that maximizes the revenue. [5]

Ans: $Q = 20, P = \frac{400}{3}$

9. 2071 Set D Q.No. 13

Find the maximum, minimum values and point of inflection of:

$$x^3 - 2x^2 - 4x - 1.$$

Ans: Max = 13/27 at $x = -2/3$; Min = -9 at $x = 2$; Point of inflection = 2/3 [5]

10. 2070 Supp Q.No. 13

The demand equation for a certain commodity is $P = \frac{1}{3}Q^2 - \frac{15}{2}Q + 50$. Find the value of Q and the corresponding value of P that maximizes the revenue. [5]

Ans: $Q = 5, P = \frac{125}{6}$

11. 2070 Set C Q.No. 13

A firm has a demand function $P = 108 - 5Q$ and cost function $C = -12Q + Q^2$. Find the price at which the profit is maximum. Find the maximum profit. [5]

Ans: 58 and 600

12. 2070 Set D Q.No. 13

Examine whether the function $f(x) = x^3 - 3x^2 - 24x + 8$ is increasing or decreasing at the points $x = 1$ and $x = 3$. Also find the point of inflection. [5]

Ans: Decreasing at $x = 1, x = 3$ and $x = 1$;

13. 2069 Supp Q.No. 13

A firm has a demand function $P = 108 - 5Q$ and the cost function $C = -12Q + Q^2$. Find the price at which the profit is maximum. Also find the maximum profit. [5]

Ans: 58 and 600

14. 2069 Q.No. 13

Given the demand function $P = 20 - Q$ and the total cost function $C = Q^2 + 8Q + 2$, determine the optimal output Q , price P and total profit under profit maximization. [5]

Ans: $Q = 3, P = 17, \text{profit} = 16$

15. 2068 Q.No. 13

A firm estimates that its daily total cost function is $C(x) = x^3 - 6x^2 + 13x + 15$ and its total revenue function is $R(x) = 28x$. Find the value of x that maximizes the daily profit. [5]

Ans: $x = 5$

16. 2067 Q.No. 13

If the revenue function is $R = Q - 3Q^2$ and cost function $C = Q^2 - 2Q$. Find the value of the maximum profit. [5]

Ans: $\frac{9}{16}$

17. 2067 Supp Q.No. 13

Find the points where the function $x^3 - 3x^2 - 9x$ is increasing or decreasing, also find the point of inflection of the function. [5]

Ans: Increasing at $x < -1, x > 3$ decreasing at $-1 < x < 3$; point of inflection is $x = 1$

18. 2066 Q.No. 13

The demand equation for a certain commodity is $P = \frac{1}{12}Q^2 - 10Q + 300$ ($0 \leq Q \leq 60$). Find the value of Q and the corresponding price P that maximize the revenue. [5]

Ans: $Q = 20; P = \frac{400}{3}$

19. 2066 Old Q.No. 8 a

The demand equation for a certain commodity is $P = \frac{1}{12}q^2 - 10q + 300$. Find the value of q and the corresponding price P that maximize the revenue. [6]

Ans: $q = 20; P = \frac{400}{3}$

20. 2066 Supp Q.No. 13

Find the value of x for which the function: $y = (x - 2)^3(x - 3)^2$ is a maximum or minimum. [5]

Ans: $x = 2, 3, \frac{13}{5}$

21. 2065 Q.No. 13

If the revenue function is $R = Q - 3Q^2$ and cost function is $C = Q^2 - 2Q$, find the value of maximum profit. [5]

$$\text{Ans: } \frac{9}{16}$$

22. 2065 Old Q.No. 7 b

The demand equation for a certain commodity is $P = \frac{1}{12} Q^2 - 10Q + 300$. Find the value of Q and the corresponding price P that maximize the revenue. [6]

$$\text{Ans: } Q = 20 \text{ and } P = \frac{400}{3}$$

23. 2064 Q.No. 13

Examine whether the function $f(x) = 2x^3 - 6x + 10$ is increasing or decreasing at $x = 2$, also, find the stationary points and the point of inflection (if any). [5]

Ans: Increasing at $x = 2$, Stationary points are $x = \pm 1$, Point of inflection $x = 0$

24. 2064 Q.No. 7 b Old

Given the demand function $P = 20 - Q$ and the total cost function $C = Q^2 + 8Q + 2$, determine the optimal output Q , price P , total profit and total revenue R under profit maximization. [6]

Ans: Optimal output (Q) = 3, Price (P) = 17, Profit (π) = 16 and Revenue (R) = 51

25. 2063 Q.No. 7 b

The demand function for a certain commodity is $P = \frac{1}{3} Q^2 - 10Q + 75$

Find the value of Q and the corresponding value of P that maximizes the revenue. [6]

$$\text{Ans: } Q = 5, P = 33\frac{1}{3}$$

26. 2062 Q.No. 7 b

The demand equation for a certain commodity is $P = \frac{1}{3} Q^2 - \frac{15}{2} Q + 50$

Find the value of Q and the corresponding value of P that maximizes the revenue. [6]

$$\text{Ans: } Q = 5, P = 20\frac{5}{6}$$

27. 2061 Q.No. 7 b

If the revenue function is $R = Q - 3Q^2$ and the cost function $C = Q^2 - 2Q$, find the value of the maximum profit. [6]

$$\text{Ans: } \frac{9}{16}$$

28. 2060 Q.No. 7 b

Find the points where the function $x^3 - 3x^2 - 9x$ is increasing or decreasing? Also find the stationary and point of inflection of the function. [6]

Ans: Increasing at $x < -1$, $x > 3$ decreasing at $-1 < x < 3$; stationary points are $x = -1$, $x = 3$, point of inflection is $x = 1$

29. 2059 Q.No. 7 a

Find the increasing or decreasing of the function, $f(x) = x^3 - 4x^2 + 5$ at $x = 1$ and $x = 4$. [6]

Ans: Decreasing at $x = 1$, increasing at $x = 4$

30. 2059 Q.No. 7 b

Given the demand function $P = 16 - Q$ and the cost functions, $C = 2 + Q^2$, find the value of maximum profit. [6]

$$\text{Ans: } 30$$

31. 2058 Q.No. 7 a

Find the point of inflection-of: $y = x^3 - 2x^2 + 3x$ [6]

$$\text{Ans: } 2/3$$

32. 2057 Q.No. 7 a

Find the increasing or decreasing of the function $f(x) = 2x^3 - x^2 + 5$ at $x = 1$ and $x = 3$ [6]

Ans: Increasing at $x = 1$ and $x = 3$

33. 2057 Q.No. 7 b

Find the maximum, minimum values and points of inflection of $y = x^3 - 2x^2 - 4x - 1$. [6]

Ans: max = 13/27, min = -9, Point of inflection = 2/3

UNIT 11: INTEGRATION

A. INTEGRATION

1. 2073 Set C Q.No. 7a

Integrate: $\int \frac{3x^2}{\sqrt{x^3-9}} dx$. [3]

Ans: $2\sqrt{x^3-9} + C$

2. 2073 Set D Q.No. 7a

Evaluate: $\int \frac{1}{x} \left(x + \frac{1}{x} \right) dx$. [3]

Ans: $x - \frac{1}{x} + C$

3. 2072 Supp Q.No. 7a

Evaluate: $\int \frac{3x^2 + 4x + 6}{4x} dx$. [3]

Ans: $\frac{3x^2}{8} + x + \frac{5}{4} \log x + k$

4. 2072 Set C Q.No. 7a

Evaluate the integral $\int \frac{6x^2 - 6x + 5}{\sqrt{2x^3 - 3x^2 + 5x - 7}} dx$ [3]

Ans: $2\sqrt{2x^3 - 3x^2 + 5x - 7} + C$

5. 2072 Set D Q.No. 7a

Evaluate $\int \frac{6x-7}{(3x^2-7x+5)^3} dx$ [3]

Ans: $-\frac{1}{2(3x^2-7x+5)^2} + C$

6. 2072 Set E Q.No. 7a

Evaluate: $\int \frac{x^3}{\sqrt{x^4-1}} dx$ [3]

Ans: $\frac{1}{2}\sqrt{x^4-1} + c$

7. 2071 Supp Q.No. 7a

Evaluate: $\int \frac{4x+1}{2x^2+x+1} dx$. [3]

Ans: $\log(2x^2+x+1) + C$

8. 2071 Set C Q.No. 7a

Evaluate: $\int \frac{3x+1}{x-2} dx$ [3]

Ans: $3x + 7 \log(x-2) + C$

9. 2071 Set D Q.No. 7a

Evaluate: $\int \frac{3x^2}{x^3-1} dx$ [3]

Ans: $\log(x^3-1) + C$

10. 2070 Supp Q.No. 7a

Evaluate: $\int \frac{2x+3}{2x-5} dx$ [3]

Ans: $x + 4 \log(2x-5) + C$

11. 2070 Set C Q.No. 7 a

Evaluate: $\int (2x - 3)(x + 4) dx$.

[3]

Ans: $\frac{2}{3}x^3 + \frac{5}{2}x^2 - 12x + C$

12. 2070 Set D Q.No. 7 a

Evaluate: $\int \frac{3x+1}{x-2} dx$.

[3]

Ans: $3x + 7 \log(x - 2) + c$

13. 2069 Supp Q.No. 7 a

Evaluate: $\int \frac{x+2}{x-2} dx$

[3]

Ans: $x + 4 \log(x - 2) + C$

14. 2069 Q.No. 7 a

Evaluate: $\int (x + \frac{1}{x})^2 dx$

[3]

Ans: $\frac{x^3}{3} + 2x - \frac{1}{x} + c$

15. 2068 Q.No. 7 a

Evaluate: $\int \frac{x^2-4}{x+2} dx$

[3]

Ans: $\frac{x^2}{2} - 2x + C$

16. 2067 Q.No. 7 a

Find the indefinite integral $\int \frac{3x+1}{x-2} dx$

[3]

Ans: $3x + 7 \log(x - 2) + c$

17. 2067 Supp Q.No. 7 a

Evaluate: $\int \frac{2x+5}{\sqrt{x^2+5x}} dx$

[3]

Ans: $2\sqrt{x^2+5x} + C$

18. 2066 Q.No. 7 a

Evaluate the integral: $\int (2x+1)\sqrt{x^2+x} dx$

[3]

Ans: $\frac{2}{3}(x^2+x)^{3/2} + C$

19. 2066 Old Q.No. 7 b

Evaluate: $\int_0^1 \frac{3dx}{\sqrt{3x+1}}$

[3]

Ans: 2

20. 2066 Supp Q.No. 7 a

Evaluate: $\int \frac{3x+4}{\sqrt{x+1}} dx$

[3]

Ans: $2(x+2)(x+1)^{1/2} + C$

21. 2065 Q.No. 7 a

Evaluate: $\int \frac{3x+1}{x-2} dx$

[3]

Ans: $3x + 7 \log(x - 2) + c$

22. 2065 Old Q.No. 8 a

Evaluate: $\int_0^a x \sqrt{x^2 + a^2} dx$ [3]

Ans: $\frac{a^3}{3} (2\sqrt{2} - 1)$

23. 2064 Q.No. 7 a

Evaluate the indefinite integral: $\int \frac{ax^2 + bx + c}{x} dx$ [3]

Ans: $\frac{ax^2}{2} + bx + c \log x + k$

24. 2064 Q.No. 8 a Old

Evaluate: $\int_0^a \frac{x}{\sqrt{a^2 - x^2}} dx$ [3]

Ans: a

25. 2063 Q.No. 8 a

Integrate: $\int \frac{1}{x} \left(x + 1 + \frac{1}{x} \right) dx$ [3]

Ans: $x + \log x - \frac{1}{x} + C$

26. 2062 Q.No. 8 a

Integrate: $\int \frac{1}{\sqrt{x+a} - \sqrt{x-a}} dx$ [3]

Ans: $\frac{1}{3a} [(x+a)^{3/2} + (x-a)^{3/2}] + C$

27. 2061 Q.No. 8 a

Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$ [3]

Ans: $\frac{x^2}{2} + 2x + \log x + C$

28. 2060 Q.No. 8 a

Integrate: $\int (2x + 3) \sqrt{2x - 1} dx$ [3]

Ans: $\frac{(2x-1)^{5/2}}{5} + \frac{4(2x-1)^{3/2}}{3} + C$

29. 2059 Q.No. 8 a

Integrate the following: $\int \frac{5x + 4}{\sqrt{x+1}} dx$ [3]

Ans: $\frac{10}{3} (x+1)^{3/2} - 2 (x+1)^{1/2} + C$

30. 2059 Q.No. 8 b

$\int_0^1 x^3 \sqrt{2 + 3x^4} dx$ [3]

Ans: $\frac{1}{18} (5\sqrt{5} - 2\sqrt{2})$

31. 2058 Q.No. 8 a

Integrate the following: $\int \frac{x+2}{2x-5} dx$ [3]

$$\text{Ans: } \frac{x}{2} + \frac{9}{4} \log(2x-5) + C$$

32. 2057 Q.No. 8 b

Integrate the following $\int_0^1 (2t^2 - 3)^{3/2} t \cdot dt$ [3]

$$\text{Ans: } \frac{1}{6} [(-1)^{3/2} - (-3)^{3/2}]$$

B. APPLICATION OF INTEGRATION

1. 2073 Set C Q.No. 7b

If the marginal cost of a product is given by $MC = 36 - 20x + 6x^2$, and the initial cost is Rs 30, find the total cost and average cost function. [3]

$$\text{Ans: } C = 2x^3 - 10x^2 + 36x + 30; AC = 2x^2 - 10x + 36 + \frac{30}{x}$$

2. 2073 Set D Q.No. 7b

If for 20 units of production, total revenue is 200, find the total revenue function given that $MR = 10 + 20x - 3x^2$. [3]

$$\text{Ans: } 10x + 10x^2 - x^3 + 4000$$

3. 2072 Supp Q.No. 7b

If for 2 units of production, total revenue is 200, find the total revenue function given that marginal revenue function $(MR) = 10 + 20x - 3x^2$. [3]

$$\text{Ans: } 10x + 10x^2 - \frac{3x^3}{4} + 152$$

4. 2072 Set C Q.No. 7b

The marginal cost of a product is found to be $M_c = 2000 - 40x + 3x^2$, where x is the number of units produced. The fixed cost of production is Rs.18,000. Find the cost function and the average function. [3]

$$\text{Ans: Total cost (C)} = x^3 - 20x^2 + 2000x + 18000 \text{ and Average cost } \left(\frac{C}{x}\right) = x^2 - 20x + 2000 + \frac{18000}{x}$$

5. 2072 Set D Q.No. 7b

The marginal cost function of manufacturing x shoes is $6 + 10x - 6x^2$. The total cost function of producing a pair of shoes is Rs.12, find the total cost and the average cost functions. [3]

$$\text{Ans: Total cost (C)} = 6x + 5x^2 - 2x^3 - 4 \text{ and Average cost } \left(\frac{C}{x}\right) = 6 + 5x - 2x^2 - \frac{4}{x}$$

6. 2072 Set E Q.No. 7b

If the marginal revenue function for output x is $MR = x^3 + 4x - 2$, find the total revenue function. [3]

$$\text{Ans: } \frac{x^4}{4} + 2x^2 - 2x$$

7. 2071 Supp Q.No. 7b

The marginal cost function of manufacturing x units of commodity is $6 + 10x - 6x^2$. If cost of producing one unit is Rs. 12, find the total average cost function. [3]

$$\text{Ans: } 6 + 5x - 2x^2 + \frac{3}{x}$$

8. 2071 Set C Q.No. 7b

If for 2 units of production, the total revenue is 200, find the total revenue function given that marginal revenue function $(MR) = 10 + 20x - 3x^3$. [3]

$$\text{Ans: } 10x + 10x^2 - \frac{3x^4}{4} + 152$$

9. 2071 Set D Q.No. 7b

The marginal cost of a product is $x^2 - 3x + 2$ where x is the number of units produced. If the cost of producing one unit is Rs 10, find the total cost function. [3]

$$\text{Ans: } \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \frac{55}{6}$$

10. 2070 Supp Q.No. 7b

The marginal cost function of a firm is $Q^2 + Q + 2$, where Q is output. Find the total cost function and average cost function when the fixed cost is Rs. 50. [3]

$$\text{Ans: } C = \frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + 50; \text{ AC} = \frac{Q^2}{3} + \frac{Q}{2} + 2 + \frac{50}{Q}$$

11. 2070 Set C Q.No. 7 b

If the marginal cost function of a product is $x^2 - 3x + 2$ where x is the number of units produced and if the cost of producing one unit is Rs. 10, find the total cost function. [3]

$$\text{Ans: } \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \frac{55}{6}$$

12. 2070 Set D Q.No. 7 b

The marginal revenue function is $4 - 8x - 3x^2$, x being the output. Find the total revenue function if for 3 units of production, the total revenue is Rs. 100. [3]

$$\text{Ans: } -x^3 - 4x^2 + 4x + 151$$

13. 2069 Supp Q.No. 7 b

The marginal revenue function is $4 - 8x - 3x^2$, x being the output. Find the total revenue function if for 3 units of production, the total revenue is 100. [3]

$$\text{Ans: } -x^3 - 4x^2 + 4x + 151$$

14. 2069 Q.No. 7b

The marginal cost of a product is $MC = 36 - 20x + 6x^2$ and the initial cost is Rs. 20. Find the total cost function. [3]

$$\text{Ans: } 2x^3 - 10x^2 + 36x + 20$$

15. 2068 Q.No. 7b

If the marginal cost (MC) for a product is $6x + 4$ and the cost of producing 100 items is Rs. 31,400. Find the total cost function and also the fixed cost. [3]

$$\text{Ans: } 3x^2 + 4x + 1000 \text{ and } 1000$$

16. 2067 Q.No. 7 b

If the marginal cost of a product is given by $Mc = 36 - 20x + 6x^2$ and the initial cost is Rs. 20, find the total cost and average cost function. [3]

$$\text{Ans: Total cost (C)} = 2x^3 - 10x^2 + 36x + 20 \text{ and Average cost } \left(\frac{C}{x}\right) = 2x^2 - 10x + 36 + \frac{20}{x}$$

17. 2067 Supp Q.No. 7b

The marginal revenue function is $4 - 8x - 3x^2$, x is being the output. Find the total revenue function if for 3 units of production the total revenue is 100. [3]

$$\text{Ans: } -x^3 - 4x^2 + 4x + 151$$

18. 2066 Q.No. 7 b

The marginal cost function $C'(x)$ for the production of x is $C'(x) = 5 + 8x$. Find the cost function $C(x)$ if $C(0) = 40$, i.e. when $x = 0$ the fixed cost $C(0) = 40$. [3]

$$\text{Ans: } C(x) = 4x^2 + 5x + 40$$

19. 2066 Old Q.No. 8 b

The annual rate of repair cost is given by $\frac{dc}{dt} = 10t + 100$ where t is the age of the machine in years and $\frac{dc}{dt}$ is in rupees per year. Find the total repair cost after 5 years. [3]

$$\text{Ans: Rs. 625}$$

20. 2066 Supp Q.No. 7 b

If the marginal cost of a product is given by $MC = 36 - 20x + 6x^2$ and the initial cost is Rs. 20, find the total cost and average cost function. [3]

$$\text{Ans: Total cost (C)} = 2x^3 - 10x^2 + 36x + 20 \text{ and Average cost } \left(\frac{C}{x}\right) = 2x^2 - 10x + 36 + \frac{20}{x}$$

21. 2065 Q.No. 7 b

If the marginal cost of a product is given by $MC = 36 - 20x + 6x^2$ and the initial cost is Rs. 20, find the total cost and average cost function. [3]

$$\text{Ans: Total cost (C)} = 2x^3 - 10x^2 + 36x + 20 \text{ and Average cost } \left(\frac{C}{x}\right) = 2x^2 - 10x + 36 + \frac{20}{x}$$

22. 2065 Old Q.No. 8 b

If the marginal cost of a product is given by $MC = 36 - 20x + 6x^2$ and the initial cost is Rs. 20, find the total cost and average cost function. [3]

$$\text{Ans: Total cost (C)} = 2x^3 - 10x^2 + 36x + 20 \text{ and Average cost } \left(\frac{C}{x}\right) = 2x^2 - 10x + 36 + \frac{20}{x}$$

23. 2064 Q.No. 7 b

If the marginal cost of a product is $x^2 - 3x + 2$ where x is the number of units produced, and if the cost of producing one unit is Rs. 10, find the total cost function. [3]

$$\text{Ans: } \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \frac{55}{6}$$

24. 2064 Q.No. 8 b Old

If the marginal cost function for a product is $MC = 6x + 4$ and the cost of producing 100 items is Rs. 31,400; find the fixed cost and the total cost function. [3]

$$\text{Ans: Rs. 1,000 and } 3x^2 + 4x + 1000$$

25. 2063 Q.No. 8 b

If the marginal cost of a product is given by $MC = 8 + 6x - 3x^2$ and the fixed cost is Rs. 150, find the cost function. [3]

$$\text{Ans: } -x^3 + 3x^2 + 8x + 150$$

26. 2062 Q.No. 8 b

If for 2 units of production total revenue is 300, find the total revenue function given that $MR = 10 + 20x - 3x^2$. [3]

$$\text{Ans: } -x^3 + 10x^2 + 10x + 248$$

27. 2061 Q.No. 8 b

The marginal revenue function is $4 - 8x - 3x^2$, x being the output. Find the total revenue function if for 3 units of production, the total revenue is 100. [3]

$$\text{Ans: } -x^3 - 4x^2 + 4x + 151$$

28. 2060 Q.No. 8 b

The marginal cost function of a firm be $Q^2 + Q + 2$, where Q is output. Find the total cost function and average cost function when fixed cost is Rs. 50. [3]

$$\text{Ans: } C = \frac{Q^3}{3} + \frac{Q^2}{2} + 2Q + 50 \text{ and } \frac{C}{Q} = \frac{Q^2}{3} + \frac{Q}{2} + 2 + \frac{50}{Q}$$

29. 2058 Q.No. 8 b

If the marginal cost of a product is $x^2 - 3x + 2$ where x is the no. of units produced. If the cost of producing one unit is Rs. 10, find the total cost function. [3]

$$\text{Ans: } \frac{x^3}{3} - \frac{3x^2}{2} + 2x + \frac{55}{6}$$

30. 2057 Q.No. 8 a

Find the total revenue function and demand function when the marginal revenue function for output x is $MR = x^3 - 3x^2 - 4x - 2$. [3]

$$\text{Ans: } R = \frac{x^4}{4} - x^3 - 2x^2 - 2x \text{ and } D = \frac{x^4}{4} - x^2 - 2x - 2$$

UNIT 12: LINEAR PROGRAMMING PROBLEMS

1. 2073 Set C Q.No. 14

Find the extreme values of the objective function $F = 16x - 2y + 40$ subject to constraints $3x + 4y \leq 24$, $0 \leq y \leq 4$ and $0 \leq x \leq 7$. [5]

$$\text{Ans: Max. } F = 152 \text{ at } (7, 0); \text{ Min. } F = 32 \text{ at } (0, 4)$$

2. 2073 Set D Q.No. 14

Find the maximum value of the objective function, $Z = 9x + 7y$ subject to $x + 2y \leq 7$; $x - y \leq 4$; $x \geq 0$, $y \geq 0$ [5]

$$\text{Ans: Max. value} = 52 \text{ at } (5, 1)$$

3. 2072 Supp Q.No. 14

Find the minimum value of the objective function $F = 7x + 6y$ subject to $4x - 9y \leq 36$, $x \geq 9$, $y \leq 4$ [5]

$$\text{Ans: Minimum value: } 63 \text{ at } (9, 0)$$

4. 2072 Set C Q.No. 14

Minimize $F(x, y) = 100x + 600y$, subject to [5]
 $x + y \leq 10$, $x + 3y \leq 16$, $x \geq 3$, $y \geq 2$.

Ans: Min. value of $F = 1500$ at (3, 2)

5. 2072 Set D Q.No. 14

Maximize the objective function $F(x, y) = 5x + 2y$ subject to the constraints: [5]
 $2x + y \leq 4$, $x - 2y \leq 2$, $x \geq 0$, $y \geq 0$

Ans: Max. value of $F = 10$ at (2, 0)

6. 2072 Set E Q.No. 14

Find the extreme values of the function F defined by $F = 10x + 15y$ subject to constraints [5]
 $x + 2y \leq 20$, $x + y \leq 16$, $x \geq 0$, $y \geq 0$.

Ans: Max. value of $F = 180$ at (12, 4) and Min. value of $F = 0$ at (0, 0)

7. 2071 Supp Q.No. 14

Maximize the objective function $Z = 60x + 15y$ subject to constraints $x + y \leq 50$, $3x + y \leq 90$, $x \geq 0$, $y \geq 0$. [5]

Ans: Max. value of $Z = 1800$ at (30, 0)

8. 2071 Set C Q.No. 14

Find the maximum value of the objective function. [5]
 $Z = 5x + 8y$ subject to $2x + 5y \leq 15$, $x \geq 2$, $y \geq 1$.

Ans: Max. value of $F = 33$ at (15, 1)

9. 2071 Set D Q.No. 14

Maximize and Minimize the objective functions $F = x + 2y$ subjected to the constraints. [5]
 $2x + y \leq 14$, $x + 2y \leq 10$ and $x \geq 0$, $y \geq 0$.

Ans: Max. value = 10 at (6, 2) and (0, 5), Min. value = 0 at (0, 0)

10. 2070 Supp Q.No. 14

Maximize $z = 10x + 15y$ subject to the constraints. $x + 2y \leq 25$; $2x + y \leq 20$; $x \geq 0$, $y \geq 0$ [5]

Ans: Max. $Z = 200$ at (5, 10)

11. 2070 Set C Q.No. 14

Find the maximum value of the objective function. [5]
 $G = x + 2y$ subject to $x + y \geq 2$, $2x - y \leq 4$, $y \leq 2$.

Ans: Max. value 7 at (3, 2) and Min value 2 at (2, 0)

12. 2070 Set D Q.No. 14

Minimize $z = x + y$ subject to the constraints. $3x + 2y \geq 12$, $x + 3y \geq 9$, $x \geq 0$, $y \geq 0$. [5]

Ans: Min. value of $z = \frac{33}{7}$ at $(\frac{18}{7}, \frac{15}{7})$.

13. 2069 Supp Q.No. 14

Maximize $Z = 6x + 5y$ subject to $x + y \leq 6$, $x - y \geq -2$, $x \geq 0$, $y \geq 0$. [5]

Ans: Max. value of $z = 36$ at (6, 0)

14. 2069 Q.No. 14

Solve the following L.P. problem graphically. Maximize $Z = x + y$ subject to constraints. [5]
 $x + y \geq 3$, $2x + 3y \leq 18$, $x \leq 6$, $x \geq 0$, $y \geq 0$.

Ans: Max. value = 8 at (6, 2)

15. 2068 Q.No. 14

Find the maximum value of the objective function. [5]

$Z = 6x + 15y$ subject to:
 $4x - 3y \geq -15$; $x \leq 3$; $y \geq 5$

Ans: Max. value of $z = 153$ at (3, 9)

16. 2067 Q.No. 14

Maximize and minimize the objective function $F = x + 2y$ subject to the constraints [5]

$x + y \geq 2$; $2x - y \leq 4$; $y \leq 2$.

Ans: Max value 7 at (3, 2) and Min value 2 at (2, 0)

17. 2067 Supp Q.No. 14

Maximize and minimize $G = x + 2y$ subject to the constraints: [5]
 $x + y \geq 2$; $2x - y \leq 4$; $y \leq 2$.

Ans: Max value 7 at (3, 2) and Min value 2 at (2, 0)

18. 2066 Q.No. 14

Find the extreme values of the function $F = 10x + 15y$ over the convex polygon determined by the system of inequalities: $x + 2y \leq 20$, $x + y \leq 16$, $x \geq 0$, $y \geq 0$. [5]

Ans: Max: 180 at (12, 4); Min: 0 at (0, 0)

19. 2066 Supp Q.No. 14

Find the extreme values of the function $\phi(x, y) = 34x + 6y$ subject to $x + y \leq 6$, $x + y \geq 1$, $1 \leq x \leq 3$. [5]
 Ans: maximum value of $\phi = 120$ at (3, 3) and the minimum value of $\phi = 34$ at (1, 0)

20. 2065 Q.No. 14

Maximize $Z = 2x + 3y$ subject to the constraints: [5]
 $2x + y \leq 14$; $x + 2y \leq 10$; $x, y \geq 0$

Ans: $Z = 18$ at (6, 2)

21. 2064 Q.No. 14

Draw the graph of the following inequalities: $6x + 5y \leq 30$; $x \geq 1$, $y \geq 2$
 Find the feasible region and also find the vertices of the feasible region.. [5]

Ans: A (1, 2) B (10/3, 2) and C (1, 24/5)

UNIT 13: MEASURES OF DISPERSION

1. 2073 Set C Q.No. 8a

Two factories A and B pay their employees as follows: [3]
 $\bar{X}_A = 34.5$ and $\sigma_A = 5$; $\bar{X}_B = 28.5$ and $\sigma_B = 4.5$. Determine which factory has greater variability in wages.

Ans: Factory B

2. 2073 Set C Q.No. 15

Find the standard deviation for the following distribution. [5]

Age	20-25	25-30	30-35	35-40	40-45	45-50
N. of persons	170	110	80	45	40	45

Ans: 8.23

3. 2073 Set D Q.No. 8a

Find the quartile deviation from the following data: 158, 156, 162, 161, 163, 164, 165, 159 [3]
 Ans: 2.75

4. 2073 Set D Q.No. 15

Calculate the standard deviation from the following data: [5]

Profit (in Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	8	13	16	8	5

Ans: 11.88

5. 2072 Supp Q.No. 8a

Obtain the mean deviation from mean for the following data: [3]

Height (in cms)	10	20	30	40
No. of plants	2	3	9	21

Ans: 7.2

6. 2072 Supp Q.No. 15

Find the standard deviation of the following frequency distribution. [5]

Profit (in Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	8	13	16	8	5

Ans: Rs. 11.88

7. 2072 Set C Q.No. 8a

From the following data, obtain Q.D. and the coefficient of Q.D.: [3]
 20, 25, 23, 18, 17, 26, 22

Ans: 3.5; 0.162