10. MATHS EDUCATION

(a) Algebra & Vector Analysis (Math. Ed. 331)

Exam 2068

Group 'A' 20

Attempt ALL the questions. Tick (√) the best answers.

1. Which of the following is an algebraic structure?
   a) (N, +)
   b) (Q, ×)
   c) (1, –)
   d) all of the above

2. Which of the following is true?
   a) a transposition is always an even permutation
   b) a transposition if always an odd permutation
   c) a transposition is even and odd both permutation
   d) the identity permutation is also a transposition

3. Which of the following is a trivial action?
   a) G be a group and A be a non–empty set then the action ϕ: G × A → A is defined by ϕ(g,a) = ga = a∀g∈G, a ∈ A.
   b) If Z be the group of integers then the action Z × Z → Z is defined by ϕ(z,a) = z + a∀z, a∈Z
   c) V be a vector space over a field F and F* (set of all non–zero elements of E) then the action ϕ: F* × V → V is defined by ϕ(s,v) = sv∀s∈F*, v∈ V
   d) none of the above.

4. Which of the following is not true statement?
   a) Every group of prime order is cyclic
   b) If G is finite group of order n and a ∈ G then $a^n = e$
   c) A finite group of prime order has no proper subgroups
   d) all of the above

5. Which of the following may not be the 4 order of a simple group?
   a) 2
   b) 3
   c) 4
   d) 5

6. Which of the following is the correct statement?
   a) the relation of isomorphism in the set of groups, is an equivalence relation.
   b) every finite cyclic group is isomorphic to (Z, +)
   c) any two cyclic groups are isomorphic
   d) there exists at least one subgroup of (Z, +) which is not normal.

7. Which of the following group is cyclic?
   a) $Z_9 \times Z_{14}$
   b) $Z_{12} \times Z_{49}$
   c) $Z_{15} \times Z_{28}$
   d) all of the above

8. A ring (R,+, .) with multiplication defined by ab = 0∀a, b ∈ R is called
   a) zero ring
   b) trivial ring
   c) division ring
   d) ring without zero divisors

9. The characteristics of the ring (R, +, .) of real numbers is
   a) 0
   b) 1
   c) 2
   d) ∝

10. If R is a field then which of the following is maximal ideal in R?
    a) {0}
    b) R
    c) an ideal M s,t. M≠ R
    d) there exists no maximal ideal MR

11. If R be a commutative integral domain and a, b, u ∈ R then which of th4e following may not be true?
    a) a|b, iff(b) < (a)
    b) a and b are associates, iff (a) = (b)
    c) u|r ∀ r∈R
    d) "a is an associated of b" is an equivalence relation on R

12. Let R be a Euelidean domain and a (≠0)∈R then which of the following is true?
    a) d(a) ≤ d(I) ∀a∈R
    b) d(I) ≤ d(a) ∀a∈R

c) $d(a) = d(I)\ \forall a \in R$ d) none of the above.

13. Which of the following is the remainder when $3x^4 - 5x^3 + 10x^2 + 11x - 61$ divided by $x - 3$?
a) –61 b) 77
c) 22 d) 170

14. The number of negative roots of the equation $x^5 + x^3 - 2x^2 + x - 2 = 0$ cannot exceed
a) 0 b) 2
c) 3 d) 4

15. Which of the following statements is true?
a) every finite extension on E of a field F is an algebraic extension of F
b) the filed R of real numbers is a finite extension of R its / and F
c) the field C of complex numbers is a finite extension of R of real numbers
d) all of the above

16. If A is non–singular square matrix then which of the following is not true?
a) A^{-1} also non–singular b) $|A| \neq 0$
c) $(A^{-1})^{-1} = A^{-1}$ d) $(A^n)^{-1} = (A^{-1})^n$

17. Let W_1 and W_2 be two subspaces of a vector space V over the field F then $W_1 \cup W_2$ is a subspace of V if
a) $W_1 = W_2$ b) $W_1 \cup W_2$ is a subspace of $W_1 \subset W_2$
c) $W_2 \subset W_1$ d) all of the above

18. Let V be real vector space3 and $u, v, w \in V$ and c is the real number. Which of the following is not a necessary condition for the scalar product on V?
a) $\langle u, v\rangle = \langle v, u\rangle$ b) $\langle u, v + w\rangle = \langle v, u\rangle + \langle u, w\rangle$
c) $\langle cu, v\rangle = c\langle u, v\rangle$ d) $\langle u, v\rangle \geq 0$

19. If $\vec{v}$ be vector function of a scalar variables t then $\vec{v}$ is said to be irrotational if
a) grad $\vec{v} = 0$ b) div $\vec{v} = 0$
c) curl $\vec{v} = 0$ d) $\vec{v}$ is a constant function

20. If $\vec{i}$, $\vec{j}$ and $\vec{k}$ are the orthogonal unit vectors then which of the following is the reciprocal of $\vec{v}$?
a) $\vec{i}$ b) $\vec{j}$
c) $\vec{k}$ d) $\vec{j}$ and $\vec{k}$ both

Attempt ALL the questions.

Group 'B' 8×7=56

1. Define cyclic group. Prove that the set of cube roots of unity forms a cyclic group with respect to the multiplication operation.

2. Let G be group and H be subgroup of G, K a normal subgroup of G. Prove that $\dfrac{H}{H \cap K} \cong \dfrac{HK}{K}$

OR

If G_1 and G_2 be two cyclic groups of order m and n respectively, prove that $G_1 \times G_2$ is cyclic if and only if m and n are respectively prime.

3. Prove that every finite integral domain is field.

4. Define ring homomorphism. If $f: R \to \bar{R}$ be ring homomorphism of R into $\bar{R}$, then prove that F is 1 – 1 if and only if ker. $f = \{0\}$

OR

Define irreducible element in an integral domain. Prove that every associate of an irreducible element of an integral domain R is irreducible.

5. Define polynomial equation. Show that the equation $x^5 + x^3 - 2x^2 + x - 2 = 0$ has at least one pair of imaginary roots.

6. Let F, E and K be fields satisfying $F \subseteq E \subseteq K$. If [K : E] and [E : F] are finite, prove that [K : F] is finite and [K : F] = [K : E][E : F]

OR

If A and B be two matrices conformable for the sum and product then show that $(AB)^T = B^T A^T$.

7. Define direct sum of subspaces of a vector spaces. Prove that vector space V is the direct sum of its subspaces U and W if and only if

(i) V = U + W and

(ii) $U \cap W = \{0\}$

8. If $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that $\vec{a} = a_1\vec{l} + a_2\vec{m} + a_3\vec{n}$, $\vec{b} = b_1\vec{l} + b_2\vec{m} + b_3\vec{n}$ and $\vec{c} = c_1\vec{l} + c_2\vec{m} + c_3\vec{n}$ prove that

$$\left[\vec{a}\ \vec{b}\ \vec{c}\right] \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_3 & c_3 \end{vmatrix} \left[\vec{l}\ \vec{m}\ \vec{n}\right]$$

Group 'C'

2×12=24

9. Prove the following

(i) Let (G,0) be a finite group and (H,0) be a subgroup of G, then 0(H) divides 0(G).

(ii) A finite group of prime order has no proper subgroup.

10. Prove that every principal ideal R is a unique factorization domain.

OR

Let V be a vector space over the field F and $(v_1, v_2, \ldots\ldots v_n)$ be a basis of V. If $w_1, w_2, \ldots\ldots w_n$ be elements of V within n > m, prove that $w_1, w_2, \ldots\ldots w_n$ are linearly independent.

Exam 2069

Group "A"

20

Attempt all the questions. Tick (√) the best answers

1. If (G,o) be a group and a, b, c ∈ G, which ofthe following property is not true?
   a. $(a^{-1})^{-1} = a$ b. $(a \circ b)^{-1} = a^{-1}$
   c. $(a \circ b)^2 = a^2 \circ b^{-1}\ \forall\ a, b \in g \Rightarrow G$ is abelian d. $a^2 = e\ \forall a \in G \Rightarrow G$ is abelian
2. Which of the following is true?
   a. The product of two even permutations is an odd permutation
   b. the product of two odd permutation is an even permutation
   c. the product of an even permutation and an odd permutation
   d. A transposition is always an even permutation
3. Which of the following element of S3=1(11(123), (132), (12), (13) 1 is not its own inverse?
   a. (12) b. (23)
   c. (13) d. (123)
4. Which is the following statements is true?
   a. If H is a subgroup of a group G than (H)=H
   b. If A and B are two subsets of a group with A ≤ then (A)<(b)
   c. The subgroup generated by two distinct elements of order 2 in S_3 is all of S_3
   d. All of the above
5. Which of the following may be the order of non-abelian group?
   a. 6 b. 5
   c. 4 d. 3
6. Let G be a group acting on set S and s is some fixed element of S then the set Gs={g∈ G: g s=s} is called
   a. orbit of G b. stabilizer of s
   c. normalizes of s d. centralizer of s
7. Which of the following wing rroup is cyclic?
   a. $Z_2 \times Z_4$ b. $Z_2 \times z_3$
   c. $z_2 \times z_6$ d. $Z_3 \times Z_6$
8. Which of the following rings is an integraldomain?
   a. the ring of residue classes modulo 4, b. the ring of residue classes modulo 5
   c. the ring of residue classes modulo 6 d. the ring of residue classes modulo 9
9. The ring(R,+,-) satisfying a ∀a ∈ R is called
   a. idempotent ring b. nilpotent ring

c. Boolean ring
d. trivial ring

10. Which of the following is, true. statement?
a. Every subring of a ring R is an ideal of R
b. Every ideal of a ring R is a subring of R
c. The product of two ideals of a ring may not be an ideal
d. If I is a proper ideal of a ring R then every element of I has multiplicative inverse.

11. Let R be a commutative integral domain and let a, b, u R then which of the following is true?
a. a/b iff (b)⊂(a)
b. u is unit iff u/∀α∈R
c. u is unit iff (u)=R
d. all of the above

12. Let R be a commutative raing with regular elements. Let S be the set of all regular elements in R then which of the following is true?
a. R is embeddable in R_s
b. each element of R is invertible in R_s
c. each element of Rs is of the form r-1 s where ∈R, S ∈ S
d. R, is a field

13. Which of the following is not the symmetric functions of the roots α,β and y of the equation $x^3+px^2+qx+r=0$?
a. $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$
b. $\frac{\beta^2+\gamma^2}{\beta+\gamma}+\frac{r^2+\alpha^2}{r+\alpha}+\frac{\alpha^2+\beta^2}{\alpha+\beta}$
c. $\alpha^2\beta+\beta^2\gamma+\gamma^2\alpha$
d. $\alpha^3+\beta^3\gamma+\gamma^3$

14. The polynomial a 2+2 ℜ is[x)
a. reducible over ℜ
b. reducible over Q
c. reducible over C
d. reducile over R and C both

15. If is a unit matrix then which of the following is true?
a. I is a triangular matrix
b. I is a diagonal matrix
c. I is a scalar matris
d. all of the above

16. Let V be a vector space of all 2×3 matrices over the real field R. Then dim V is equal, 10
a. 2
b. 3
c. 5
d. 6

17. Which of the following pairs of vectors in $ℜ^2$ are not linearly independent?
a. (a, 2) and (2, 1)
b. (1, 0) and (0, 1)
c. (a', 3) and (3,1)
d. (a, 2) and (2, 4)

18. Which of the following vectors in %³ is perpendicular to the vector (1, 2, 3)?
a. (1, 2, 3)
b. (2, 2,-2)
c. (3, 2, 1)
d. (2, -2, 2)

19. Which of the following is the condition for the vector ii to have a constant direction?
a. (0, 1)
b. (0, 0)
c. (0, 9)
d. (0, 8)

20. If 1.∫=(x+y+1) ∫ (–x–y)k then which of the following is div *f* ?
a. 0
b. 1
c. 2
d. $\bar{i}$ $\bar{f}+\bar{k}$

Attempt all questions.

Group "B"

8×7=56

1. Define Aelian group. If G be an Aelian group and n be an integer, provethat $(ao)^n=1^n$∀α, b ∈G.

OR

Define permutation group. Prove that the set S_n dftrpermulations on a symbols is a group w.r. to the permulation multiplication

2. Let (H,o) and (H_2 0) be two subgroups of a group (g.o). Prove that (H_1 U H_2 o) is also a sugroup if and only if $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$.

3. If *f*: R → $\bar{R}$ be a ring homomorphism of R into $\bar{R}$ prove that (i) (0)O=where 0 and 0 are the zero elements of $\bar{R}$ and respectively. (ii) f(-a)=f(a) α,∈R.

4. Let R be an Euclidean domain and a0) c-R. Thenprove 'that
(i) $d(I_R) \leq d(a)$ ∀α∈ R
(ii) d(a)=d(Ir) iff a is aunit in r.

5. Define primitive polynomial. If the primitive polynomial *f*(x) can be factored as the product of two polynomials having rational coefficients then prove that it can be factored as the product of two polynomials having integral coefficient.

6. Discuss transformation of an equation. Transform the equation $x-6x^2+4x-7=0$ into one which lacks the second term.

7. Prove that a non-empty subsct U of a vector space V over the field F is a subspace of V if and only if (i) $u_1-u_2 \in U \forall u_1, u_2 \in U$ (ii) $u \in U_1$ s $F \Rightarrow s \in u$.

OR

Define orthogonal vectors. Prove that a set (u1, u2 ... un) of non-zero orthogonal vectors of a vector space V over a field F is linearly inderpendent,

8. show that the necessary and sufficient condition for the vector a of a scalar variale t to have a constant direction is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$

Group "C" 2×12=24

9. State and prove the third isomorphism, theorem on groups.

OR

Let $(G_1 \times G_2, .)$ be the direct product of the groups (G_1, o) and (G_2, o_2) then prove that

i. $\overline{G_1} = \{(g_1, e_2) | g_1 \in G_1, e_2$ is the edentity element of $G_2\}$ and $\overline{G_2} = \{(e_1, g_2) | g_2 \in G_2$, e_1 is the edentity element of $G_1\}$ are normal subgroups of $(G_1 \times G_2)$

ii. $(\overline{G1}) \cong (G_1, o_1)$ and $(\overline{G_2}, .) \cong (g_2, o_2)$

iii. If $g_1^1 = (g_1, e_2) \in \overline{G_1}$ and $g_2^1 = (e_1, g_2) \in \overline{G_2}$ then $g_1^1.g_2^1 = g_2^1.g_1^1$

iv. Each $g \in G_1 \times G_2$ is uniquely expressed as $g_1^1.g_2^1$ where $g_1^1 \in \overline{G_1}, g_2^1 \in \overline{G_2}$

10. Discuss the consistency of simultaneous equations. Test the consistency and solve the following:

a. $x + y + z = 6$
$x + 2y - 3z = -4$
$-x - 4y + 9z = 18$

b. $x - 4y + 7z = 8$
$3x + 8y - 2z = 6$
$7x - 8y + 26z = 31$

Exam 2070

Group 'A' [20]

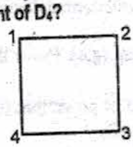
Attempt ALL the questions. Tick (√) the best answers.

1. Which of the following is not a binary operation on the set Q of rational numbers?
a. +
b. –
c. x
d. ÷

2. Which of the following statement is true?
a. If G be a group and $a \in G$ then $o(a) = o(a^{-1})$
b. If a is a generator of a group G then a^{-1} is also a generated of G.
c. If a, b, c be the elements of a group (G, o) then $(a.b.c.)^{-1} = c^{-1}.c^{-1}.b^{-1}.a^{-1}$
d. All of the above

3. Let G be a group the action $\phi : G \times G$ defined by $\phi(g, a) = gag-1, \forall g, a \in G$ is called
a. trivial action
b. conjugation
c. complement
d. general action

4. If the vertices of a square represented by 1, 2, 3, and 4 be as shown in the figure then which of the following is not an element of D_4?



a. (13)
b. (1234)
c. (23)
d. (24)

5. If G is a group of order 6 and N is a normal subgroup of G of order 2 then which of the following is the index of N in G?
a. 2
b. 3
c. 6
d. 2 and 3 both

6. Which of the following is the Cayley's theorem?

a. The order of a subgroup of a finite group is a divisor of the order of the group
b. If H and K are finite subgroups of a finite group G then

$$0(HK) = \frac{0(H)0(K)}{0(H \cap K)}$$

c. Every finite group is isomorphic to a group of permutations.
d. Any two cyclic groups of same order are isomorphic.

7. If $G = G_1 \times G_2$ is an internal direct product of the normal subgroups G_1 and G_2 then which of the following is true?

a. $\frac{G}{G_1} \approx G_1$
b. $\frac{G}{G_1} \approx G_2$
c. $\frac{G}{G_2} \approx G$
d. $\frac{G}{G_1} \approx \frac{G}{G_3}$

8. Which of the following ring has no unit element?

a. (z, +, ')
b. (2z, +,')
c. (C, +, .)
d. The ring of residue classes modulo – 5.

9. The characteristic of the ring of residue classes modulo 6 is

a. 0
b. 2
c. 3
d. 6

10. Let Z be the ring of integers, then which of the following is a maximal ideal in Z?

a. 2Z
b. 3Z
c. 5Z
d. All of the above

11. Which of the following is not true?

a. Every principle ideal domain is a unique factorization domain.
b. Every unique factorization domain is a principle ideal domain.
c. Every equation of an odd degree whose constant term is negative has at least two real roots of opposite signs.
d. The number of negative roots of the equation f(x) = 0 can not exceed the number of changes of signs in the terms occurring in f(x).

12. In Which of the following, R-{0} is a regular multiplicative set?

a. R is a ring
b. R is a ring without unit element
c. R has no zero divisors.
d. R is an integral domain

13. Which of the following is true?

a. If $a+ib(b \neq 0)$ is is a root of $f(x)=0$ then $a-ib$ is also a root of $f(x)=0$
b. Every equation of an even degree has at least one real root of the sign opposite to that of its constant term.
c. Every equation of an odd degree whose constant term is negative has at least two real roots of opposite signs.
d. The number of negative roots of the equation $f(x)=0$ can not exceed the number of changes of signs in the terms occurring in $f(x)$.

14. Which of the following field is a prime field?

a. The field C of complex numbers.
b. The field R of real numbers.
c. The field Q of rational numbers.
d. None of the above

15. If V be a vector space with a scalar product (u, v) and the norm $||u|| = (u, u)^{½}$ then which of the following is the Cauchy in equality?

a. $||C u|| < |C| \, ||u||$ for any $C \in R$
b. $|<u,v>| \leq ||u|| \, ||v||$
c. $||u+v|| \leq ||u|| + ||v||$
d. $(u, v + w) \leq (u, v) + (u, w)$

16. Let V and W be two vector spaces over the same field F and $f: V \to W$ be a linear transformation, then which of the following is true?

a. f is one-one if and only if im$f$ = {0}
b. f is one-one if and only if ker f = {0}
c. f is one-one if and only if f maps V onto W.
d. f is onto if and only if f is one-one.

17. Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$ and let $X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ then t_{XAY} is equal to

a. $x_1y_1 + 2y_1x_2 + x_1y_2 + 3x_2y_2$
b. $x_1y_1 + x_1y_2 + 2y_1x_2 + 3x_2y_2$

c. $x_1y_1+3x_1y_2+2x_2y_2$ d. $x_1y_1+x_2y_2$

18. If and $\vec{c}$ be any three vectors in space then which of the following is true?

a. $|\vec{a}\ \vec{b}\ \vec{c}| = |\vec{c}\ \vec{a}\ \vec{b}|$

b. $|\vec{a}\ \vec{b}\ \vec{c}| = |\vec{a}\ \vec{c}\ \vec{b}|$

d. $|\vec{a}\ \vec{b}\ \vec{c}| = (\vec{a}\cdot\vec{c})\times\vec{b}$

d. $|\vec{b}\ \vec{a}\ \vec{c}| = -\vec{b}(\vec{a}\times\vec{c})$

19. If $\vec{r} = a\cos t\ \vec{i} + b\sin t\ \vec{j} + ct\vec{k}$ then which of the following is the value of $\frac{d\vec{r}}{dt}$ at t = 0?

a. $b\vec{j} + c\vec{k}$

b. a

c. $-a\vec{i} + c\vec{k}$

d. $-a\sin t\,\vec{i} + b\cos t\,\vec{j}$

20. A linear transformation $T:V \rightarrow W$ is said to be nonsingular if

a. Ker T = {0}

b. Im T = {0}

c. ImT = W

d. Ker T = V

Group 'B' [8×7 = 56]

Attempt ALL the questions

1. Define group with example. Prove that the order of a cyclic group is the same as the order of its generator.

OR

Define subgroup of a group with example. Prove that every finite group of composite order possesses a proper subgroup.

2. Define group homomorphism. Let $f: G \rightarrow \overline{G}$ be a group homomorphism of G onto $\overline{G}$ and let K = ker *f*. Prove that $\frac{G}{K}$ is isomorphic to $\overline{G}$.

3. Let $G - G_1 \times G_2$ be an internal direct product of the normal subgroups G_1 and G_2 of G. Prove that $\frac{G}{G_2} \approx G_1$ and $\frac{G}{G_1} \approx G_2$.

4. Define prime ideal. Prove that an ideal P in a ring R is a prime ideal if and only if $\frac{R}{P}$ is an integral domain.

OR

Let L is a finite extension of a field F and K is a subfield of L which contains F. Prove that [K:F] is a divisor of [L:F]

5. Test for consistency and solve the following equations.

$x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2.$

OR

Prove that the rank of the product matrix AB of two matrices A and B cannot exceed the rank of either A or B.

6. Define maximal set of linearly independent vectors of a vector space V. Prove that the maximal set of linearly independent vectors of V is a basis of V.

7. Discuss the singular and non-singular transformations. Prove that the inverse of a linear transformation is linear.

8. Define Gradient, divergence and curl. If $f = x^3 + y^3 + z^3 - 3xyz$ find div (grad *f*) and curl (grad *f*).

Group "C" [2×12 = 24]

9. Define permutation group and alternatively group.
If S be a set of n symbols, then prove the followings:
(i) every $f \in S_n$ can be expressed as a product of disjoint cycles.
(ii) the set An of all even permutations is a subgroup of S_n and 0 $(A_n) = \frac{n!}{2}$.

OR

Define group action with example. Let G be a group acting on a non-empty set A. The relation on A is defined by a ~ b if and only if a = g.b for some g ∈ G is an equivalence relation. Further more for each a ∈ A, the number of elements in the equivalence class containing a is $[G_1.G_{a1}]$ the index of the stabilizer of a. Prove it.

10. Let a, b and u be elements of a commutative integral domain with identity then prove the following:
(i) a/b/ if and only if (b) < (a)
(ii) a and b are associates, if and only if (a) = (b)
(iii) u is unit in R if and only if u/r ∀ r ∈ R.

Exam 2071

Group "A"

Attempt ALL the questions. Tick (√) the best answers. 20

1. The set G = {1, w, w^2} forms a cyclic group under multiplication generated by....
   a. 1 b. w
   c. w^2 d. w and w^2 both
2. The order of symmetric group is not of degree n is ....
   a. n b. n!
   c. $\frac{n!}{2}$ d. (n – 1) !
3. Which of the folio following is not true?
   a. The product of two even permutations is even permutation.
   b. The product of two odd permutations is even permutation.
   c. The product of an even and an odd permutation is even permutation.
   d. A transposition is always are odd permutation.
4. What is the order of an identity element of a group G?
   a. 0 b. 1
   c. 2 d. infinite
5. A homomorphism $\phi : G \to G'$ is called epimorphism if
   a. ϕ is one-one b. ϕ is onto
   c. ϕ is one-one and onto d. all of the above
6. If G is a finite group and H is a normal subgroup of G, then which of the following is true?
   a. $0\,(G/H) = \frac{o\,(G)}{o\,(H)}$ b. o (G/H) = o (G) o (H)
   c. $o\,(G/H) = \frac{o\,(H)}{o\,(G)}$ d. o (G/H) = o (G) + o (H)
7. If $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 2 & 5 & 6 & 4 & 7 \end{pmatrix} \in S_7$, which of the following is orbit of 4 ?
   a. (234) b. (345)
   c. (567) d. (456)
8. Which of the following statement is false?
   a. The set (Q, +, •) is a division ring b. The set (R, +, •) is a division ring
   c. M_2 (R) is a division ring d. The set (Z, +, •) is not a division ring
9. A commutative division ring is called
   a. an integral domain b. a skew field
   c. a field d. a Boolean ring
10. An element of a ring (R, –, •) is said to be nilpotent if then exists a positive integer n such that
    a. $a^n = 0$ b. $a^n = I$
    c. $a^n = a$ d. $a^n = a^2$
11. Which of the following statement is true?
    a. Every Euclidean domain R has a unit element
    b. Every Euclidean domain is a principal ideal domain
    c. Every irreducible element of UFD is a prime
    d. All of the above
12. The characteristic of the field of real numbers is
    a. 1 b. 2
    c. 0 d. ∞

13. If α, β, and y be the roots of an equation of degree 3, which of the following is not symmetric function?

a. $\alpha\beta + \beta\gamma + \gamma\alpha$ | b. $\alpha^2 + \beta^2 + \gamma^2$
c. $\alpha + \beta + \gamma$ | d. $\alpha^2\beta + \beta^2\gamma + \gamma^2\alpha$

14. What is the rank of the matrix $\begin{pmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \end{pmatrix}$?

a. 0 | b. 1
c. 2 | d. 3

15. If X = (1, 2) and Y = (3,4) be any two vectors in $\Re^2$, what is the value of X Y?

a. 24 | b. 11
c.21 | d. 10

16. If P = (1, 2) and Q + (1, 1) be any two vectors in R^2, what is the scalar projection of Q onto P?

a. $\frac{3}{\sqrt{5}}$ | b. $\frac{3}{5}$
c. $\frac{\sqrt{3}}{5}$ | d. $\sqrt{\frac{3}{5}}$

17. If $T : R^2 \rightarrow R^2$ is a linear transformation defined by T(x, y) = (x + y, y) for (x, y) $\in R^2$, then kernel of T is given by

a. {(0,0)} | b. {(1,0)}
c. {(0,1)} | d. {(1,1)}

18. A set of vectors {u_1, u_2, u_3, ... un} in a real vector space V is said to be orthonormal if ....

a. $<u_i, u_j> = 0$ for $i \neq j$ | b. $<u_i, u_j> = 0$ for i = j
c. $<u_i, u_j> = 1$ for $1 \neq j$ | d. $<u_i, u_j> = 1$ for i = j

19. A vector a is irrotational if ....

a. $\nabla . a^{TM} = 0$ | b. $a^{TM} = 0$
c. $\nabla a^{TM} = 0$ | d. $\nabla \times a^{TM} = 0$

20. The divergence of V^{TM} is given by ....

a. $i^{TM} . \frac{\partial v^{TM}}{\partial x} + j^{TM} . \frac{\partial v^{TM}}{\partial y} + k^{TM} . \frac{\partial v^{TM}}{\partial z}$ | b. $i^{TM} . \frac{\partial v^{TM}}{\partial x} + j^{TM} . \frac{\partial v^{TM}}{\partial y} + k^{TM} \frac{\partial v^{TM}}{\partial z}$
c. $i^{TM} \times \frac{\partial v^{TM}}{\partial x} + j^{TM} \times \frac{\partial v^{TM}}{\partial y} + k^{TM} . \frac{\partial v^{TM}}{\partial z}$

Attempt ALL the questions.

Group "B"

8×7=56

1. Define commutative group with an example. If (G, *) be a group such that $(a * b)^2 = a^2 * b^2$ for all a, b $\in$ G, prove that G is commutative.
2. Define normal subgroup of a group G. If Ni and N2 are normal subgroups of a group G, prove that $N_1 \cap N_2$ also a normal subgroup of G.

OR

Prove that the normalizer N(a) of any element a of a group G is a subgroup of G.

3. Define integral domain. Prove that every field is an integral domain.
4. Prove that in a commutative ring R with unity, and ideal M is maximal iff R/M is a field.

OR

Prove that if $f : R \rightarrow R'$ is a ring homomorphism of R into R' then the kernel k_f of f is an ideal of R.

5. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of (i) $\alpha^2, \beta^2, \gamma^2$ and (iii) $\frac{1}{a} + \frac{1}{b} + \frac{1}{g}$.
6. Explain algebraic extension of a field F. Prove that every finite extension K of a field F is algebraic extension of F.

OR

Test for consistency and solve the equations
$x - 2y + 3z = 2$, $2x - 3z = 3$, $x + y + z = 0$.

7. Let V be a vector space over a field F and $\{v_1, v_2, ... v_n\}$ be maximal set of linearly independent vectors of V.
Prove that $\{v_1, v_2, ... v_n\}$ is a basis of V.

8. If a is a constant vector, prove that $(a^{TM} \times V) \times r^{TM} = -2a^{TM}$.

Group "C"

2×12=24

9. Prove:

(a) If $\phi : G \to G'$ is a homomorphism with kernel k_ϕ then k_ϕ is a normal subgroup of G.

(b) A homomorphism $\phi : G \to G'$ with kernel k_ϕ is an isomorphism of G into G' iff $k_\phi = \{e\}$.

10. Prove the following:

(a) Every Euclidean domain is a principal ideal domain;

(b) If R be a principal ideal domain then an element $P \in P$ is prime iff it is irreducible.

OR

What is a linear transformation? If $T: R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (x, o, z)$, find (a) Ker T and (b) Im T.

If $T: V \to W$ be a linear transformation then prove that Ker T and Im T are subspaces V and W respectively.

Exam 2072

Group "A"

20

Attempt ALL the questions. Tick (√) the best answer.

1. For the Klein's four group G of elements e, a, b, c, which of the following is not true?
   a. $a^2 = b^2 = c^2 = e$
   b. $ab = ba = c$
   c. $bc = cb = a$
   d. G is a cyclic group

2. Which of the following is false?
   a. every cyclic group is abelian
   b. isomorphic image of a cyclic group is not always cyclic
   c. every subgroup of a cyclic group is cyclic
   d. every group of prime order is cyclic

3. If $\sigma = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$, what is the value of σ^3?
   a. $\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 3 & 4 & 2\end{pmatrix}$
   b. $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix}$
   c. $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$
   d. $\begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix}$

4. Which of the following group <u>is not</u> cyclic?
   a. $Z_9 \times Z_{14}$
   b. $Z_{12} \times Z_{49}$
   c. $Z_4 \times Z_6$
   d. $Z_{15} \times Z_{28}$

5. What is the order of the alternating group A_4?
   a. 12
   b. 6
   c. 24
   d. 8

6. A homomorphism $\phi : G \to G'$ is called isomorphic if .....
   a. ϕ is onto
   b. ϕ is one-one and onto
   c. ϕ is one-one
   d. ϕ is into

7. IF $N \Delta G$, which of the following is true for all $x \in G, n \in N$?
   a. $_xN = N_x$
   b. $x^{-1}N_x = N$
   c. $xN_x^{-1} = N$
   d. all of the above

8. Which of the following statement <u>is not</u> true?
   a. every field is an integral domain
   b. a finite integral domain is a field
   c. a skew field has zero divisors
   d. an arbitrary intersection of sub rings is c subring

9. A ring R is called simple if it has ....
   a. no proper ideals
   b. proper ideals
   c. no ideals
   d. ideals

10. The ring $(Z_6, +, .)$ of integers modulo 6 has characteristic

a. 0 b. 2
c. 3 d. 4

11. $x^2 + 2 \in R[X]$ is a minimal polynomial of ....
a. $I\sqrt{2}$ over R b. $\sqrt{2}$ over Q
c. 2i over $\Re$ d. -2 over R

12. What is the characteristics of the field of real numbers?
a. 1 b. 1
c. 2 d. ∞

13. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, what is the value of $\alpha + \beta + \gamma$?
a. p b. -p
c. p^2 d. $q^2 - 2pr$

14. A square matrix A is called nilpotent of order n if ....
a. $A^n = A$ b. $A^n = 0$
c. $A^{n-1} \neq 0$ d. $A^n = O$ but $A^{n-1} \neq 0$

15. If $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$, what is the value of P.Q?
a. $x_1x_2+y_1y_2+z_1z_2$ b. $\sqrt{x_1x_2+y_1y_2+z_1z_2}$
c. $\sqrt{x_1^2+y_1^2+z_1^2}\sqrt{x_2^2+y_2^2+{}_2^2}$ d. $\sqrt{(x_1x_2)^2+(y_1y_2)^2+(z_1z_2)^2}$

16. The dimension of the vector space of all 2×3 metrices over R is
a. 2 b. 3
c. 5 d. 6

17. If $\pi : R^3 \to R^3$ be the projection mapping on xy - plane defined by $x(x, y, z) = (x, y, o)$, then ker π is
a. $\{(o, c, o) : c \in \Re\}$ b. $\{(o, o, c) : c \in \Re\}$
c. $\{(c, o, o) : c \in \Re\}$ d. $\{(o, o, o)\}$

18. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ be reciprocal system of vectors then which of the following is true?
a. $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 0$ b. $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$
c. $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 0$ d. $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 1$

19. The necessary condition for vector function $\vec{f}$ of scalar variable t to have a constant magnitude is ..
a. $\vec{f}\frac{d\vec{f}}{dt} = 0$ b. $\vec{f} \times \frac{d\vec{f}}{dt} = 0$
c. $\vec{f}.\frac{d\vec{f}}{dt} = 0$ d. $\frac{d\vec{f}}{dt} = 0$

20. Which of the following is not meanifgful.
a. $\nabla\vec{r}$ b. $\nabla.\vec{r}$
c. $\nabla \times \vec{r}$ d. $\nabla^2\vec{r}$

Attempt ALL the questions

Group "B" 8×7=56

1. Define cyclic group. Prove that every cyclic group is abelian.
2. If H and K are subgroups of a group G, prove that HK is a subgroup of G if and only if HK = KH.

OR

3. A non-empty subset S of a ring R is a subring of R iff $\forall$ a, b $\in$ S, a - $\in$ S and $\in$ S, prove.
4. If $f : R \to R'$ is a ring homomorphism of R into R', prove that the kernel k_f of f is an ideal of R.

OR

Let R be a principal ideal domain and p $\in$ R. Prove that p is prime if it is irreducible.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, if the ratio of two of the roots is 3 : 2.
6. Explain the characteristics of a field. Prove that the characteristics of a field F is either zero or a prime.

OR

Test for consistency and solve the equations $3x - 4y = 2$, $5x + 2y = 12$, $x - 3y = 1$.

7. Prove that the union of two subspaces of a vector space V is a subspace of V if and only if one is contained on the other.

8. Prove that the necessary and sufficient condition for a vector function $\vec{a}$ of scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

Group "C" 2×12 = 24

9. Define internal direct product of the normal subgroups. $N_1, N_2, \ldots\ldots N_n$ of a group G. If G be the internal direct product of normal subgroups $N_1, N_2, \ldots\ldots N_n$ prove that $G \approx N_1 \times N_2 \times \ldots\ldots N_n$.

10. Define Euclidean domain and show that the set of integers is an Euclidean domain. Also prove that:
(a) Every Euclidean domain is a principal ideal domain.
(b) Every Euclidean domain R has a unit element.

OR

10. (a) Prove that a set of non-zero orthogonal vectors is linearly independent.
(b) If $T : V \rightarrow W$ be a linear transformation then prove that Ker T and Im T are subspace of V and W respectively.

(b) Advanced Calculus (Math. Ed. 333) Elective Group A

Exam 2068

Group "A" 20

Attempt ALL the questions. Tick (√) the best answers.

1. The improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if

a. $n > 1$ b. $n < 1$ c. $n = 1$ d. $n \neq 0$

2. Which one of the following is not true?
a. point wise convergence ⇒ uniform convergence
b. uniform convergence ⇒ point wise convergence
c. non-point wise convergence ⇒ non- uniform convergence
d. all of the above

3. Which of the following series is uniformly convergent for all values of θ and $0 < r < 1$?
a. $\Sigma r^n \cos n\theta$ b. $\Sigma r^n \sin n\theta$ c. $\Sigma r^n \cos^2 n\theta$ d. all of the above

4. The series $\Sigma \frac{a_n x^n}{1 + x^{2n}}$ converges uniformly for all real values of x if,
a. Σa_n is convergent b. Σa_n is absolutely converbent
c. Σa_n is uniformly convergent d. all of the above

5. In which of the following condition the series $\Sigma \frac{\cos_n q}{n^p}$ us uniformly and absolutely convergent for all real values of θ?
a. $P = 1$ b. $P < 1$ c. $P > 1$ d. $P \geq 1$

6. Which is the sufficient condition for a function f(x, y) to be differentiable at (a, b)?
a. f_x is continuous at (a, b)
b. f_x exists at (a, b)
c. f_x exists at (a, b)
d. f_x is continuous at (a, b) and f_y exists at (a, b)

7. f(a, b) is an extreme value of f(x, y) if $f_x(a, b) = 0$ $f_y(a, b)$ and ....
a. $f_{xx}(a, b).f_y(a, b) - [f_{xy}(a, b)]^2 < 0$ b. $f_{xx}(a, b).f_{yy}(a, b) - [f_{xy}(a, b)]^2 = 0$
c. $f_{xx}(a, b).f_{yy}(a, b) - [f_{xy}(a, b)]^2 < 0$ d. $f_{xx}(a, b).f_{yy}(a, b) - [f_{xy}(a, b)]^2 \leq 0$

8. For $f(x, y) = 2x^2 - xy + 2y^2$, what is the value of $f_y(1, 2)$?
a. 7 b. 6 c. 5 d. 0

9. Which of the following is equal to $\frac{\partial(y_1, y_2, \ldots\ldots y_n)}{\partial(x_1, x_2, \ldots\ldots x_n)}$?

a. $\frac{\partial(y_1, y_2, \ldots\ldots y_n, x_{m+1}, \ldots\ldots\ldots\ldots x_n)}{\partial(x_1, x_2, \ldots\ldots x_n, x_{m+1}, \ldots\ldots x_n)}$ b. $\frac{\partial(y_1, y_2, \ldots\ldots y_n, y_{m+1}, \ldots\ldots\ldots\ldots x_n)}{\partial(x_1, x_2, \ldots\ldots x_n, x_{m+1}, \ldots\ldots x_n)}$

c. $\frac{\partial(y_1, y_2, \ldots\ldots y_n, x_{m+1}, \ldots\ldots\ldots\ldots x_n)}{\partial(x_1, x_2, \ldots\ldots x_n, y_{m+1}, \ldots\ldots x_n)}$ d. $\frac{\partial(x_1, x_2, \ldots\ldots x_m)}{\partial(y_1, y_2, \ldots\ldots y_m)}$

10. If the function f, f_1, f_2 of x and y are integrable on a rectangle R, then which of the following is not true?

a. $\iint_R |f|\, dx\, dy \le \iint_R f\, dx\, dy|$

b. $\iint_R (f_1 + f_2)\, dx\, dy = \iint_R f_1\, dx\, dy + \iint_R f_2\, dx\, dy$ sz

c. the product f_1, f_2 is integrable over R

d. the quotient $\frac{f_1}{f_2}$ is integrable over R, if $|f_2| \ge 0$ or R

11. Which of the following theorem is different from others?
a. Gauss's theorem
b. Divergence theorem
c. Second generalization of Green's theorem
d. Stoke's theorem

12. if $\iint R\, f\, dx\, dy$ and $\int_a^b f dx$ both exist, then the double integral can be expressed as

a. $\iint R\, f\, dx\, dy = \int_a^b dy \int_c^d f dx$

b. $\iint R\, f\, dx\, dy = \int_d^c dy \int_a^b f dx$

c. $\iint R\, f\, dx\, dy = \int_a^b dx \int_c^d f dy$

d. $\iint R\, f\, dx\, dy = \int_c^d dx \int_a^b f dy$

13. in spherical polar co-ordinates, the volume of a solid is given by

a. $\iiint r^2 \sin\theta\, dr\, \theta\, d\phi$

b. $\iiiint r^2 \sin^2\theta\, dr\, d\theta\, d\phi$

c. $\iiint r^2 \cos\theta\, dr\, d\theta\, d\phi$

d. $\iiint r \cos\theta\, dr\, d\theta\, d\phi$

14. Which of the following is not true?
a. every closed sphere is a closed set
b. every closed interval is a closed set
c. every finite subset of a metric space is closed
d. every open sphere is a closed set

15. Which of the following is the diameter of empty set ϕ?
a. 0
b. 1
c. ∞
d. $-\infty$

16. If (X, d) be a metric space and A, B be subsets of X then which of the following in not true?
a. $f_r(A) = \bar{A} \cap (\bar{A})^c$
b. $F_r(A) = \bar{A} - \text{int } A$
c. $\bar{A} \cap (\bar{A}) - \text{int } A$
d. $f_r(A \cap B) \subseteq F_r(A) \cup F_r(B)$

17. If (X, d_1) and (Y, d_2) be any two metric spaces then f: X→Y is continuous if and only if
a. $f^{-1}(G)$ is closed in X, Wherever G is open in Y
b. $f^{-1}(G)$ is closed in X, Wherever G is open in Y
c. $f^{-1}(G)$ is closed in X, Wherever G is closed in Y
d. $f^{-1}(G)$ is closed in X, Wherever G is intersected in Y

18. A subset A of a compact metric space (X, d) is itself compact if and only if
a. it is open in (X, d)
b. it is closed in(X, d)
c. it is compact in (X, d)
d. it is bounded in (X, d)

19. Which of the following represents the first backward differences in difference table at the argument x = a ?

a. $\Delta f(a)$ b. $\Delta f(afh)$ c. $\nabla f(a)$ d. $\nabla f(afh)$

20. What is the degree of the approximation polynomial to the Trapezoidal rule?

a. 0 b. 2 c. 1 d. n

Attempt ALL the questions.

Group 'B'

8×7=56

1. Define absolute convergence of $\int_a^\infty f\,dx$. Show that the integral $\int_1^\infty \frac{\sin x}{x^p}dx$ is convergent for P > 0.

2. If a sequence $\{f_n\}$ converges uniformly in [a, b], and x_0 is a point of [a, b] such that $\lim_{x\to x_n} f(x) = a_n(a=1,2.......)$ then prove that (i) $\{a_n\}$ converges (iii) $\lim_{x\to x_n} f(x) = \lim_{n\to\infty} a_n$

OR

State Abel's test for uniform convergences of series of functions.

Show that the series $\sum\frac{(-1)^n}{n}|x|^n$ is uniformly convergent in $-1 \le x \le 1$.

3. If a power of series $\sum_{n=0}^{\infty} a_n x^n$ converges at the end point x = R of the interval of convergence (–R,R), then prove that it is uniformly convergent in the closed interval [0,R].

4. State the sufficient condition for differentiability of function of two variables. Show that the function

$$f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & , if (x^2+y)^2\neq 0 \\ 0 & , if \quad x=y=0 \end{cases}$$

is differentiable at the origin?

5. If the roots of the equation in λ,

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ are u, v, w, prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(x-x)(x-y)}{(v-w)(w-u)(u-v)}$$

6. If (X_1, d_1) and (Y_2, d_2) be two metric spaces, then, $f: X \to Y$ is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.

7. If a double integral I = $\iint f\,dxys$ exists over a rectangle R = [a, b; c_2, d_2] and if $\int_c^d f dy$exists for each fixed in x in [a, b], then prove that the integrated integral $\int_a^b dx \int_e^d f dy$ exists and is equal to the double integral.

OR

Use cylindrical coordinated and find the volume common to the sphere $x^2 + y^2 + z^2 = 4$ and cylinder $x^2 + y^2 = zy$.

8. Use method of false position to find the root of the equations $3xe^x = 1$ to 3 decimal places.

OR

Estimate $\int_1^{13}\sqrt{x}dx$ from the given data, using Simpson's rule.

(h = 0.15, 0.05)

X	1.0	1.05	1.10	1.15	1.20	1.25	1.30
f(x)	1.0000	1.02470	1.04881	1.07238	1.09545	1.11803	1.14018

Group 'C'

2×12=24

9. If f(x, y), $\begin{cases} \frac{x^3-y^3}{x^2+y^2} & , for\ (x,y) \neq (0,0) \\ 0 & , for\ (x,y) = (0,0) \end{cases}$

then show that $f(x,y)$ is continuous at (0, 0) and possesses partial derivatives at (0, 0) but not differentiable at (0, 0)

10. Define compact set with an example. Prove that every subset F of a metric space (X, d) is closed. Also shoe by an example that every closed set may not be compact.

OR

Let (X, d) be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subset of X such that $d(F_n) \to 0$ as $n \to \infty$, then prove that $F = \bigcap_{n=1}^{\infty} F_n$ contains exactly one point.

Exam 2069

Group 'A'

[20]

Attempt ALL the questions. Tick the best answers.

1. The improper integral $\int_a^b f dx$ is absolutely convergent if
   a) $\int_a^b |f| dx$ is convergent
   b) $\int_a^b |f| dx$ is divergent
   c) $\left|\int_a^b f dx\right|$ is convergent
   d) $\left|\int_a^b f dx\right|$ is divergent

2. What is the interval of uniform convergence of a sequence of functions $\{f_n\}$?
   a) open interval
   b) closed interval
   c) half-closed interval
   d) half-open interva

3. The series $\Sigma \frac{a_n}{n^x}$ converges uniformly in [0, 1] if
   a) Σn^x converges
   b) Σn^x diverges
   c) Σa_n converges
   d) Σa_n diverges

4. Which of the following series is uniformly convergent for all values of θ and $0 < r < 1$?
   a) $\Sigma r^n \cos n\theta$
   b) $\Sigma r^n \sin n\theta$
   c) $\Sigma a_n \cos^2\theta$
   d) all of the above

5. If a sequence $\{f_n\}$ be defined on [0, 1] by $f_n(x) = x^n$ its limit function if given by
   a) $f(x) = \begin{cases} 0 \text{ if } 0 \le x < 1 \\ 1 \text{ if } x = 1 \end{cases}$
   b) $f(x) = \begin{cases} 0 \text{ if } 0 < x \le 1 \\ 1 \text{ if } x = 0 \end{cases}$
   c) $f(x) = \begin{cases} 0 \text{ if } 0 \le x \le 1/2 \\ 1 \text{ if } 1/2 \le x \le 1 \end{cases}$
   d) $f(x) = 1 \ \forall \ x \in [0, 1]$

6. Which theorem gives sufficient condition for the equality of f_{xy} and f_{xy}?
   a) Euler's theorem
   b) Lagrange's theorem
   c) Schwarz's theorem
   d) Taylor's theorem

7. Which of the following is not a condition for the existence theorem of the function of two variables?
   a) $f(a, b) = 0$
   b) f_x and f_y exists and are continuous in the neighbourhood of (a, b)
   c) $f_y(a, b)$
   d) $b = \phi(a)$

8. A stationary point of f is an extreme point of f if
   a) d^2f is positive and minimum
   b) d^2f is positive and minima
   c) d^2f has opposite sings alternatively
   d) d^2f keeps the same sign

9. The value of the limit $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$ is
   a) 0
   b) 1
   c) 2
   d) does not exists

10. If $f(x,y) = 1$ for all (x, y) in the rectangle $R = [a, b] \times [c, d]$ then the value of $\iint_r f\,dxdy$ is

a) $\frac{bd}{ac}$
b) $(b + a)(c + d)$
c) abcd
d) $(b - a)(d - c)$

11. If $\iint_r f(dxdy)$ and $\int_a^b fdx$ both exists, then the double integral can be expressed as

a) $\int_a^b dy \int_c^d f\,dx$
b) $\int_a^b dx \int_c^d f\,dy$
c) $\int_c^d dy \int_a^b f\,dx$
d) $\int_c^d dx \int_a^b f\,dy$

12. If P^* is a refinement of a partition for a bounded function f, then which of the following is not true?

a) $L(P^*, f) \geq L(P, f)$
b) $U(P^*, f) \leq U(P, f)$
c) $U(P^*, f) - L(P^*, f) \leq U(P, f) - L(P, f)$
d) $U(P^*, f) - L(P^*, f) \geq U(P, f) - L(P, f)$

13. In spherical polar co-ordinates, the volume of a solid is given by

a) $\iiint r^2 \sin\theta\, dr\, d\theta\, d\phi$
b) $\iiint r^2 \sin^2\theta\, dr.\, d\theta.\, d\phi$
c) $\iiint r^2 \sin\theta\, dr\, d\theta. d\phi$
d) $\iiint r \cos\theta\, dr. d\theta. d\phi$

14. Which of the following functions $d : R \times R \rightarrow R$ is called usual metric on R?

a) $d(x,y) = \begin{cases} 1, & (x \neq y) \\ 1, & (x = y) \end{cases}$
b) $d(x, y) = |x - y|$
c) $d(x, y) = \sum_{i=1}^{n} (x_i - y_i)$
d) $(x, y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{\frac{1}{2}}$

15. Which of the following set is open in the metric subspace. [0, 1]. of the Euclidean metric space R^2?

a) [0,1/2)
b) (1/3, 1/2]
c) (1/3, 1/2)
d) all of the above

16. A subset of A of metric space (X, d) is said to be closed if

a) A contains all limits points
b) A contains all adherent points
c) A contains all interior points
d) A is a neighbourhod of its points

17. Which one of the following is not an open covering of the real line R?

a) $\{(-n, n) : n \in Z^+\}$
b) $\{(n, n+ 2): n \in Z\}$
c) $\{(a, b) : a, b \in R\}$
d) $\{(n, n+1) : n \in Z^+\}$

18. Which of the following set is compact?

a) (0, 1)
b) (0, 1]
c) [0, 1]
d) R

19. When K^{th} order differences are constant, the tabular function may be approximated by a polynomial of

a) degree K – 1
b) degree K
c) degree K + 1
d) degree 2K

20. The degree of the approximation polynomial related to the Simpson's rule is

a) 0
b) 1
c) 2
d) 2n +1

8×7=56

Group"B"

Attempt all the questions.

1. If $\emptyset$ bounded and monotonic in $[a, \infty)$ and $\int_\alpha^\infty f\,dx$ is convergent at ∞, then prove that $\int_\alpha^\infty f\,\emptyset\,dx$ is convergent at ∞
2. A sequence of functions $\{f_n\}$ defined on [a, b] converges uniformly on [a, b] if and only if for every ε >0 and for all x ∈[a, b], there exists an integer N such that $|f_{mp}(\chi)-f_n(\chi)|<\in, \forall n\geq N, P\geq 1$~,, P(x)—f,, (x) < cVn~ !N,P?l.

OR

If a series $\sum f_n$ converges uniformly to f in an interval [a, b] and its terms f_n are continuous at α point x_o of the interval, then the sum function f is also continuous at x_o

3. $\sum \alpha_n \chi^n$ diverges for $x=x^1$, then prove that it diverges for every x=x", where |x"| >| x".
4. Define the differentiability of a function of two variables. Show that the function. is not differentiable at the origin.
5. Evaluate
6. Evaluate $\int\int$(y-2x,)dx dy, over R=[1,2,3,5].

OR

Define the partition of a rectangular parallelepiped. Prove that a bounded function is integrable over a rectangular parallelepiped R if and only if for every F,>0 there is a partition P of R such that

U(P,f)-L(P,f)<∈

7. Define Cauchy sequence in a metric space. Prove that every convergent sequence is a Cauchy sequence, but the converse need not be true.
8. If f (x) be a polynomial of degree: n in x, then prove that the nth difference of f(x) is constant and $\Delta A n^{+1} f(x)=0$.

OR

Evaluate (h 0.20, Q. 10, 0.5)

x	0.10	0.15	0.20	0.25	0.30
f(x)	1.10517	1.16123	1.22140	1.28403	1.34986

Group "C"

2×12=24

9. Show that for the function

$f_{xy}(0,0)= f_{yx}(0.0)$, even though the conditions of Schwarz's theorem and also of Young" S theorem are not satisfied.

10. Define continuous function on a metric space. Let (x_1d_1) and (y_1d_2) be any two metric spaces and f is a function from X into Y, Then prove that f is continuous at a ∈X if and only if, for every, sequence

(a_n) converging to a we have $\lim_{n\to\alpha} f(a_n)= f(a)$-"-c

OR

Define compact set. Prove that every closed and bounded subset of the real numbers is compact.

Exam 2070

Group "A"

[20]

Attempt all the questions. Tick (√) the best answers.

1. The improper integral $\int_a^b \frac{dx}{x(x-a)^n}$ converges if and only if

a. n=1 b. n<1 c. n>1 d. n≠1

2. If ϕ is bounded and monotonic in [a, ∞) and $\int_a^\infty f\,dx$ is convergent at ∞ then $\int_a^\infty f\,\phi\,dx$ is

a. convergent at a
b. divergent at a
c. convergent at ∞
d. divergent at ∞

3. Which of the following is not true?

a. point-wise convergence ⇒ uniform convergence
b. non-point-wise convergence ⇒ non-uniform convergence

c. uniform convergence $\Rightarrow$ point-wise convergence
d. all of the above

4. The interval of uniform convergence of a sequence of function $\{f_n\}$ is
a. half-open interval
b. open interval
c. half-closed interval
d. closed interval

5. If a power series $\sum a_n x^n$ diverges for x=x', then it diverges for every x=x", where
a. $|x'|<|x''|$
b. $|x'|\leq|x''|$
c. $|x'|>|x''|$
d. $|x'|\geq|x''|$

6. If a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at the end point x=R of the interval of convergence (-R,R), then it is uniformly convergent in
a. (0,R)
b. (0,R]
c. [0,R)
d. [0,R]

7. An error due to use of approximate formula is called
a. round- off error
b. relative error
c. truncation error
d. absolute error

8. The term "interpolation" is defined as
a. the art of reading the missing values between the lines in tables
b. the special case of process of curve fitting
c. the process by which non-tabulated talues of a tabular function are estimated
d. all of the above

9. What is the degree of the approximating polynomial corresponding to the Simpson's rule?
a. linear
b. quadratic
c. cubic
d. biquadratic

10. A stationary point of f will be an extreme point of F if
a. d^2F have positive sign
b. d^2F have negative sign
c. d^2F have same sign
d. d^2F have opposite sign

11. f_{xy} and f_{yx} are both continuous at (a,b) then
a. $f_{xy} = f_{yx}$
b. $f_{xy} \neq f_{yx}$
c. $f_{xy}(a,b) = f_{yx}(a, b)$
d. $f_{xy}(a,b) \neq f_{yx}(a, b)$

12. A necessary condition for $f(x, y)$ to have an extreme value at (a, b) is
a. $f_x(a, b) = 0$
b. $f_y(a, b) = 0$
c. $f_x(a, b)$ and $f_y(a, b)$ exist
d. all of the above

13. A function f is said to be continuous at a point (a, b) of its domain of definition if
a. $\lim_{x\to a} f(x) = f(a)$
b. $\lim_{x\to b} f(x) = f(b)$
c. $\lim_{(x\,y)\to(a\,b)} f(x, y) \neq f(a, b)$
d. $\lim_{(x\,y)\to(a\,b)} f(x, y) = f(a, b)$

14. If $u_1, u_2, \ldots, u_n$ be n differentiable functions of n variables $x_1, x_2, \ldots x_n$ then which of the following is not a Jacobian?
a. $\dfrac{\partial(u_1,u_2,\ldots,u_n)}{\partial(x_1,x_2,\ldots,x_n)}$
b. $J\dfrac{(u_1,u_2,\ldots,u_n)}{(x_1,x_2,\ldots,x_n)}$
c.
d. all of the above

15. Which of the following theorem is different from others?
a. Stoke's theorem
b. Gauss's theorem
c. Divergence theorem
d. Second generalization of Green's theorem

16. $\iint_R f\,dx\,dy$ and $\int_c^d f\,dy$ both exist, the double integral can be expressed as
a. $\iint_R f\,dxdy = \int_a^b dy \int_c^d f\,dx$
b. $\iint_R f\,dxdy = \int_a^b dx \int_c^d f\,dy$

c. $\iint_R f\,dxdy = \int_c^d dy \int_a^b f\,dx$ d. $\iint_R f\,dxdy = \int_c^d dx \int_a^b f\,dy$

17. If the function f of x, y is integrable on a rectangle R so is $|f|$ then which one of the following is true?
    a. $\left|\iint_R f\,dx\,dy\right| = \iint_R |f|\,dx\,dy$
    b. $\left|\iint_R dx\,dy\right| \leq \iint_R |f|\,dx\,dy$
    c. $\left|\iint_R f\,dx\,dy\right| \neq \iint_R |f|\,dx\,dy$
    d. $\left|\iint_R f\,dx\,dy\right| \leq \iint_R |f|\,dx\,dy$

18. A subset F of a metric space (X,d) is said to be closed if
    a. F contains all its limit points
    b. F contains all interior points
    c. F contains all adherent points
    d. F is a nbhd of its points

19. Let (X,d) and (Y,d') be any two metric spaces. T function $f: X \to Y$ is said to be a homeomorphism if
    a. f is both one-one and onto
    b. f is continuous
    c. f^{-1} is continuous
    d. all of the above

20. A subset A of a compact metric space (X,d) is itself compact if and only if
    a. it is open in (X,d)
    b. it is compact in (X,d)
    c. it is closed in (X,d)
    d. it is bounded in (X,d)

Attempt ALL the questions.

Group 'B' [10×6 = 60]

1. Define the absolute and conditional convergence of the improper integral $\int_a^\infty f\,dx$. Show that the integral $\int_1^\infty \frac{\sin x}{x^p}\,dx$ is convergent for p>0.

OR

Prove that an infinite integral which converges (not necessarily absolutely) will remain convergent after the insertion of a factor which is bounded and monotonic.

2. Explain point-wise convergence of a sequence of functions. Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval [a,b], b>0.

OR

If a series $\sum_{n=1}^{\infty} f_n$ converges uniformly to f in [a,b] and x_0 is a point in [a,b] such that $\lim_{x \to x_0} f_n(x) = a_n$ (n = 1,2,3...) then prove that (i) $\sum_{n=1}^{\infty} a_n$ converges, and (ii) $\lim_{x \to x_0} f(x) = \sum_{n=1}^{\infty} a_n$

3. Define the radius of convergence of a power series. If a power series $\sum a_n x^n$ diverges for x = x', then prove that it diverges for every x = x', where |x''| > |x'|.

4. Use the Newton - Raphson method to find the root of the equation $x^4 - x - 10 = 0$ to three decimal places.

OR

If $f(x)$ be a polynomial of n^{th} degree in x then prove that the n^{th} difference of f(x) is constant and $\Delta^{n+1} f(x) = 0$

5. If (a,b) be a point of the domain of definition of a function f such that (i) f_x is continuous at (a,b), (ii) fy exists at (a,b), then prove that f is differentiable at (a,b).

6. If x = rsinθ sinϕ, z = rcosθ, then show that $\frac{\partial(xyz)}{\partial(r\theta\phi)} = r^2 \sin\theta$.

7. Prove that a bounded function f is integrable over a rectangle R, if and only if to every ε > 0, there corresponds δ > 0, such that for every partition φ of R with norm μ (φ) < δ, the oscillatory sum, U (φ,f) – L (φ,f) <ε.

8. Compute the integral: $\iiint_3$ xyzdxdydz over a domain bounded by

$x = 0, y = 0, z = 0, x + y + z = 1.$

9. Let (x, d) be a metric space and $Y \subseteq X$, then prove that a subset S of Y is open in (Y, d_Y) if and only if there exists a set G open in (x,d) such that $S = G \cap Y$.

OR

Show that the function d defined by $d(\{x_n\},\{y_n\}) = \left(\sum_{n=1}^{\infty}(x_n - y_n)^2\right)^{1/2}$, $\{x_n\}, \{y_n\} \in l_2$ is a metric on l_2.

10. Define compact metric space. Prove that every closed subset of a compact metric space is compact.

Group 'C' [2×10 = 20]

11. If f_x and f_y are both differentiable at a point (a,b) of the domain of definition of a function f, then prove that $f_{xy}(a,b) = f_{yx}(a,b)$.

12. Define complete metric space. let (x,d) be a complete metric space and y be a subspace of X. Then Y is complete if and only if it is closed in (x,d) prove it.

OR

Let (x,d_1) and (Y,d_2) be two metric spaces, then $f:X \rightarrow Y$ is continuous if and only if $f^{-1}(G)$ is open in X, wherever G' is open in Y, prove it.

Exam 2071

Group "A"

Attempt ALL the questions. Tick (√) the best answers. 20

1. The integral $\int_0^1 e^{-mx}\,dx$ converges for ....
   a. n>l
   b. n>–l
   c. n>l
   d. n>0
2. Every absolutely convergent integral is
   a. convergent
   b. conditionally convergent
   c. uniformly convergent
   d. non-convergent
3. Which of the following is not true?
   a. uniform convergence implies pointwise convergence
   b. non uniform convergence implies non pointwise convergence
   c. pointwise convergence implies uniform convergence
   d. non pointwise convergence implies non uniform convergence
4. The power series '$\Sigma a^n x^n$' which converges for all values of x is called
   a. region of convergence
   b. nowhere convergence
   c. everywhere convergence
   d. somewhere convergence
5. The radius of convergence of the power series $l + 2x + 3x^2 + 4x^3 +$ is
   a. 1
   b. –1
   c. ∞
   d. not exist
6. A series of functions ΣM_n converges uniformly on [a, b] if there exists a convergent series ZMn of positive numbers such that
   a. $|f_n(x)| \leq M_n$, for $x \in [a, b]$
   b. $|f_n(x)| \geq M_n$, for all n, and $x \in [a, b]$
   c. $|f_n(x)| \leq M_n$, for all n, and $x \in [a, b]$
   c. $|f_n(x)| \leq M_n$, for all n, and $x \in [a, b]$
   d. $|f_n(x)| \geq M_n$, for some $x \in [a, b]$
7. If a series $\sum_{n=1}^{\infty} f_n$ converges uniformly to f in [a, b] and x_0 is a point in [a, b] such that $\lim_{x \circledR x_0} f_n(x) = a_n$, n = 1,2......... then

a. $\sum_{n=1}^{\infty} a_n$ converges

b. $\sum_{n=1}^{\infty} a_n$ diverge

c. a_n converges

d. a_n diverges

8. A function f (x, y) is continuous at (a, b) if

a. $\lim_{(x, y) ® (a, b)} f(x, y) = f(a, b)$

b. $\lim_{(x, y) ® (a, b)} f(x, y) - f(a, b) < \varepsilon$

c. $\lim_{(a, b) ® (x, y)} f(x, y) = f(a, b)$

d. $\lim_{(a, b) ® (x, y)} f(x, y) - f(a, b)$

9. Let f(x, y) is a function of two variables and y = ϕ (x) is the root of f (x, y) = 0 then the implicit equation defined

a. y = ϕ (x)

b. ϕ (x) = 0

c. x = ϕ (y)

d. ϕ (y) = 0

10. Let F (x, y, z) = 0 is a function subject to the constraint G (x, y, z) = 0 then which of the following condition is satisfied at stationary points?

a. $F_xG_x - F_yG_y = 0$

b. $F_{xx}G_{yy} - F_{xy}G_{xy} = 0$

C. $F_xG_y - F_yG_x = 0$

d. $F_{xy}G_{xy} - F_{yy}G_{xx} = 0$

11. A closed point set consisting of a bounded domain plus boundary points is called

a. region

b. bounded domain

c. compact domain

d. compact region

12. If P* is refinement of a partition P then for a bounded function/, which of the following is not true?

a. $L(P^*, f) \geq L(P, f)$

b. $L(P^*, f) \geq U/(P, f)$

c. $U(P^*, f) \leq U(P, f)$

d. $L(P, f) \leq U(P^*, f)$

13. If the functions f_1, f_2, are integrable then which of the following is not integrable?

a. $f_1 + f_2$

b. $\frac{f_1}{f_2}$

c. $f_1 . f_2$

d. $|f_1 . f_2|$

14. The jaccobian for the x = r cos θ, y = r sin θ, z = z is

a. r

b. sin θ

c. r^2

d. r cos θ

15. The surface integrals taken over the opposite side of surface have

a. different sign

b. same sign

c. are positive

d. are negative

16. In any metric space, which of the following is not true ?

a. the union of any arbitrary family of open sets is open

b. The union of finite number of open sets is open

c. the intersection of any arbitrary family of closed sets is closed

d. the union of finite number of closed sets is closed

17. In a discrete metric space (X, d), which of the following is not true ?

a. union of open set is open

b. intersection of open set is open

c. every set is open

d. every set is closed

18. The numerical difference between the true value and approximate value is called

a. mistake

b. absolute error

c. round off

d. truncation

19. The method of false position is also known as

a. methods of chords

b. bisection

c. Newton-Raphson

d. iteration

20. Which of the following is related to Newton-Raphson Method ?

a. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

b. $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

c. $x_n = x_{n+1} - \frac{f(x_n)}{f'(x_n)}$ b. $x_n = x_{n+1}$

Attempt ALL the questions.

Group "B" 20

1. What is the statement for the Dirichlet's test for convergent ? Use this statement to show that $\int_1^{\infty} \frac{\sin x}{x^p}$ dx converges for p > 0.
2. If f and g are positive and f (x) $\leq$ g (x) for al x in [a, x] then prove that

   (a) $\int_a^{\infty} f\, dx$ converges if $\int_a^{\infty} g\, dx$ converges and

   (b) $\int_a^{\infty} g\, dx$ diverges if $\int_a^{\infty} f\, dx$ diverges
3. If a series Σf_n converges uniformly to f in an interval [a, b] and its term f_n are continuous at x_0 of the interval then the sum function f is also continuous at x_0. Prove it.

OR

Prove that a series of functions Σf_n defined on [a, b] converges uniformly on [a, b] if and only if for every $\varepsilon > 0$ and for all $x \in [a, b]$ there exists an integer K such that $| f_{n+1}(x) + f_{n+2}(x) + ... + f_{n+p}(x) | < \varepsilon$ for all $n \geq K, p \geq 1$.

4. Define power series. Explain the region of convergence of the power series with example.
5. Define differentiability of function of two variables. If a function is defined in (a, b) such that f_x is continuous at (a, b) and f_y exists at (a, b) then prove that f is differentiable at (a, b).

OR Define implicit function with example.

If $u = \frac{x^2+y^2+z^2}{x}, v = \frac{x^2+y^2+z^2}{y}, w = \frac{x^2+y^2+z^2}{z}$ then show that $\frac{\partial (x, y, z)}{\P (u, v, w)} = \frac{x^2y^2z^2}{(x^2+y^2+z^2)^3}$

6. Evaluate $\iint (y-2)\, dx\, dy$, over R = [1, 2; 3, 5]

OR

Define the partition of a rectangle. Prove that a bounded function f is integrable over a rectangle R is if and only if for $\varepsilon > 0$ there is a partition P of R such that U (P, f) – L (P, f) < ε.

7. In any metric space (X, d), prove that
   (i) the intersection of any arbitrary family of closed sets is closed
   (ii) the union of finite number of closed sets is closed.
8. Evaluate $I = \int_0^1 \frac{1}{1+x}$ correct to three decimal places with h = 0.125 using (i) trapezoidalrule (ii) Simpson's rule.

Group "C" 2×12=24

9. Define volume integral and evaluate. Compute the volume of the solid bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the surface of the praboloid $x^2 + y^2 = 3z$ where the surfaces intersect at z= 1.
10. Define metric space. Show that n-dimensional space $\Re^n$ is metric space. OR
    Define compact set with example. Prove that every compact subset of A of a metric space is compact.

Exam 2072

Group "A" 20

Attempt ALL the questions. Tick (√) the best answer.

1. For the Klein's four group G of elements e, a, b, c, which of the following is not true?

   a. $a^2 = b^2 = c^2 = e$ b. ab = ba = c

c. bc = cb = a

d. G is a cyclic group

2. Which of the following is false?

a. every cyclic group is abelian

b. isomorphic image of a cyclic group is not always cyclic

c. every subgroup of a cyclic group is cyclic

d. every group of prime order is cyclic

3. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, what is the value of σ^3?

a. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$

b. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \end{pmatrix}$

c. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$

4. Which of the following group <u>is not</u> cyclic?

a. $Z_9 \times Z_{14}$

b. $Z_{12} \times Z_{49}$

c. $Z_4 \times Z_6$

d. $Z_{15} \times Z_{28}$

5. What is the order of the alternating group A_4?

a. 12

b. 6

c. 24

d. 8

6. A homomorphism $\phi : G \to G'$ is called isomorphic if .....

a. ϕ is onto

b. ϕ is one-one and onto

c. ϕ is one-one

d. ϕ is into

7. IF $N \Delta G$, which of the following is true for all $x \in G$, $n \in N$?

a. $_xN = N_x$

b. $x^{-1}N_x = N$

c. $xN_x^{-1} = N$

d. all of the above

8. Which of the following statement <u>is not</u> true?

a. every field is an integral domain

b. a finite integral domain is a field

c. a skew field has zero divisors

d. an arbitrary intersection of sub rings is c subring

9. A ring R is called simple if it has ....

a. no proper ideals

b. proper ideals

c. no ideals

d. ideals

10. The ring $(Z_6, +, .)$ of integers modulo 6 has characteristic

a. 0

b. 2

c. 3

d. 4

11. $x^2 + 2 \in R[X]$ is a minimal polynomial of ....

a. $I\sqrt{2}$ over R

b. $\sqrt{2}$ over Q

c. 2i over $\Re$

d. -2 over R

12. What is the characteristics of the field of real numbers?

a. 1

b. 1

c. 2

d. ∞

13. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, what is the value of $\alpha + \beta + \gamma$?

a. p

b. -p

c. p^2

d. $q^2 - 2pr$

14. A square matrix A is called nilpotent of order n if ....

a. $A^n = A$

b. $A^n = 0$

c. $A^{n-1} \neq 0$

d. $A^n = O$ but $A^{n-1} \neq 0$

15. If $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$, what is the value of P.Q?

a. $x_1x_2+y_1y_2+z_1z_2$

b. $\sqrt{x_1x_2+y_1y_2+z_1z_2}$

c. $\sqrt{x_1^2 + y_1^2 + z_1^2} . \sqrt{x_2^2 + y_2^2 + z_2^2}$

d. $\sqrt{(x_1x_2)^2 + (y_1y_2)^2 + (z_1z_2)^2}$

16. The dimension of the vector space of all 2×3 metrices over R is

a. 2

b. 3

c. 5

d. 6

17. If $\pi : R^3 \to R^3$ be the projection mapping on xy - plane defined by

$\pi(x, y, z) = (x, y, o)$, then ker π is

a. $\{(o, c, o) : c \in \Re\}$
b. $\{(o, o, c) : c \in \Re\}$
c. $\{(c, o, o) : c \in \Re\}$
d. $\{(o, o, o)\}$

18. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ be reciprocal system of vectors then which of the following is true?

a. $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 0$
b. $\vec{a}.\vec{a}' = \vec{b}.\vec{b}' = \vec{c}.\vec{c}' = 1$
c. $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 0$
d. $\vec{a}.\vec{a}' + \vec{b}.\vec{b}' + \vec{c}.\vec{c}' = 1$

19. The necessary condition for vector function $\vec{f}$ of scalar variable t to have a constant magnitude is ....

a. $\vec{f}\frac{d\vec{f}}{dt} = 0$
b. $\vec{f} \times \frac{d\vec{f}}{dt} = 0$
c. $\vec{f}.\frac{d\vec{f}}{dt} = 0$
d. $\frac{d\vec{f}}{dt} = 0$

20. Which of the following is not meanifgful.

a. $\nabla\vec{r}$
b. $\nabla.\vec{r}$
c. $\nabla \times \vec{r}$
d. $\nabla^2\vec{r}$

Attempt ALL the questions

Group "B"

8×7=56

1. Define cyclic group. Prove that every cyclic group is abelian.
2. If H and K are subgroups of a group G, prove that HK is a subgroup of G if and only if HK = KH.

OR

3. A non-empty subset S of a ring R is a subring of R iff $\forall$ a, b $\in$ S, a - $\in$ S and $\in$ S, prove.
4. If $f : R \to R'$ is a ring homomorphism of R into R', prove that the kernel k_f of f is an ideal of R.

OR

Let R be a principal ideal domain and p $\in$ R. Prove that p is prime if it is irreducible.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$, if the ratio of two of the roots is 3 : 2.
6. Explain the characteristics of a field. Prove that the characteristics of a field F is either zero or a prime.

OR

Test for consistency and solve the equations $3x - 4y = 2$, $5x + 2y = 12$, $x - 3y = 1$.

7. Prove that the union of two subspaces of a vector space V is a subspace of V if and only if one is contained on the other.
8. Prove that the necessary and sufficient condition for a vector function $\vec{a}$ of scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$.

Group "C"

2×12 = 24

9. Define internal direct product of the normal subgroups. $N_1, N_2, ...N_n$ of a group G. If G be the internal direct product of normal subgroups $N_1, N_2, ...N_n$ prove that $G \approx N_1 \times N_2 \times ..... N_n$.
10. Define Euclidean domain and show that the set of integers is an Euclidean domain. Also prove that:
(a) Every Euclidean domain is a principal ideal domain.
(b) Every Euclidean domain R has a unit element.

OR

10. (a) Prove that a set of non-zero orthogonal vectors is linearly independent.
(b) If $T : V \to W$ be a linear transformation then prove that Ker T and Im T are subspace of V and W respectively.

(c)Differential Equation & Number Theory (Math. Ed. 334) Elective Group B

Exam 2068

Group "A"

20

Attempt ALL the questions. Tick (√) the best answers.

1. How many arbitrary constants are there in the general equation of the form Mdx + Ndy = 0 where M and N are function of x and y respectively?

a. one
b. two
c. three
d. four

2. If the solution of equation $P_0\frac{d^ny}{dx^n} + P_1\frac{dx^{n-1}y}{dx^{n-1}} + \dots\dots P_nY = 0$ is $Y = (C_1 + C_{2x} + C_{3x}{}^2 0 \dots\dots + C_nX^{n-1})_x$ mx then the nature of roots are

a. real and distinct
b. real and equal
c. non-repeated are imaginary
d. irrational and imaginary

3. The binomial expansion of $(1 + D)^{-2}$ is

a. $1 + D + D^2 + D^3 + \dots$
b. $1 - 2D + 4D^2 - 4D^3 + \dots\dots$
c. $1 - 2D + 3D^2 - 4D^2 + \dots$
d. $1 - 2D + 3D^2 - 4D^3 + \dots\dots$

4. The solution of differential equation with no arbitrary constant is

a. complete solution
b. particular solution
c. singular solution
d. general solution

5. If P + Q + 1 = in the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = x$ then a part of complementary function is

a. e^{-x}
b. e^x
c. e^{ax}
d. $y = x$

6. If the auxiliary equation of differential equation $D^2 + dD^1 = 0$ is $m^2 + 4 = 0$ then its complementary function C.F. is

a. C.F = $C_1 \cos x + C_2 \sin 2x$
b. C.F = $C1 \cos e^x + C_2 \sin e^{-x}$
c. C.F = $C_1 \cos 2x + C_2 \sin 2x$
d. C.F = $C_1 \cos 2x + C_2 \sin 2x$

7. Which one of the following is condition for the differential equation Pdx + Qdy + Rdz = 0 to be integrable?

a. $\left(\frac{dQ}{dz} - \frac{dR}{dy}\right) + Q\left(\frac{dR}{dx} - \frac{dP}{dz}\right) + R\left(\frac{dP}{dy} - \frac{dQ}{dx}\right) = 0$

b. $\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

c. $\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right)\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right)\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

d. $\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = 0$

8. The differential equation in the for $P_p + Q_p = R$ is

a. Charpit's equation
b. Largrage's equation
c. Monge's equation
d. Clairaut's equation

9. The particular integral of the differential is in the form $\frac{1}{(bD + aD')^n}\phi(ax + by)$ if F (a, b) = 0 is

a. $\frac{x^n}{b^n n!}\phi(ax + by)$
b. $\frac{x^{r+1}}{b(r+1)}\phi(ax + by)$
c. $\frac{1}{F(a, b)}\int\int\dots\dots\int\phi(u)\, du\, du \dots\dots du$
d. $\frac{x^r}{b(r + 1)}\phi(ax + by)$

10. Which one of the following triple is the Pythagorean triple?

a. 3, 5, 7
b. 3, 4, 5
c. 5, 6, 7
d. 1, 2, 3

11. Any prime p can be written as the sum of

a. two squares
b. three squares
c. four squares
d. one squares

12. An equation of the form $x^2 - dy^2 = 1$ is called the

a. Diophantine equation
b. Pell's equation
c. Lagrange's equation
d. Pythagorean equation

13. What is the value of τ (180)?

a. 180
b. 0
c. 9
d. 18

14. How many prime numbers can be formed of the type 4K + 1?

a. finite primes
b. infinite prime
c. no prime
d. only one prime

15. A positive integer n is equal to the sum of all its positive divisors excluding n itself in called

a. prime number
b. perfect number

c. rational number d. triangular number

16. If P be an odd prime and a, b, be integer which are relatively prime to p, then which one of the following symbols is not Legendre symbols?

a. If $a \equiv b \pmod p$ then $\left(\frac{a}{p}\right) \equiv \left(\frac{b}{p}\right)$ b. $\left(\frac{a^2}{p}\right) = 1$

c. $\frac{a}{p} = a^{\frac{R-1}{2}} \pmod p$ d. $\left(\frac{1}{p}\right) = -1$

17. The number in the form of $F_n = 2^{2N} + 1$, $n \geq 0$ is called

a. Mersenne numbers b. Perfect numbers

c. Fermat numbers d. Square numbers

18. Which one of the following sequence is Fibonacci sequence?

a. 1, 2, 3, 4,............ b. 1,1, 2, 3, 5, 8,...........

c. 2, 4, 8, 16,............ d. 1, –1, 1, –1, 1, –1,............

19. If n has a primitive root cf r and ind denote the index of a relative to r then which one of the following relation is not correct?

a. $\text{ind } ab \equiv \text{ind } a + \text{ind } b \pmod{\phi(n)}$

b. $\text{ind } a^k \equiv k \text{ ind } a \pmod{\phi(n)}$ for $k > 0$

c. $\text{ind } 1 \equiv 0 \pmod{\phi(n)}$ $\text{ind } r \equiv 1 \pmod{\phi(n)}$

d. $\text{ind } ab = \text{ind } a . \text{ind } b \pmod{\phi(N)}$

20. Let P be an odd prime and gcd (a, P) = 1. If the congruence $x^2 \equiv a$(mod) has a solution then a is said to be a

a. quadratic reciprocity b. quadratic residue

c. quadratic non residue d. quadratic quotient

Time: 3hrs

Attempt ALL the questions.

Group 'B' 8×7=56

1. Distinguish between general and singular solution. Find the singular and general solution of $y = x\frac{dy}{dx} + \frac{1}{2}\frac{dx}{xy}$.

2. Solve the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y$ given that $x + \frac{1}{x}$ is one integral.

OR

Solve the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx}\tan x + y\cos^2 x = 0$

3. Solve

$\frac{dx}{dt} + 4x + 3y = t$ $\frac{dx}{dt} + 2x + 5y = e^t$

4. Define complete and particular integral and solve Px + qy + Pq = 0 by Charpit's method.

OR

Solve (q + 1) = (p + 1)t by Monge's method.

5. Define primitive root of n. for $K \geq 3$, prove that the integer's 2^K has no primitive roots.

6. Define quadratic residues and quadratic non residues. Prove that if p is an odd prime then $\sum_{a=1}^{p=4}\left(\frac{p}{q}\right) = 0$. Hence show that there are precisely $\frac{p-1}{2}$ quadratic residues and $\frac{p-1}{2}$ quadratic non-residues of p.

7. Prove that there are infinitely many primes of the form 8K–1.

OR

Define Pythagorean triple and also prove that the area of a Pythagorean triangle can never be equal to a perfect integral.

8. Define Fibonacci numbers. Prove that the greatest divisor of two Fibonacci numbers is again a Fibonacci number, specifically

$\text{ged}(u_m, u_n) = u_d$ where $d = \text{ded}(m,n)$

Group 'C' 2×12=12

9. Solve (a) $r + s - 6t = y\cos x$. (b) $(4r - 4s + t) = 16\log((x+2y)$

OR

(a) Find the general solution of the equations

$$D^2 - 5DD^2 + 4D^{12})z = \sin(4x+y)$$

(b) Solve the partial differential equation

$$(D^2 - 2aDD^1 + a^2D^{12})z = f(y+ax)$$

10. (a) If p and q are distinct odd prime numbers then prove that

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left(\frac{p-1}{2}\right)\left(\frac{q-1}{2}\right)}$$

(b) For any n > 1, the positive integers less than n and relative prime to n is $\frac{1}{2}n\,\phi(n) = \sum k$

$$\frac{1}{2}n\,\phi(n) = \sum_{\substack{\gcd(k,n)\\ 1\le k<n}} k$$

Exam 2069

Group "A" 20

Attempt all the questions. Tick (√) the best answers.

1. What is the order and degree of $\frac{d^2y}{dx^2} = 5y + 3$?
   a. degree 1, order 1 b. degree 2, order 2
   c. degree 2, order 1 d. degree 1, order 2
2. The homogenous differential equation can be solved by putting
   a. $y=c^x$ b. $y=c$
   c. $y=vx$ d. $y=x^2$
3. IF Q e^{2x} V, where V is a function of x then the particular integral of differential equation is in the form.
   a. $\frac{1}{\int(d+a)}V$ b. $\frac{1}{\int(d-a)}.V$
   c. $\frac{1}{\int(d+a)}e^{m}$ d. $\frac{1}{\int(D)}.V$
4. The solution of differential equation with no arbitrary constants is
   a. complete solution b. particular solution
   c. singular solution d. general solution
5. If $y=e^{ax}$ is a part of complementary function of the linear differential equation of second order then which of the following relation is true?
   a. P+Q+1 b. 1-P+Q=O
   c. P+Qx d. $1+\frac{P}{a}+\frac{P}{a2}=0$
6. The two subsidiary equation of the equation Rr +Ss +Tt=V in solving Monge's method are
   a. $R.dx^2+Sdx.dy+Tdy^2$ 0 and Rdgdx+Tdpdy-Vdxdy=0
   b. Rdy^2-Sdex.dy+$Tdx^2$0 and Rdpdy+Tdgdx-Vdxdy=0
   c. R(ly^2+Sdxdy+$TdX^2$0 and Rdpdy+Tdgdx-Vdxdy=0
   d. Rdx^2+Sdx.dy-$Tdx^2$0 and Rdpdy-Tdgdx+Vdsdy=0
7. Which one of the following is equal to d[^{log}(X^2+y^2)]
   a. $\frac{1}{2}\log(x^2+y^2)$ b. $\frac{2xdy+2yd}{2x^2+y^2}$
   c. $\frac{2xdx+2ydy}{x^2+y^2}$ d. $\frac{xdy+ydx}{x^2}$
8. The complementary function of $(D^2+2DD'+d'^2)z=0$ is
   a. $\phi_1(y-x)+\phi_2(y+x)$ b. $\phi_1(y+x)+\phi_2(y+x)$
   c. $\phi_1(y+x)+x\phi_2(y+x)$ d. $\phi_1(y-x)+\phi_2(y-x)$
9. The normal form equation

a. $\frac{d^2y}{dx^2} + \left[Q - \frac{1}{4}p^2 \frac{1}{2}\frac{dp}{dx}\right] = 0$

b. $\frac{d^2v}{dx^2} + p\left[Q - \frac{1}{4}p^2 \frac{1}{2}\frac{dp}{dx}\right] = 0$

c. $\frac{d^2v}{dx^2} + p\left[q - \frac{1}{4}p^2 \frac{1}{2}\frac{dp}{dx}\right] = Xe\int^{\frac{p}{2}dx}$

d. $\frac{d^2v}{dx^2} + p\left[Q - \frac{p^2}{4} - \frac{1}{2}\frac{dp}{dx}\right] = e\int p^{dx}$

10. Which of the following is the particular integral $\frac{1}{D + D'}e^{2x+3y}$?

a. $\frac{1}{25}e^{2x+3y}$

b. $\frac{1}{5}e^{2x+3y}$

c. $\frac{1}{25}e^{2x+3y}$

d. e^{2x+3y}

11. Which of the following is the complementary equation of $\frac{\partial^2 u}{dx^2} - a^2\frac{\partial^2 u}{dx^2} = 0$?

a. $\phi_1(y - ax) = \phi_2(y + ax)$

b. $\phi_1(y + ax) + x\phi_2(y + ax)$

c. $\phi_1(y - ax) + \phi_2(y - ax)$

d. $\phi_1(Y + ax) + \phi_2(y - ax)$

12. What is the value of $\phi(100)$?

a. 100

b. 10

c. 40

d. 1

13. The value of $\sum_{d|10}\phi(d)$ is

a) 10

b) 9

c) 20

d) 5

14. In the language of cryptography, the codes are called

a) Plain text

b) eipher text

c) ciphers

d) encrypting

15. What is the value of ?(180)?

a) 18

b) 546

c) 180

d) 80

16. If P is the prime and pla then which one of the following relation is true?

a) $a^{P-1} \equiv 1(\text{Mod } P)$

b) $a^{P-1} \equiv -1(\text{Mod } P)$

c) $a^{P-1} \equiv p(\text{Mod } a)$

d) $a^{P+1} \equiv 1(\text{Mod } p)$

17. What is the gcd of (250, 500)?

a) 25

b) 125000

c) 250

d) 500

18. If gcd (a, n) = 1 and a is of order ϕ (n) modulo n then a is

a) square root

b) composite number

c) primitive root

d) prime number

19. Which of the following ratio is called Golden ratio?

a) $\frac{1+\sqrt{5}}{2}$

b) $\frac{1-\sqrt{5}}{2}$

c) $\frac{1+\sqrt{5}}{3}$

d) $\frac{1-\sqrt{5}}{32}$

20. If P is a prime and d|(P – 1), then how many solutions are there in $x^d - I\_ \equiv 0$ (MOd P)?

a) P

b) 0

c) 1

d) d

Attempt all the questions.

Group "B" 8×7=56

1. Define modal locus with suitable diagram. Obtain the general and singular solution of $y^2 - 2pxy + p^2(x^2-1)m^2$.

2. Solve the differential equation.

$$x\frac{d^2y}{dx} + (1-x)\frac{dy}{dx} - y = e^2$$

3. Solve the differential equation by changing into normal form:

$$\frac{d^2y}{dx} + (1-x)\frac{dy}{dx} - y = e^2$$

Solve by operational method:

4. Solve:

OR

Define regular singular points and irregular singular points in differential equation and find the solution in series of

$$\frac{d^2y}{dx} + (1-x)\frac{dy}{dx} - y = e^2$$

5. If n is a positive integer and gcd (a,n)1 then $\alpha\ \emptyset_{(n)} \equiv 1(MOd\ n)$ Prove it.

6. If P is a prime and K>0, prove that $\phi(p^k)$-p^k-p^{k-1}.

OR

For each positive integer n≥1, prove that n=

The sum being extended over all positive divisors of n.

7. Let P be an odd prime and gcd (a,p)=1, prove that a is a quadratic Residue of P if and only if .

8. Prove that the radius of the inscribed circle of a Pythagorean triangle is always an integer.

9. Solve the partial differentialequation

(a) $(D^2+DD^1+D^1-1)z=\sin(x+2y)$

(b) $r=a^2t$ by Monge's method.

OR

Find the general surface satisfying $t=6x^3y$, containing the two lines y=0=z, y=1=z.

10. (a) Prove that there are infinitely many primes of the form 4K+1.

(b) Prove that the greatest common divisor of two Fibonacci numbers is again a Fibonacci number. i.e. $Gcd(u_{m2},u_n)u_d$ where d gcd (m,n).

Exam 2070

Group "B"

8× 7 = 56

1. Define Clairaut's equation with an example. find the general and singular solution of $P^2 + px - y = 0$.

2. Solve the differential equation.

$$x^2\frac{d^2y}{dx^2} - (x^2+2x)\frac{dy}{dx} + (x+2)y = x^3e^x$$

OR, Solve $\frac{d^2y}{dx^2} + (\tan x - 3\cos x)\frac{dy}{dx} + 2\cos^2 x.y = \cos^4 x$.

3. Solve $\frac{d^2y}{dx^2} - 3x - 4y = 0,\ \frac{d^2y}{dt^2} + x + y = 0$

4. Define complete and particular integral and solve $P_q = px + qy$ by using Charpit's method.

OR, Solve $x^2r + 2xys + y^2t = 0$ by monge's method.

5. Define number theoretic function. How is the number theoretic function multiplicative? Prove that the function τ and σ are both multiplicative.

6. If n ≥ 1 and gcd (a, n) = 1 then prove $a^{(n)} \equiv 1 \pmod n$.

7. Define Euler's phi-function with an example. Prove that, if P is prime and k > 0 the $\phi(P^k) = P^k - P^{k-1}$

$$= P^k\left(1 - \frac{1}{P}\right).$$

OR, Define prime number with example. Prove that there are infinitely many primes of the from 4k + 1.

8. Define inscribed circle. Prove that the radius of inscribed circle of a Pythagorean triangle is always an integer.

Group "C"

2×12=24

9. Solve

(a) $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$

(b) $\frac{yzdx}{y-z} = \frac{yxdy}{z-x} = \frac{xydz}{x-y}$.

OR, (a) $\left(\frac{d^2y}{dx^2}+y\right)\cot x + 2\left(\frac{dy}{dx} + y\tan x\right) = \sec x$

(b) $(r - 2s + t) = \sin(2x + 3y)$.

10. Define Pythagorean triples with an example. Prove that all solution of the Pythagorean equation $x^2 + y^2 = z^2$ satisfying the condition gcd (x, y, z) = 1, 2|x, x > 0, y > 0, z > 0 are given by

$x = 2xt, y = s^2 - t^2$ $\quad$ $z = s^2 + t^2$

for integers $s > t > 0$ such that, gcd $(s, t) = 1$ and $\neq t \pmod 2$.

Group "A" 20

Attempt ALL the questions. Tick (√) the best answers.

1. The differential equation is in the form $M_{dx} + N_{dy} = 0$ and the equation is homogenous with $M_x + N_y \neq 0$ then its integrating factor is in the form ....

   a. $\frac{1}{M_x - N_y}$ b. $\frac{1}{M_x + N_y}$ c. $e^{\int f(x)dx}$ d. $e^{\int f(x)dx}$

2. If the solution of the equation $P_o \frac{d^ny}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + p_2 \frac{d^{n-2}y}{dx^{n-2}} \ldots\ldots p_n y = 0$ are real and equal then its solution is in the form ....

   a. $y = C_1 e^{m_1x} + C_2{}^{m_2x} + \ldots\ldots\ldots + C_n e^{m_nx}$

   b. $y = (C_1 + C_2X + C_3X^2 + \ldots\ldots\ldots + C_nX^{n-1}) e^{m1x}$

   c. $y = (C_1 \cos \beta x + C_2 \sin \beta x) e^{ax}$

   d. $y = C_1 e^{ax} \sinh (x\sqrt{\beta} + c_2)$.

3. What are the degrees and order of the differential equation $\left(\frac{d^2y}{dx^n}\right)^2 = \sqrt{\left(1 + \frac{dy}{dx}\right)^2}$?

   a. order 1, degree 2 b. order 2, degree 1
   c. order 2, degree 2 d. order 2, degree 0

4. What is the binomial expansion of $(1 + d)^{-1}$?

   a. $1 - D + D^2 - D^3 + \ldots\ldots\ldots$ b. $1 + D + D^2 + D^3 + \ldots\ldots\ldots$
   c. $1 + D - D^2 - D^3 + \ldots\ldots\ldots$ d. $1 - 2D + 3D^2 - 4D^3 + \ldots\ldots\ldots$

5. What is the condition for the integrability of the total differential equation of $Pdx + Qdy + Rdz = 0$?

   a. $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

   b. $P\left(\frac{\partial Q}{\partial z} - \frac{\partial Q}{\partial z}\right) + Q\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + R\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = 0$

   c. $P\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + Q\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

   d. $P\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + Q\left(\frac{\partial P}{\partial x} - \frac{\partial R}{\partial x}\right) + R\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = 0$

6. What is the value of $d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$?

   a. $\frac{x\,dy - y\,dx}{x^2}$ b. $\frac{x\,dx - x\,dy}{x^2 + y^2}$ c. $\frac{x\,dy - y\,dx}{x^2 + y^2}$ d. $\frac{x\,dx + y\,dx + z\,dz}{x^2 + y^2 + z^2}$

7. If $Q = e^{ax} V$ where V is a function of x, then the particular integral of differential equation is in the form

   a. $\frac{1}{f(D-a)}V$ b. $\frac{1}{f(D+A)}e^{ax}$ c. $\frac{1}{f(D)}V$ d. $\frac{1}{f(D+a)}V$

8. If $P + Qx = 0$ then a part of complementary function of the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$ is

   a. $y = x$ b. e^{ax} c. e^x d. e^{-x}

9. What is the name of differential equation which is in the form $P_p + Q_p = R$ where P, Q, R are function of x, y, z ?

   a. Pell's equation b. Fermat equation
   c. Larange's equation d. General equation

10. What are the pair of two subsidiary equation of $R_r + S_s + T_t = V$ by solving Monge's Method?

    a. $Rdx^2 + Sdxdy + Tdy^2 = 0$; $Rdpdy + Tdp.dy - Vdxdy = 0$

    b. $Rdy^2 - Sdxdy - Tdx^2 = 0$; $Rdpdy - Td.qdx - Vdxdy = 0$

    c. $Rdy^2 - Sdxdy + Tdx^2 = 0$; $Rdpdy + Tdp.dy - Vdxdy = 0$

    d. $Rdx^2 + Sdxdy - Tdx^2 = 0$; $Rdpdy - Tdq.dx + Vdxdy = 0$

11. If σ (n) denotes the sum of positive divisors of n, then what is the value of σ (15)?
a. 15 b. 4 c. 24 d. 150

12. If n = 30, a = 11 then which of the following relation is true?
a. $11^{\phi(30)} \equiv 1 \pmod{30}$
b. $11^{\phi(30)} \equiv 30 \pmod{11}$
c. $11^{\phi(30)} \equiv 0 \pmod{30}$
d. $11^{\phi(30)} \equiv -1 \pmod{30}$

13. If the integer a has order K modulo n and h > 0 then what is the order of a^h modulo n?
a. $\frac{k}{\gcd(h,k)}$ b. k c. h d. $\frac{\gcd(h, K)}{k}$

14. For $k \geq 3$, how many primitive roots does 2^k has?
a. 0 b. 1 c. 2 d. 3

15. If P and q are distinct odd prime then what is the value of $\left(\frac{P}{q}\right)\left(\frac{q}{P}\right)$?
a. 0 b. 1 c. $(-1)^{\left(\frac{P+1}{2}\right)\left(\frac{q+1}{2}\right)}$ d. $(-1)^{\left(\frac{P-1}{2}\right)\left(\frac{q-1}{2}\right)}$

16. Let a be an odd integer then how many solution are there of $x^2 \equiv a \pmod 2$?
a. 0 b. 1 c. 2 d. 3

17. In the language of cryptography the codes used to represent plain text is called
a. decrypting b. cipher text c. Plain text d. ciphers

18. Which of the following is Fermat's last theorem?
a. $x^n = y^n = z^n$ Where n > 2
b. $x^n = y^n = z^n$ Where n = 2
c. $x^n = y^n = z^n$ Where n < 2
d. all of the above

19. How many solution does Diophantine equation $x^4 + y^4 = z^2$ has in positive integers x, y, z?
a. 0 b. 1 c. 2 d. 4

20. The set of numbers is in the form 1, 1, 2, 3, 5, 8, 13, ... is .....
a. Fermat number b. Mersenne number
c. Fibonacci number d. Prime number

Exam 2071

Group "A" 20

Attempt ALL the questions. Tick (√) the best answers.

1. How many independent variables are there in an ordinary differential equation?
a. 0 b. 1
c. 2 d. 3

2. The integrating factor (I.F) of the equation Mdx + Ndy = 0 where $M_x + N_y \neq 0$ is
a. $\frac{1}{M_x - N_y}$
b. $\frac{1}{M_x + N_y}$
c. $e^{\int f(x)dx}$
d. $e^{\int f(y)\,dy}$

3. The binomial expansion of $(1 + D)^{-1}$ is
a. $I + D + D^2 + D3^+ +$ ....
b. $1 + D^2 + D^3 +$ ......
c. $1 - D + D^2 - D^3 +$ ....
d. $1 + 2D + 3D^2 +$......

4. The differential equation in the form y = px + f(p) with $p = \frac{dy}{dx}$ is ....
a. homogenous equation
b. general equation
c. Clairaut's equation
d. partial equation

5. The locus of points of intersection of the consecutive curves of the system which are obtained by assigning different values of c in $\phi(x, y, c) = 0$ is
a. curvature
b. the parabola
c. the envelope
d. hyperbola

6. If P + Qx = 0 then a part of complementary function of the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$ is.
a. y = x
b. e^{ax}
c. e^x
d. e^{-x}

7. The Monge's method of solving the equation is in the form Rr + Ss + Tt = V where R, S, T & V are functions of x, y, z, p and q then the value of r is given by

a. $\frac{dp - dy}{dx}$ b. $\frac{dq - sdx}{dy}$

c. $\frac{dp - sdy}{dx}$ d. $\frac{dp - sdy}{dy}$

8. An equation of the form $Pdx + Qdy + Rdz = 0$ where P, Q, R are functions of x, y, z is known as
a. partial differential equation b. total differential equation
c. ordinary differential equation d. general differential equation

9. The complementary function of the partial different equation $(D^2 + 2DD' + D'^2)z = 0$ is the form
a. $\phi_1(y - x) + \phi_2(y + x)$ b. $\phi_1(y + x) + x\,\phi_2(y + x)$
c. $\phi_1(y + x) - \phi_2(y + x)$ b. $\phi_1(y - x) + \phi_2(y + x)$

10. The complementary function of $r - s - \sigma t = xy$ is
a. $\phi_1(y + 3x) + \phi_2(y + 2x)$ b. $\phi_1(y + x) + \phi_2(y - x)$
c. $\phi_1(y - x) - \phi_2(y + x)$ b. $\phi_1(y + 3x) + \phi_2(y + 2x)$

11. Any function whose domain is the set of positive integer is called
a. objective function b. definite function
c. number theoretic function d. numeric function

12. Which one of the following pair of numbers are relatively prime?
a. 2,4 b. 5, 15
c. 3,9 d. 4,7

13. What is the value of ϕ (5)?
a. 4 b. 5
c. 6 d. 10

14. The value of $\sum_{d/10} \phi(d)$ is
a. 10 b. 9
c. 20 d. 5

15. In cryptograph, the process of converting from plain text to ciphertext is called
a. encrypting b. plain text
c. ciphers d. decrypting

16. If p is an odd prime and $\sum_{q-1}^{p-1} \left(\frac{p}{q}\right) = 0$ then how many quadratic residues are there?
a. $\frac{p-1}{2}$ b. $p - 1$
c. p d. $\frac{p+1}{2}$

17. Which one of the following numbers is not the form of odd prime?
a. 8K+1 b. 8K+3
c. 8K+5 d. 8K+4

18. The number which in the form $M_n = 2^n - 1, n > 1$ are called
a. Fermat numbers b. mersenne numbers
c. prime numbers d. composite numbers

19. Which one of the following Fermat numbers is divisible by 641 ?
a. F_2 b. F_3
c. F_4 d. F_3

20. What are the degrees and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\left(1 + \frac{dy}{dx}\right)^2}$?
a. order 1, degree 2 b. order 2, degree 1
c. order 2, degree 2 d. order 2, degree 0

Attempt ALL the questions.

Group "B"

8×756

1. Define singular solution. Find the singular solution of $y = x\frac{dy}{dx} + a\sqrt{1 + p^2}$.

2. Solve the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = 0$

3. Solve : $\frac{dx}{dt} + 5x + y = e^t$
$\frac{dy}{dt} - x + 3y = e^{2t}$

OR

Solve the differential equation $x^2\frac{d^2y}{dx^2} - (x^2 + 2x)\frac{dy}{dx} + (x + 2)\,y = x^3e^x$.

4. Test the condition of condition of integrability and solve
$(x^2 + yz)\,dx + (z^2 + zx)\,dy \ (y^2 - xy)\,dz = 0$

OR

Solve $r + (a + b)\,s + abt = xy$ by Monge's method.

5. Prove that the function τ and σ are both multiplicative functions.

6. If n is a positive integer and gcd (a, n) = 1 then prove that $a^{\phi(n)} \equiv 1$ (Mod n)

7. Prove that there are infinitely many primes of the form 4K + 1.

OR

Define perfect number with an example. Prove that if $2^k - 1$ is a prime (k >1) then $n = 2^{k-1}(2^k - 1)$ is perfect and every perfect number is of this form.

8. Define pythagorean triple and also prove that the area of a Pythagorean triangle can never be equal to a perfect integral.

Group "C"

2×12=24

9. (a) Define particular integral and solve
$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{dy^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = xy + e^{x+2y}$$
(b) Solve : $(D^3 - 4D^2D^1 + 4DD^{12})\,u = \cos(y + 2x)$.

OR

Find the surface satisfying $t = 6x^3y$, containing the two lines $y = 0 = z$, $y = 1 = z$.

10.(a) Define quadratic residue and quadratic non-residue of odd prime p. Let P be an odd prime and gcd(a, p) = 1. Prove that a is a quadratic residence of p is and only if
$a^{\frac{p-1}{2}} \equiv 1(\text{Mod}P)$

(b) Prove that if a has order Kmodulo n then $a^i \equiv a^j$ (Modn) if and only if $i \equiv j$ (Mod K)

Exam 2072

Group "A"

Attempt ALL the question. Tick (√) the best answers.

1. What is the order and degree of the differential equation
$\frac{d^2y}{dx^2} = \left[7 + 2\left(\frac{d^2y}{dx^2}\right)\right]^{1/2}$?
a. degree 1, order 1
b. degree 1, order 2
c. degree 2, order 2
d. degree 2, order 1

2. The solution which is obtained from the general solution by giving some particular values to arbitrary constants is called?
a. particular solution
b. general solution
c. particular integral
d. general integral

3. What is the expanded form of $(1 + D)^{-2}$?
a. $1 - 2D + 3D^2 + \ldots\ldots$
b. $1 + 2D + 3D^2 + 4D^3\ldots\ldots$
c. $1 + D + D^2 + D^3 + \ldots\ldots$
d. $1 - D + D^2 + D^3 \ldots\ldots$

4. The locus of ultimate point of intersection of consecutive curve is called
a. cuspidal locus
b. nodal locus
c. tac locus
d. envelope

5. The differential equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = X$ and $P + Qx = 0$ then wha is a part of complementary function?

a. $y = e^{ax}$

b. $y = x$

c. $y = x^2$

d. $y = e^{-x}$

6. What is the condition of integrability of the differential equation of the form $Pdx + Qdy + Rdz = 0$?

a. $\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} + \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = 0$

b. $P\frac{\partial Q}{\partial z} + Q\frac{\partial R}{\partial x} + R\frac{\partial P}{\partial y} = 0$

c. $P^{\partial R}\sqrt{\partial y} + Q\frac{\partial P}{\partial z} + R\frac{\partial Q}{\partial x} = 0$

d. $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$

7. The locus $Pdx + Qdy + Rdz = 0$ is orthogonal to the locus of

a. $\frac{dx}{1} = \frac{dy}{2} = \frac{dz}{3}$

b. $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

c. $\frac{P}{dx} = \frac{Q}{dy} = \frac{R}{dz}$

d. $Pdx + Qdy + Rdz = 0$

8. What is the complete integral of $Pq = k$?

a. $z = ax + c$

b. $z = ax + ky + c$

c. $z = ax\left(\frac{k}{a}\right)y + c$

d. $z = x\left(\frac{k}{a}\right)y + c$

9. What is the complementary function (C.F) of $(D^3 - 6DD' + 9D'^2)Z = 12x^2 + 36xy$?

a. $\phi_1(y + 3x) + x\,\phi_2(y + 3x)$

b. $\phi_1(y - 3x) + \phi_2(y + 3x)$

c. $\phi_1(y + 3x) - x\,\phi_2(y + 3x)$

d. $\phi_1(y - 3x) + x\,\phi_2(y - 3x)$

10. To solve the equation $Rr + Ss + Tt = V$ by Monge's method, r stands for

a. $\frac{dP - Sdy}{dx}$

b. $\frac{dq - Sdx}{dy}$

c. $\frac{dP + dy}{dx}$

d. $\frac{dp - Sdy}{dy}$

11. Any function whose domain is the set of positive integers is called

a. objective function

b. definite function

c. numeric function

d. number theoretic function

12. Which of the following numbers are not relatively prime

a. (2, 5)

b. (3, 15)

c. (3, 7)

d. (4,19)

13. If n = 30, a = 11 then which of the following relation is true?

a. $11^{\phi(30)} \equiv 1 \ (\text{Mod } 30)$

b. $11^{\phi(30)} \equiv 30 \ (\text{Mod } 11)$

c. $11^{\phi(30)} \equiv 0 \ (\text{Mod } 30)$

d. $^{\phi(30)} \equiv -1 (\text{Mod } 30)$

14. If P is an odd prime and $\sum_{a-1}^{p-1} (a/p) = 0$ then how many quadratic residue of P are there?

a. $P - 1$

b. $P + 1$

c. $\frac{P + 1}{2}$

d. $\frac{P - 1}{2}$

15. Let a be an odd integer then how many solutions are there if $x^2 \equiv a(\text{mod } 2)$?

a. 0

b. 1

c. 2

d. infinite

16. The process of changing cipher text back to plain text is called

a. decrypting

b. ciphering

c. enciphering

d. encrypting

17. The number in the form $F_n = 2^{2n} + 1, n \geq 0$ is

a. Prime number

b. perfect number

c. Fermat number

d. Mersunn number

18. An odd prime p is expressible as a sum of two squares if and only if

a. $P \equiv 1$ (Mod 2)
b. $P \equiv 1$ (Mod 3)
c. $P \equiv -1$ (Mod 4)
d. $P \equiv 1$ (Mod 4)

19. How many incongruent solution does $x^3 \equiv 11$ (Mod 19) have?
a. 0
b. 1
c. 2
d. 3

20. Which of the following is not the form of an odd prime?
a. 8k + 1
b. 8k + 3
c. 8k + 4
d. 8k + 5

Attempts ALL the questions.

Group "B" 8×7=56

1. Define general and singular solution. Find the general and singular solution of the equation $y = x\frac{dy}{dx} + \frac{1}{2}\frac{dx}{dy}$.

2. Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = 0$ given that $y = x^3$ is a solution.

3. Solve: $\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + y\cos^2x = 0$.

OR

Find the series solution of the equation $(1 - x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$.

4. Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 12(x + y)$

OR

Find the complete integral of $p(q^2 + 1) = q(z - b)$ by Charpit method.

5. If P is a prime and d/p-1 then prove that there are exactly $\phi(d)$ incongruent integer having d mod p.

6. Let the integer 'a' have order k modulo n then prove that $a^h \equiv 1$ (Mod n) if and only if k/h.

OR

Prove that the function ϕ is multiplicative.

7. Define primitive root and also find the primitive root of P = 41.

8. If P is an odd prime then prove that $\sum_{a=1}^{p-1}\left(\frac{a}{p}\right) = 0$.

Group "C" 2×12 = 24

9. Solve (a) $\frac{d^2y}{dx^2} - 2x^2\frac{dy}{dx} + 4xy = x^2 + 2x + 2$ in power of x.

(b) $P\tan x + q\tan y = \tan z$.

Solve $r - 2s + t = \sin(2x + 3y)$ by Monge's method.

10. (a) Define a finite continued fraction and prove that any rational number can be written as a finite continued fraction.

(b) Define Fermat number. Prove that the Fermat number F_5 is divisible by 641.