5. State the principal of virtual work for a system of coplanar forces acting on a particle. Five weightless rods of equal length are joined together so as to form a rhombus ABCD with diagonal BD. If a weight W be attached to C and the W system be suspended from A, show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$ [1+4]

6. A given length 2s of a uniform chain has to be hung between two pts at the same horizontal level. The tension has not to exceed the weight of the length b of the chain. Show that the greatest span is $\sqrt{b^2 - s^2}\log\left(\frac{b+s}{b-s}\right)$ [5]

OR

A uniform chain of length ℓ is suspended from two points A and B in the same horizontal line. If the tension at A is n times that at the lowest point, show that span AB is $\frac{l}{\sqrt{n^2-1}}\log(n+\sqrt{n^2-1})$. [5]

7. Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$ the axis x and latus rectum. [5]

8. In a S.H.M. of amplitude a and period T, prove that

$\int_0^T V^2\,dt = \frac{2N^2a^2}{T}$, where the symbols have their usual meaning. [5]

9. If the radial and transverse velocities of a particle are always proportional to each other, prove that the path in an equiangular spiral

OR

A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always zero. Prove that $\ddot{Q} = 2\theta^2\cot\theta$.

10. A particle slides down the arc of smooth cycloid whose axis is vertical and vertex downwards. Discuss the motion. [5]

11. Define central force and central orbit. If the orbit is a cardoid $r = a(1 + \cos\theta)$, find the law of force. [5]

12. Determine M.I. of a hollow sphere about its diameter, a & b being external and internal diameter.

Or

Determine MI of truncated cone about its axis a, b being radii of ends.

Tribhuvan University, 2072

Attempt ALL the questions

Group "A" $3\times10=30$

1. A carpenter has 90, 80 and 50 running feet respectively of teak, plywood and rosewood. The product A requires 2, 1 and 1 running feet and product B requires 1, 2 and 1 running feet of teak, plywood and rosewood respectively. If A would sell for Rs. 480 and B would sell for Rs. 400 per unit, how much of each should he make and sell in order to obtain the maximum gross income out of his stock of wood?

(a) Formulate this problem as a linear programming problem.

(b) Use graphical method to solve the problem indicate clearly the feasible region on a graph. [5+5]

2. What is meant by a basic solution to the system of m linear non-homogeneous equations in n unknown ($m < n$)? Prove that every extreme point of the convex set of feasible solution is a basis feasible solution to an L.P.P. [2+8]

Or

Define extreme point of convex set. Show that $S = \{(x_1, x_2) : 2x_1 + 3x_1 = 7\}$ in R^2 is a convex set. Find the convex hull of the set of points (3, 0), (0, 4), (0, 0) and (1, 1). Express the point (1, 1) of the convex hull as convex combination of the extreme points. [1+3+3+3]

3. Consider the following transportation problem.

Warehouse

Factory		W_1	W_2	W_3	W_4	Factory Capacity
	F_1	19	30	50	10	7
	F_2	70	30	40	60	9
	F_3	40	8	70	20	18
	Warehouse requirement	5	8	7	14	

(a) Determine whether the given TP is balance or not.

(b) Find an initial basic feasible solution using Vogel's approximation. Is the initial basic feasible solution degenerate?

(c) Is the initial basic feasible solution on optimal solution? If not, obtain the optimal solution. [1+4+5]

Group "B" 9×5=45

4. Use simplex algorithm show that the following L.P.P.

$\text{Max}Z = 5x_1 + 2x_2$

Subject to the constraints;

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \le 0.$

has basic alternative optimal solution. [5]

5. Obtain the dual of the following linear programming problem min $Z = x_1 + x_2 + x_3$ subject to the constraints

$x_1 - 3x_2 + 4x_3 = 5;\ x_1 - 2x_2 \le 3;\ 2x_2 - x_3 \ge 4;\ x_1, x_2, x_3 \ge 0$ [5]

6. Show that the number of basic variables in a transportation problem is at most $m + n - 1$, where m = number and n = number of destination. [5]

OR

What is degeneracy in transportation problem? In which stages the problem of degeneracy may occur? Explain how to resolve the degeneracy in different stage. [1+1+3]

7. Consider the problem of assigning four works to four persons. The assignment cost are given as follows.

		Persons			
		A	B	C	D
Works	I	12	30	21	15
	II	18	33	9	31
	III	44	25	24	21
	IV	23	30	28	14

How should the works be allotted to persons so s to optimize the total cost?

8. Six jobs go first over machine I and then over machine II. The order of the completion of jobs has no significance. The following table gives the machine times in hours for six jobs and the two machines:

Job No. →	1	2	3	4	5	6
Time on machine I	5	9	4	7	8	6
Time on Machine II	7	4	8	3	9	5

Find the sequence of jobs that minimizes the total elapsed time to complete the jobs. Also find the idle time for each machine. [2+3]

9. Solve that following 2×3 games graphically

Player B

$$\text{Player A} \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

OR

Use dominance principle to solve the following 3×3 game: [5]

Player B

$$\text{Player A} \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

10. Explain for maxima and minima of the function

$Z = f(x_1, x_2, x_3) = x_1^2 + x_2^2 + {}_3^2$

Subject to the constraints $= x_1 + x_2 + x_3 = 1$. [5]

OR

Investigate the function

$f(x, y, z) = -x^3 + 3xz + 2y - y^2 - 3z^2$

for maximum or minimum. [5]

11. Prove that the set of all feasible solutions to an L.P.P. is a convex set. [5]

12. Find the solution of the difference equation.

$y_{x+2} - 4y_{x+1} + 3y_x = 5^x$

satisfying the initial conditions $y_0 = 2$ and $y_1 = 1$. [5]

Tribhuvan University, 2072

Attempt ALL the questions

Group "A" 3×10=30

1. State the condition under which A.M., G.M. and H.M. are most appropriate to describe the measure of central tendencies.

Explain why standard deviation is considered as the ideal measure of dispersion.

Prove that the standard deviation is independent on the change of origin and scale. [3+2+5]

Or

Define the moment of a variable about any point.

Prove that $\bar{X} = A + \mu_1^1$ where the symbols have their usual meanings. The first four moments of a distribution about its origin are 1, 4, 10 and 46, find the mean, s.d. and β_2. [1+3+6]

2. Define Spearmen's correlation coefficient. Show that the coefficient of correlation is the G.M. between two regression coefficients. A computer while calculating the correlation coefficient between two variates x and y from 25 pairs of observation obtained the following constants.

$n = 25, \Sigma x = 125, \Sigma x^2 = 650, \Sigma y = 100, \Sigma y^2 = 460, \Sigma xy = 508$. It was however, later discovered at the time of checking that he had mistakenly copied two pairs (x, y) as (6, 14) and (8, 6) while the correct pair was (8, 12) and (6, 8). Obtain the correct value of the correlation coefficient. [1+2+7]

3. State the addition and multiplicative laws of probabilities, if A and B are not two mutually exclusive events then prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

An urn contains 8 white and 7 black balls. 4 balls are drawn one by one without replacement. What is the probability that white and black balls appear alternatively. [1+2+2+5]

Group "B"

9×5=45

4. Draw a percentage bar diagram to represent the following expenses of a certain family in one month [5]

Item	Expenditure (in Rs.)
Food	2050
Cloth	600
House rent	1200
Fuel	1300
Education	750

5. Define standard deviation. Show that the mean deviation about the mean is not greater than the standard deviation. [5]

Or

Find the mean and standard deviation of the numbers 1,2, 3,4, ... n, the frequency of each being unity.

6. What are Benoullis trials ? If the random variable X denotes the number of successes in a trial and if p and q denotes the probability of successes and failures in n number of independent trials, then prove that the probability of r successes

$P(X = r) = nc_r p^2 q^{n-r}$ r = 0, 1, 2, ..., n .

Also find the probability of no success. [1+3+1]

7. The incidence of an occupation disease in an industry is such that the workers have a 20% chance of suffering from it what is the probability that out of six workers four or more will contract the disease ? [5]

8. From the following data find out the line of regression of y on x and hence find

the coefficient of correlation.

$\Sigma x = 250$, $\Sigma y = 300$, $\Sigma xy = 7{,}900$, $\Sigma x^2 = 6{,}500$, and $n = 10$. [3+2]

Or

Prove that the correlation coefficient between the two variable is independent of the change of origin and scale. [5]

9. If X is a random variable and a, b are constants prove that

$E(aX + b) = aE(X) + b$

$var(aX + b) = a^2 \, var \, X$ [5]

10. Suppose a continuous variable x takes values between 0 and 2 with p.d.f.

$f(x) = x$ for $0 \le x < 1$

$= 2 - x$ for $1 \le x < 2$

$= 0$ for $x \ge 2$

Find the mean and variance of the p.d.f [2+3]

11. Find the most plausible values of x and y from the following equation.

$x - 5y + 4 = 0$ $\quad$ $2x - 3y + 5 = 0$

$x + 2y - 3 = 0$ $\quad$ $4x + 3y + 1 = 0$ [5]

12. What is t-distribution ? State the properties of t-distribution. [5]

Or

In a sample survey of 1000 house wives in a city, 23% preferred LG brand microwave, find 99% confidence limits for the percentage of all housewives in the city preferring that brand.

Applied Statistics (Stat.332) (New + Old)2068

Bachelor Level/III Year/Sc. & Tech. + Humanities

Full Marks: 100 **Time: 3 hrs.**

1. (Compulsory) Attempt any SIX questions. 6x5=30
   a) Compare crude death rate with standardized death rate, Which one is better and why?
   b) Define mean length of generation and net reproduction rate. Derive a relation between them.
   c) What is a life table? What are its components?
   d) Describe compound interest population growth model. If growth rate *(r)* = 0.019, find the time period for a given population to double in its size.
   e) Describe national income and its components.
   f) Distinguish between control charts for attributes and variables.
   g) Explain the method of moving averages in time series analysis. Also specify its drawbacks.
   h) Distinguish between un-weighted and weighted aggregate methods in the construction of index numbers.

Group "A"

Attempt any FIVE questions. 5x7=35

2. What are the sources of demographic data in Nepal? Describe about the information usually collected in population census of Nepal.
3. Which population model do you think is appropriate for short period

population projection and why? Also derive the model.

4. Describe how abridged life table is constructed using census or demographic sample survey data.

5. Compute net reproduction rate (NRR) from the following data assuming sex ratio at birth is 101 males per 100 females.

Age group	Children born to 1000 women	Mortality rate per 1000
15-19	80	12
20-24	205	16
25-29	140	15
30-34	82	20
35-39	39	22
40-44	20	25
45-49	8	30

6. Distinguish between crude birth rate (CBR), age specific birth rate (ASBR) and total fertility rate (TFR).

7. Write short notes on:
a. Whipple's index
b. Age dependency ratio
c. Exponential model

Group "B"

Attempt any FIVE questions. 5x7=35

8. What do you mean by control charts? Describe how $\bar{X}$ and R charts 'are

9. Explain process control in industrial statistics. In sampling inspection plans discuss (a) consumers risk (b) producers risk and (c) average sample number (ASN).

10. Define index number. Compare between fixed based and chain based index numbers.

11. Construct the cost of living index number for the year 2009 taking base year as 2007.

Item	Unit	Price (2007)	Price (2009)	Weight in %
A	Kg.	0.6	0. 75	10
B	Litre	0.8	0.90	25
C	Dozen	2.0	2.50	20
D	Kg.	0.7	0.95	40
E	One pair	9.0	11.0	5

12. Distinguish between ratio to moving average and ratio to trend methods for analyzing seasonal fluctuations in time series data.

13. Discuss what you known about coverage, sources and limitations of official statistics in Nepal.

Bachelor Level/Sc. & Tech. + Huma./III Year **Full Marks: 75**
Mathematical Analysis (Math. 331) **Time: 3 hrs.**

Attempt All the questions.

Group "A" 5×7=35

1. Define absolute and conditional convergence of a series. Prove that absolute convergence of Σa_n implies the convergence. Let Σa_n be a given series with real-valued terms and define.

$$Pn = \frac{|a_n| + a_n}{2},\ q_n = \frac{|a_n| - a_n}{2}\ (n = 1, 2, ...)$$

Then prove that if Σa_n is conditionally convergent, both Σp_n and Σq_n diverge.

2. Prove that every Cauchy sequence in $\Re^n$ is convergent. Show that if a real sequence $\{a_n\}$ is given by

$|a_{n+2} - a_{n+2}| \leq \frac{1}{2}\ |a_{n+1} - a_n|$ for all $n \geq 1$ then it converges. [5+2]

OR

State and prove the sign preserving property of a continuous function.

Let $f: [a, b] \to \Re$ be continuous on [a, b]. Prove that

(i) If $f(a) f(b) \leq 0$, then $\exists$ a point $c \in (a, b)$ such that $f(c) = 0$.

(ii) If $f(a) \neq f(b)$, then f takes every value between f (a) & f (b) in (a, b).

[3+3+1]

3. Define directional derivative and total derivative at a point. Obtain the relation between total and directional derivative.

4. Define Riemann - Stieltijes sum of bounded function.

If $f \in R(\alpha)$, $g \in R(\alpha)$ on [a, b], then $c_1 f + c_2 g \in R(\alpha)$ on [a, b], where c_1 and c_2 are two constants and also we have

$$\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b f d\alpha + c_2 \int_a^b g d\alpha$$

5. Define primitive of f and hence state and prove second fundamental theorem of integral calculus. [1+6]

OR

Define a function of bounded variation on [a, b]. Prove that if f is of bounded variation on [a, b] and $c \in (a, b)$, then f is of bounded variation on [a, c] and on [c, b] and we have

$$V_f(a, b) = V_f(a, c) + V_f(c, b)$$ [1+6]

Group "B" 10×4=40

6. Assume that $f_n \to f$ uniformly on S and that each f_n is bounded on S. Prove that $\{f_n\}$ is uniformly bounded on S. [4]

OR

Define an open set in $\Re^n$.

Prove that the intersection of a finite collection of open sets is open. [1+3]

7. Define a compact subset in $\Re^n$.

Let X be a closed subset of a compact metric space M. Then prove that X is compact.

8. What do you mean by Cauchy sequence in $\Re$? Show that the sequence in Euclidian metric $\Re$. [1+3]

9. Let f be a function of bounded variation on [a, b]. Let V be defined on [a, b] as follows :

$V(x) = V_f(a, x)$ for $a < x \leq b$ and $V(a) = 0$. Then

(i) V is increasing function on [a, b]

(ii) V − f is an increasing function on [a, b]. [2+2]

OR

Determine whether or not the function

$f(x) = x^2 \sin(^1/_x)$ if $x \neq 0$

$f(x) = 0$ is of bounded variation on [0, 1]. [4]

10. Let $\alpha \nearrow$ on [a, b]. If $f \in R(\alpha)$ on [a, b], then prove that $f^2 \in R(\alpha)$ on [a, b]. [4]

11. State and prove first mean value theorem for Riemann - Stieltijes integrals.

OR [4]

State and prove first fundamental theorem of integral calculus.

12. Prove that if $\Sigma a_n = S$, then every series Σb_n obtained from Σa_n by inserting parentheses also converges on S. Illustrate with an example that removing parentheses may destroy the convergence.

13. Define the term "absolutely convergent. " Show the integral $\int_0^\infty \frac{\sin x}{x} dx$ does not converge absolutely. [1+3]

14. Let $f : f \to \Re^m$ be differentiable at an interior point c on S, where $S \subseteq \Re^n$. If $V = V_1u_1 + \ldots\ldots\ldots + V_nu_n$, where, u1, ..., un are the unit coordinate vectors in $\Re^n$, then prove that

$$f'(c)(u) = \sum_{k=1}^{n} V_k D_k f(c)$$

OR [4]

State limit comparison test for convergence of an improper integral.

Hence, show that $\int_0^\infty e^{-x^2} dx$ converges but $\int_0^\infty \frac{1}{\sqrt{1+2x^2}} dx$ diverges.

15. Let $f : S \to T$ be a function from one metric space (S, d_S) to another (T, d_T). Prove that f is continuous on S if and only if, for every open set Y in T, the inverse image $f^{-1}(Y)$ is open is S.

Bachelor Level/III Year / Sc. & Tech. + Hum. **Full Marks: 75**

Advanced Calculus VI Paper (332) **Time: 3 hrs.**

Attempt All the questions.

Group "A" 5×7=35

1. Solve : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 4$, by variation of parameter method. [7]

2. Define curvature and torsion at a point on the space. Prove that for curve r = r (t) and r = r (s)

$$k = \frac{|\dot{r} \times \ddot{r}|}{|\dot{r}|^3} \text{ and } \tau = \frac{[\dot{r}\ \ddot{r}\ \dddot{r}]}{|\dot{r} \times \ddot{r}|^3}$$ [2+5]

OR

Define osculating plane. Find the equation of the osculating plane at the point t for the curve $r = (3t, 3t^2, 2t^3)$

3. State Gauss's divergence theorem for Cartesian form and evaluate

$$\iint_S [x^2\,dy\,dz + y^2\,dz\,dx + 2z(xy - x - y)\,dx\,dy]$$

where S is the surface of the cube, $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$. [2+5]

4. Define harmonic function, harmonic conjugate and Cauchy-Riemann conditions. Show that $V(x, y) = 3x^2y - y^3$ is harmonic and find the conjugate $u(x, y)$. [1+1+1+2+2]

5. Define Fourier cosine and sine series. Find a series of sines and cosines of multiples of x which will represent $f(x) = x$ in $-\pi \le x \le \pi$. [2+5]

OR

Define Fourier series and its coefficients. Expand $f(x) = x^2$ for $\pi \le x \le \pi$ in a Fourier series. [2+5]

Group "B" 10×4=40

6. Solve $\frac{d^2y}{dx^2} - a\left(\frac{dy}{dx}\right)^2 = 0$.

7. Solve : $4x^2\frac{d^2y}{dx^2} + 4x^2\frac{dy}{dx} + (x^8 + 6x^4 + 4)\,y = 0$, by removing the first derivative.

OR

Solve : $(x + a)^2\frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$

8. Solve : $yz \log z\,dx - zx \log z\,dy + xydz = 0$

OR

Find the general solution of $(y - z)\,p + (x - y)\,q = z - x$

9. Find the equation of the integral surface of the differential equation: $2y(z - 3)\,p + (2x - z)\,q = y(2x - 3)$, which passes through the circle $z = 0, x^2 + y^2 = 2x$.

OR

Solve the Charpit's method $p^2x + q^2y = z$

10. Solve : $z(qs - pt) = pq^2$, by Monge's method.

OR

Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 12(x + y)$

11. Show that the principal normal at consecutive points do not intersect unless $T = 0$.

12. Evaluate $\int_c \vec{F}.\,d\vec{r}$ where $\vec{F} = x\vec{i} - xy\,\vec{j}$ from the origin to the point (1, 1) along the parabola $y^2 = x$

13. Evaluate $\int_r (2x + y)\,dV$, where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$.

14. If $f(z) = \frac{x^3y\,(y - ix)}{x^6 + y^2}$, $z \ne 0$, $f(0) = 0$, prove that $\frac{f(z) - f(0)}{z} \to$ as $z \to 0$ along any radius vector, but not as $z \to 0$ in any manner.

OR

Value of z, show that function $z = e^{-v}(\cos u + i \sin u)$, $w = u + iv$ ceases to be analytic.

15. Find the Fourier series for the function $f(x) = |x|, -\pi \leq x \leq \pi$.

Tribhuvan University, 2069

Group "A"

1. **(Compulsory) Attempt any SIX questions.** 6x5=30

   a. Explain what do you mean by Analysis of Variance. Write down its lasic assumptions.

   b. Explain in brief the principles of experimental design.

   c. How can the sum of squares of analysis of variance for a randomized block design with 'K' treatments and 'b' blocks in case of one observation per experimental unit be calculated?

   d. Explain what is meant by main effects and interaction effects in factorial experiment.

   e. What is a sample? In what situations sampling is inevitable?

   f. Show that in simple random sampling the sample mean is an unbiased estimate of the population mean.

   g. Describe the procedure of stratified random sampling.

Group "B"

Attempt any FIVE questions. 5x7=35

2. Discuss the advantages of two way classification over one way classification in analysis of variance technique.

3. The following a partial ANOVA table

Source	Sum of squares	d.f.	Mean sum of square	f-value
Treaments		2		
Error			20	
	500	11		

Complete the table and answer the following questions:

   i. How many treatments are there?

   ii. What is the total sample size?

   iii. What is the critical value of F?

   iv. Write down the null and alternative hypotheses.

   v. What is your conclusion regarding the null hypotheses

4. Give out layout of 4x4 Latin square design. Discuss advantages and disadvantages of Latin square design.

5. What do you mean by factorial experiment? Explain the technique of measuring main effects in 2^2 - experiment.

6. In what ways can the analysis of variance of data of a randomized block design one missing value be carried out?

7. Distinguish between partial and total confounding in factorial experiments. Discuss the main consideration in the use of confounded factorial design.

Group "C"

Attempt any FIVE questions. 5x7=35

8. Explain the different steps used in conducting a sample survey.
9. Differentiate between sampling and non sampling error. The estimate of the proportion is to be within plus or minus 0.05 with a 95 percent level of confidence. The best estimate of population proportion is 0.45. How large a sample is required?
10. In a stratified random sampling with a cost function $C = a + \sum C_h n_h$ prove that the variance of estimated mean $\bar{y}_{st}$ is i imum when nh is

    is proportional to $N_n S_n / \sqrt{c_n}$, where notations have usual meanings.
11. What do you understand by systematic sampling? Show that systematic sampling will be more efficient as compared to simple random sampling without replacement if $p < \frac{1}{(hk-1)}$ Where p is the infra-correlation coefficient

    between the units of the same systematic sample, n is the sample size and K is the sampling interval.
12. Describe probability proportional to size (PPS). Explain how PIPS samples are drawn.
13. Explain the difference, between ratio and regression method of estimation. Obtain the sample estimate of the variance of the ratio estimator.

Tribhuvan University, 2069

1. (Compulsory) Attempt any SIX questions. 6x5=30
    a. Explain the theoretical background of control charts.
    b. Discuss mathematical model of time series analysis.
    c. What is meant by deflation of index number? Write down the formula to estimate the real wage and real income index number.
    d. What do you understand by national income? What are its basic components?
    e. Describe UN's sex-age adjusted birth rate.
    f. What are the measures of mortality to express death rates and how they can be determined?
    g. How can one estimate population by simple exponential model?

Group "A"

Attempt any FIVE questions. 5x7=35

2. Discuss various components of life table and their interrelationship. Give the assumptions used in the construction of life tables.
3. Explain the various measures of fertility in common use. How does total fertility rate differ from gross reproduction rate?
4. From the data given below calculate gross and net reproduction rates.

Age group	No. of Children born to 1000 women	Mortality rate per 1000
15-19	150	120

20-24	1500	180
25-29	2000	150
30-34	800	200
35-39	500	220
40-44	200	230
45-49	100	250

Sex ratio being males : females 52 48.

5. What are the different measures of population growth rate? If growth rate of population is 2.4% per annum. Find time period for the population to be double.
6. Define stable population and stationary population. Show that with T the usual notations. (i) $m_x = \frac{2q_x}{2-q_x}$ (ii) $\overset{0}{e}_x = \frac{T_x}{l_x}$
7. What are the different errors usually encountered in age reporting. Discuss main pattern observed in Nepal.

Group "B"

Attempt any FIVE questions. **5x7=35**

8. What are the different causes of variation in statistical quality control? Describe how $\bar{x}$ and R is constructed.
9. Give the concept of sampling inspection. What do you understand by consumer's risk and producer's risk in single sampling plan?
10. Name the components of time series and illustrate them with suitable examples.
11. You are given the annual profit figures for a certain firm for the year 2005 to 2011. Fit a straight line trend to the data and estimate the expected profit for the year 2012.

Year	2005	2006	2007	2008	2009	2010	2011
Profit in lakh Rs.	25	30	42	35	44	49	45

12. Examine the various points that are to be considered in the construction of index numbers. From the following data calculate Fisher's ideal index number.

Item	price per unit (in Rs.)		Quantity used	
	2010	2011	2010	2011
A	10	12	5	6
B	8	9	10	10
C	4	7	6	7
D	2	3	4	5

13. What do you mean by official statistics? Describe the source and limitations of official statistics, in Nepal related to agriculture and industry.

Tribhuvan University, 2070

Group "A"

1. (Compulsory) Attempt any SIX questions. 6×5=30
   a. Explain control charts in statistical quality control.
   b. Discuss the method of least square in determining the trend

values of a time series.

c. Define index number. Describe Laspeyre's and Paasche's price index numbers.

d. What is meant by deflating the index number? Write down the formula to compute real wage and real wage index number.

e. What do you understand by national income? What are its basic components?

f. Describe UN's Sex-Age adjusted birth rate.

g. Prove that the time period for a population to double varies inversely to log(1 + r) where r is the population growth rate. Derive me relationship between them.

h. Define mean length of generation and net reproduction rate. Derive the relation between them.

Group "B"

Attempt any FIVE questions. 5×7=35

2. Define vital statistics. What are the various cases of vital statistics for a country?

3. Explain various measures of fertility in common use. How does total fertility rate differ from gross reproduction rate?

4. Prove the relationship

$$p_x = \frac{e_x}{1 + e_{x+1}}$$

where notations have usual meanings.

5. Fill in the blanks which are marked with a query in the following life table and explain the meaning of symbols at the heads of the column.

Age	ℓ_x	d_x	p_x	q_x	L_x	T_x	e_x^0	m_x
20	762227	?	?	?	?	?	27296632	?
21	758580	–	–	–	–	–	?	–

6. Which population model is often used for short period population projection? State the reason. Also derive the model.

7. Explain the difference between crude death rate and standardized death rate. Given below is the data regarding deaths in two districts. On the basis of given data, calculate the standardized death rates.

Age	District A		District B		Age distribution of a standard 1000
	Population	No. of deaths	Population	No. of deaths	
0-10	2000	50	1000	20	206
10-55	7000	75	3000	30	583
55 and over	1000	25	2000	40	211

Group 'C'

Attempt any FIVE questions. 5×7=35

8. Describe the single sampling plan. Obtain operating characteristic (OC) and

Acceptance Quality Level (AQL) curve for this plan. Distinguish between producer's and consumer's risk.

9. What is a time series? Name the various components of time series and illustrate them with suitable examples. What purpose is served by time series analysis?

10. What do you mean by seasonal index in analysis of time series? What are the methods of computing seasonal indices? Appliance centre sells a variety of electronic equipments and home appliances. For the last four years the following quarterly sales (in lakh Rs.) were recorded.

Year	Quarters			
	Q_1	Q_2	Q_3	Q_4
2011	5.3	4.1	6.8	6.7
2012	4.8	3.8	5.6	6.8
2013	4.3	3.8	5.7	6.0
2014	5.7	4.6	6.4	5.9

Determine seasonal indices for each of the four quarters by simple average method.

11. Explain briefly how Fisher's ideal index number is constructed. Justify its being called ideal.

12. What is cost of living index number? An enquiry into the budget of the middle class families in Kathmandu gave the following information:

Price in	Expenses on				
	Food 35%	Rent 15%	Clothing 20%	Fuel 10%	Miscellaneous 20%
2013	1500	600	1250	400	850
2014	1740	750	1325	550	1000

What changes in the cost of living figure of 2014 have taken place as compared to 2013?

13. What do you mean by official statistics? Discuss industrial and agriculture statistics available in Nepal.

Tribhuvan University, 2070

Group "A"

1. (Compulsory) Attempt any SIX questions. $6\times5=30$

   a. Discuss the basic assumptions used in analysis of variance technique. State Cochran theorem.

   b. What is the aim of design of experiment? Name the basic principles of experimental design, also give name of three basic design of experiments.

   c. How can the sum of squares of analysis of variance of Latin square design of order K be calculated? Is 2×2 Latin square design possible? Why?

   d. Discuss the advantages and disadvantages of sampling over complete enumeration.

   e. Explain sampling and non sampling errors in a sample survey.

f. Explain the difference between ratio and regression method of estimation.

g. Show that in simple random sampling without replacement the probability of selecting a specified unit of the population at any given draw is equal to the probability of selecting it at first draw.

Group "B"

Attempt any Five questions. $5 \times 7 = 35$

2. Explain how you would compare the efficiency of randomized block design over the completely randomized design.

3. A dietician who specializes in weight control has three different diets she recommends. As an experiment she randomly selected 15 patients and then assigned to each diet. After three months the following weight losses, in kg. were noted. At 5 percent significance level, can she conclude that there is difference in the mean amount of weight loss among three diets?

Diet A	Diet B	Diet C
3	3	4
4	4	5
2	4	6
2	3	5
1	2	6

4. What is meant by factorial experiment? What effects are measured in factorial experiment?

5. Give the layout of Greco-Latin square design and describe the procedure of the analysis of data in this design.

6. Explain the techniques of confounding in design of experiments. What are the advantages of confounding?

7. What do you mean by Analysis of Covariance? Write down the mathematical model for one way classification in covariance analysis.

Group "C"

Attempt any FIVE questions. $5 \times 7 = 35$

8. Explain the different steps used in conducting a sample survey.

9. A population consists of numbers 1, 3, 5, 7 and 9. Enumerate all possible samples of size two which can be drawn from the population without replacement. Show that sampling distribution of sample means is equal to the population mean. Also obtain its variance.

10. Show that $V(\bar{y}_{st})$ is minimum for fixed total size of the sample $n_i \propto N_i S_i$. The notations have their usual meanings.

11. What do you understand by systematic sampling? Discuss its advantages and disadvantages.

12. Describe probability proportional to size (PPS). Explain how PPS samples are drawn?

13. Obtain the variance of ratio estimate in simple random sampling and compare its efficiency with regression method of estimation.

Tribhuvan University, 2071

Group "A"

1. (Compulsory) Attempt any SIX questions. **6×5=30**
   a. Define control charts for mean and range.
   b. How do you fit the trend line by the method of least square?
   c. Define an index number and mention its uses.
   d. Define official statistics of Nepal. What are the major limitations of Nepalese official statistics?
   e. Explain why standardized death rate is more accurate than the crude death rate in comparing death rates.
   f. What is general fertility rate, and how can it be determined?
   g. How can one estimate population by compound interest model? Give an example.

Group "B"

Attempt any FIVE questions. **5×7=35**

2. Discuss the nature, scope and coverage- of demographic studies.Give some examples of important demographic studies in Nepal.
3. Fill in the blanks which are marked with a query in the following skelton life table and explain the meaning of symbols at the heads of the column.

Age x	ℓ_x	d_x	p_x	q_x	L_x	T_x	e_x^0	m_x
30	762227	?	?	?	?	27296632	?	?
31	758580	–	–	–	–	?	–	–

4. Define mean length of generation and net reproduction rate. Derive relation between them.
5. Explain the difference between birth rate and fertility rate. Obtain the total fertility rate for the following table:

Age Group	Number of women	Total births
15-19	5687	125
20-24	5324	276
25-29	4720	262
30-34	3933	163
35-39	2670	118
40-44	3015	27
45-49	2601	6

6. What are the different errors usually encountered in age reporting? Discuss main patterns observed in Nepal.
7. What are the different measures of population growth rate? Discuss component method of population projection.

Group "C"

Attempt any FIVE questions. **5×7=35**

8. What do you understand by control charts in statistical quality control? Explain the principles on which control chart is based.
9. What is a time series? What purpose is served by time series analysis? Throw light on main drawbacks of the time series analysis.

10. What do you understand by 'seasonal variations' in time series data? Calculate the seasonal indices from the following data by using simple average method.

Year	Q_1	Q_2	Q_3	Q_4
2008	75	60	82	89
2009	78	68	85	92
2010	80	72	89	95
2011	85	80	95	100

11. What is the cost of living index number ? Explain the difficulties in its construction. Compute the cost of living index number for the following data:

Item	Food	Clothing	Rent	Transport	Miscellaneous
Weight	50	25	8	4	13
Price relative	167	118	146	128	155

12. What are the tests to be satisfied by a good index number ? Examine how far they are met by Fisher's ideal index number.

13. What do you understand by National Income ? Describe different methods used in its computation.

Tribhuvan University 2071

Group "A"

1. (Compulsory) Attempt any SIX questions. 6×5=30

a. Describe the technique of analysis of variance. Give the basic assumptions made in analysis of variance for the validity of the F-test.

b. Define randomization in experimental design. What is the role of randomization in the process of experimentation?

c. What is meant by confounding in factorial experiment? Explain the terms complete and partial confounding.

d. Give the layout and analysis of 4 x 4 Latin square design.

e. What is a sample? In what situations is inevitable?

f. Discuss sampling and non-sampling errors in a sample survey.

g. What factors are responsible for determining the size of the sample? How can sample size be determined mathematically?

h. Explain the difference between ratio and regression method of estimation in sampling theory.

Group "B"

Attempt any FIVE questions. 5×7=35

2. Describe completely randomized design. Obtain the relative efficiency of randomized block design over this design.

3. Present the analysis of variance table for a randomized block design with 'K' treatments and 'b' blocks with one observation per experimental unit.

4. What do you mean by factorial experiment? Discuss the main effects and interaction effects in 2^3 experiments.

5. What do you mean by analysis of covariance? Illustrate the use of technique of analysis of covariance in reducing error as is applied to the randomized block design.

6. What is treatment contrast? When are two such contrasts are said to be

orthogonal?

7. A furniture company wants to know whether there are differences in stain resistance among the four chemicals used to treat three different fabrics. Table given below shows the yields on resistance to stain (a low value indicates good stain resistance).

At the $\alpha = 0.05$, is there evidence to conclude /hat there is a difference in mean resistance among the four chemicals?

Material	Chemical			
	C_1	C_2	C_3	C_4
M_1	3	9	2	7
M_2	7	11	5	9
M_3	6	8	7	8

Group "C"

Attempt any FIVE questions. 5×7=35

8. Explain different steps used in conducting a sample survey.

9. What is meant by sampling distribution of a statistic? Define standard error of statistic.

10. Write down the basic principles of stratification. In a stratification with two strata values of W_h and S_h are as follows:

Stratum	W_h	S_h
1	0.4	4
2	0.6	2

Compute the sample size for each of the stratum to satisfy following conditions:

(i) the standard error of the estimated population mean is 0.1.

(ii) the total sample size to be estimated.

11. What do you understand by systematic sampling? Show that the systematic sampling would be more efficient than simple random sampling without replacement if $\rho < -\frac{1}{nk-1}$

where p is the intracorrelation coefficient between the units of the same systematic sample, n is the sample size and k is the sampling interval.

12. Show that $Var(\bar{y}_{st})$ is minimum for fixed total size of the sample n if $n_i \propto N_iS_j$, where notations have their usual meanings.'

13. Describe probability proportional to size sampling. Prove that in probability proportional to size sampling with replacement the sample mean is an unbiased estimate of the population mean.

Tribhuvan University, 2072

Group "A"

1. (Compulsory) Attempt any SIX questions. 6×5=30

a. Define Analysis of Variance. Give the basic assumptions made in Analysis of Variance for the validity of F-test.

b. What are the tree basic principles of Design of Experiment? Explain them briefly.

c. State the mathematical model used in Analysis of Variance in two way classification. Explain the hypothesis to be used.

d. Discuss briefly the technique of Covariance Analysis. Write the model used for Analysis of covariance for Randomized Block Design and interpret the meanings of the parameter used in the model.

e. What do you understand by sample survey? Under what conditions can sample survey be preferred to complete enumeration?

f. Discuss sampling and non-sampling error in a sample survey.

g. Show that in simple random sampling without replacement, the probability of selecting a specified unit of the population at any given drawn in equal to the probability of selecting it at the first draw.

h. Explain Lahiri's method of selecting probability proportional to size sample.

Group "B"

Attempt any FIVE questions. $5\times7=35$

2. Obtain the relative efficiency of the Randomized Block Design compared to completely Randomized Block Design.

3. Physicians depend the laboratory test results when managing the medical problems such as diabetes or epilepsy. In a uniformity test glucose tolerance, three different laboratories were each sent $n_i = 5$ identical blood samples from a person who had drunk 50 mg. of glucose dissolved in water. Tha laboratory results (mg./dl) are listed here:

Lab 1	Lab 2	Lab 3
120.1	98.3	103.0
110.7	112.1	108.5
108.9	107.7	101.1
104.2	107.9	110.0
100.4	99.2	105.4

Do data indicate a difference in the average reading for the three laboratories? Use $\alpha = .05$.

4. What is the use of "missing plot technique"? Show that in a randomized block design with r blocks and t plots the analysis can be carried out by substituting the value $y = \frac{rB + tT - G}{(r-1)(t-1)}$ for the missing yield, B = the actual total of the block with the missing unit. T = the total of yields of the treatment with missing unit and G = the grand total.

5. Give the layout and analysis of 4×4 Latin Square Design.

6. What do you mean by factorial Experiments? Discuss the main effects and interaction effects in 2^3 – experiment.

7. Explain the technique of confounding in Design of experiment. What are the advantages of cofounding?

Group "C"

Attempt any FIVE Questions. $5\times7=35$

8. Discuss the basic principles of sample survey. Why it is said that simple random sampling without replacement is better than simple random sampling with replacement?

9. What is meant by sampling distribution of statistic? Define the standard error of statistic.

10. Explain the purpose of stratification in sample surveys. Obtain the estimate of the population mean by the method of stratified sample random sampling and compare its efficiency with that of simple random sampling.

11. What is the systematic sampling? Give illustrations of situations where such sampling is useful. How will you measure the sampling error of a systematic sample mean?

12. Describe probability proportional to size sampling. Prove that in probability proportional to size sampling with replacement the sample mean is an unbiased estimate of the population mean.

13. Explain the difference between ratio and regression method of estimation. Obtain the sample estimate of the variance of the ratio estimator.

Tribhuvan University, 2072

Attempt ALL the questions.

Group "A" 5×7=35

1. A curve is drawn on a parabolic cylinder so as to cut all the generators at the same angle, prove that

   $\rho = 2a\,(1 + t^2)^{3/2}\,\text{cosec}^2\alpha$

   and $\sigma = 2a(1 + t^2)^{3/2}\,\text{cosec}\alpha\,\sec\alpha$ [7]

   OR

   State Serret – Frenet formula. For the curve

   $x = a(3u - u^3),\ y = 3au^2, = a\,(3u + u^3),$

   Show that $k = \Im = \dfrac{1}{3a(1 + u^2)^2}$. [2+5]

2. Define second order partial differential equation

   Solve: $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2} = xy$ [1+6]

   OR

   Solve: $r + 3s + t + (rt - s^2) = 1$. by Monge's method. [7]

3. State Divergence theorem. Evaluate

   $$\iint_s (y^2z^2\vec{i} + z^2x^2\vec{j} + z^2y^2\vec{k})\,\vec{n}\ ds.$$

   Where s is in the part of the sphere $x^2+y^2 + z^2 = 1$ above the xy plane and bounded by this plane. [1+6]

4. Define analytic function. Show that CR equations are satisfied for $f(.) = .^2$ but not for $f(.) = |.|^2$ when $. \neq 0$.

5. Define Fourier consine and sine series.

   Find the Fourier series of $|x|$ in $-\pi \leq x \leq \pi$. [2+5]

Group "B" 10×4=40

6. Solve : $\dfrac{d^2y}{dx^2} - k\left(\dfrac{dy}{dx}\right)^2 = 0$. [4]

beats S. If the two players name the same item, then the game is a tie. Set up the game matrix A. [5]

Or

Find the saddle points, if any and the optimal solution as well as the best strategy for each player of the two person-zero--sum game whose pay off matrix A is

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & -1 \\ 1 & 3 & -2 \end{bmatrix}$$

9. Determine the optimal sequence of 5 jobs on three machines in the order ABC that minimize the total elapsed time bases on the following information. Processing time on machines is given in hours and passing is not allowed. [5]

Jobs	1	2	3	4	5
Machine A	3	8	7	5	4
Machine B	4	5	1	2	3
Machine C	7	9	5	6	10

10. Determine whether the following non-linear programming problem has maximum or minimum value.

Optimize $Z = 2x^2 + y^2 + 3z^2 + 10x + 8y + 5z - 100$

Subject to the constraints

$x + y + z = 20, x, y, z \geq 0$ [5]

Or

Develop Kunn - Tucker condition for the following non-linear programing problem :

Max $Z = 36x - 4x^2 + 16y - 2y^2$

subject to the constrains

$2x + y \leq 10;\ x, y \geq 0.$

11. Find the solution of the difference equation $y_{x+2} + y_x = 3^x$ with initial conditions $y_0 = 0, y_1 = 1$

12. What is line segment joining two given points in $\Re^n$?

Show that the line segment is a convex set? [1+4]

Tribhuvan University, 2071

Attempt ALL the questions.

Group "A" 5×7=35

1. Define a convergent double sequence. Let

$\lim_{p\to\infty} f(p, q)$ exists, then $\lim_{p\to\infty} f, (p, q)$ exists and has the value a. Investigate the existence of iterated limits and double limit of the double sequence given

by $f(p, q) = \frac{(-1)^p}{q}$ [1+3+3]

2. Define improper integral of first kind. Interpret it geometrically. State and prove Cauchy criterion for convergence. [2+1+4]

Or

Assume that (i) f is integrable over [a, t] for all t $\geq$ a and there exists a constant M > o such that $\forall$ t $\geq$ a

$$\left|\int_a^t f(x)\,dx\right| \leq M$$

(ii) g(x) is monotonic decreasing to 0 as $x \to \infty$ i.e.

g(x) $\to$ 0 as $x \to \infty$. Then the integral eq $\int_a^\infty f(x)\,g(x)$ is convergent.

[7]

3. Prove that if every infinite subset of a set S in R^n has a accumulation point in S then S is closed and bounded. [7]

4. Let f be a bounded variation on [a, b] and c $\in$ (a, b). Prove that f is f bounded variation on [a, c] and on [c, b], and $V_f[a, b] = V_f[a, c] + V_f[c, b]$. [7]

5. Let $\alpha \nearrow$ on [a, b] and a < c < b. Prove that if a and f both are discontinuous from the right or from the left at x = c, then $\int_a^b f d\alpha$ cannot exist. Also, prove that if α is continuous and α' is Riemann integrable on [a, b] then $\int_a^b f(x)\,d\alpha(x) = \int_a^b f(x)\,\alpha'(x)\,d$. [4+3]

OR

State and prove the first Mean value theorem for Riemann Stieltjes integrals. If f is continuous on [a, b] × [c, d] and g$\in$R on [a, b], show that the function F given by $F(y) = \int_a^b g(x)\,f(x, y)\,dx$ is continuous on [c, d] [4+3]

Group "B" **10×4=40**

6. Define point wise and uniform convergence of a sequenci of function on a set. Prove that $\{x^n\}_{n=1}^{\infty}$ converges point wise, but not uniformly on [0, 1]. [1+3]

7. Define open set in $\Re^n$. Show that every open Ball B (a ; r) is an open set in $\Re^n$. [1+3]

8. Let X be a closed subset of a compact metric space Then X is compact. [4]

9. If $\{a_n\}$ and $\{b_n\}$ are sequences of points in $\Re$ and $a_n \to 0$ and $b_n \to 0$. Then $\{a_n + b_n\}$ also converges to 0.

If $0 \leq c_n \leq a_n$ & $a_n \to 0$ then $\{c_n\}$ also converges to 0.

OR

Let $f : S \to \Re$ be a real valued function defined on metric space (S, d_s) to Euclidean space $\Re$. If f is continuous on a compact subset X of S, then $\exists$ points and q$\in$X S.t

f (p) = Infimum f(X) and f(q) = suprimum f(X). [4]

10. If f is monotonic on [a, b], then / is bounded variation on [a, b].

OR

Let f be defined on [a, b]. Then f is of bounded variation on [a, b] if and only if, f can be expressed as the difference of two increasing functions.

11. Let $f(x) = \begin{cases} 1, & \text{when x is rational} \\ 0, & \text{when x irrational} \end{cases}$

then show that $\bar{I}(f, \alpha) \le \bar{I}(f, \alpha)$ [4]

12. State Second Mean Value Theorem. Show by suitable example that the condition f is monotonic on [a, b] can not be dropped out. [1+3]

13. Define limit superior and limit inferior of a sequence. Find limit superior and limit inferior of a sequence $\{a_n\}$ defined by $a_n = (-1)^n (1 + 1/n)$. [2+2]

OR

If Σa_n converges absolutely, then every subseries Σb_n also converges absolutely. Moreover, we have [4]

$$\left|\sum_{n=1}^{\infty} b_n\right| \le \sum_{n=1}^{\infty} |b_n| \le \sum_{n=1}^{\infty} |a_n|$$ [4]

14. If f(x) be integrable over [a, t] for all $t \ge a$ and $\lim_{x\to\infty} x\, xf(x) = L \ne 0$ (or $= \pm\infty$). Then the integral $\int_a^{\infty} f(x)dx$ diverges. [4]

15. Let $f : S \to \mathfrak{R}^m$ be differentiable at an interior point c on S, where $S \subseteq \mathfrak{R}^n$. If $v = v_1u_1 + ... + v_nu_n$, where $u_1,....u_n$ are the unit coordinate vectors in $\mathfrak{R}^n$, then prove that $f'(c)(u) = \sum_{k=1}^{n} V_k D_k f(C)$.

Or

Let S be open in $\mathfrak{R}^v$ and $f : S \to \mathfrak{R}^u$ is differentiable at each point in S. Prove that if x and y are two points in S such that $L(x, y) \subseteq S$, then for every a in $\mathfrak{R}^m$ there is a point z in L(x, y) such that

$$a - \{f(y) - f(x)\} = a. \{f'(z)(y - x)\}$$

where L(x, y) is a line segment joining x & y. [4]

Tribhuvan University, 2072

Attempt ALL the questions.

Group "A"

1. (Compulsory) Attempt any SIX questions: 6×5=30
   a. Define control charts for mean and range.
   b. Describe different sources of demographic data in Nepal.
   c. How can one estimate the population by simple exponential model?
   d. Define mean length of generation and net reproduction rate.
   e. Discuss the mathematical model of time series analysis.
   f. How do you fit the trend line by the method of least square?
   g. Define index number and mention its uses.
   h. Describe national income and its components.

Group "B"

Attempt any FIVE questions. 5×7=35

2. Discuss the demographic sample survey in Nepal.
3. Explain the various measures of fertility in common use. How does total fertility rate differ from gross reproduction rate?
4. Fill in the blanks in a proportion of life table given below:

Age in years	ℓ_x	d_x	p_x	q_x	L_x	T_x	e_x^0
10	95000	500	?	?	?	4850300	?
11	?	400	?	?	?	?	?

5. What are the different measures of population growth rate? Discuss the component method of population projection.
6. Define stable population and stationary population. Show that with usual notations: (i) $m_x = \frac{2q_x}{2-q_x}$ (ii) $e_x^0 = \frac{T_x}{\ell_x}$
7. Compute Gross and Net Reproduction Rate from the following table:

Age in year	Female population in '00000'	Female births in '000'	Survival rate
15-24	13	19	0.9
25-34	15	71	0.8
35-44	16	24	0.7

Group "C"

Attempt any FIVE questions. 5×7=35

8. What do you understand by control charts in statistical quality control? Explain the principles on which the control charts is based.
9. Explain the sampling inspection in industrial statistics. Discuss a single sampling plan.
10. What is the time series? What purpose is served by time series analysis? Discuss main drawbacks of the series analysis.
11. The following table lists the annual amounts of glass cullet produced by Shree Glass Works.

Year	2009	2010	2011	2012	2013
Scrap (quintals)	2.0	4.0	3.0	5.0	6.0

Determine the least square trend equation. Estimate the amount of scrap for the year 2014.
12. Explain briefly how Fisher's ideal index number is constructed. Justify its being called ideal.
13. What is meant by consumer price index? The following table gives group index numbers and corresponding group weights with regard to cost of living for a given year. Construct the overall cost of living index number for the year.

Groups	Food	Fuel and Lighting	Clothing	Rent	Miscellaneous
Index Number	250	120	130	110	150
Weight	5	1	1	1	2

Tribhuvan University, 2072

Attempt ALL the questions

Group "A" 5×7=35

1. Define Riemann integral sum.
If $f \in R(\alpha)$, $g \in R(\alpha)$ on [a, b], then $c_1f + c_2g \in R(\alpha)$ on [a, b] where c_1 and c_2 are two constants and also we have

$$\int_a^b (c_1 f + c_2 g)\, d\alpha = c_1 \int_a^b df d\alpha + c_2 \int_a^b g\, d\alpha$$

Let α be a bounded variation on [a, b] and let V(x) be the total variation of α on [a, x], where $a < x \le b$, and V(a) = 0. Let f be defined and bounded on [a, b]. If $f \in R(\alpha)$ on [a, b] then $f \in R(V)$ on [a, b]. [1+6]

2. Define limit superior and limit inferior of a sequences. Find the limit superior and limit inferior of a sequence $[a_n] = n^2 \sin^2\left(\frac{n\pi}{2}\right)$

If $[a_n]$ be a sequence of real number, then we have

$\lim_{n\to\infty} \inf a_n \le \lim_{n\to\infty} \sup a_n$. [2+2+3]

3. What do you mean by integral $\int_a^\infty f(x)\, dx$ converges absolutely? If $\int_a^\infty f(x)\, dx$ converges absolutely and g is bounded on [a, ∞] then the product f(x) g(x) is also absolutely integrable over [a, ∞].

Further show that $\int_1^\infty \frac{\sin x}{x^2} dx$ converges absolutely. [1+4+2]

4. Define directional derivative at a point. If F(t) = f(c + tu) then F(t) = f(c + tu; u)

Show that the existence of finite directional derivative f(c; u) of a function f at c in all the directions given by a may not imply the continuity of that function at c. [1+1+5]

OR

State and prove chain rule in multivariable differential calculus. [7]

5. Define continuity of a function. How is it differ with uniformiy continuity? Let $f: S \to T$ be a function from a metric space (S, d_s) to another metric space (T, d_T). Let A be a subset of S. If f is uniformly continuous on A, then f is also continuous on A. [1+1+5]

Group "B" 10×4

6. Define Eucledian metric on $\Re^n$,

Let $M = \Re^n$, a metric d(x, y) defined on M by

$$d(x, y) = \sum_{i=1}^{n} |x_i - y_i|.$$

Show that d is a rectangular metric on $\Re^n$. [1+3]

OR

Show that $B_S(a; r) = B_M(a; r)$ n S.

If $S = \Re$ and S = [0, 1] what will be B_R (0, 1) and $B_S(0;1)$? [2+2]

7. Let X be a closed subset of a compact metric space M. Then X is compact. [4]

8. How do you define a Cauchy sequence? The sequence $\{x_n\}$ defined by $x_n = 1/n$

is a Cauchy sequence in $\Re$. [1+3]

9. If f is of bounded variation on [a, b], say $\Sigma|\Delta f_k| \leq M$ for all partitions of [a, b]. then f is of bounded on [a, b].
Infact, $|f(x)| \leq |f(a)| + M$ for all $x \in [a, b]$. [4]

OR

Let be a continuous function on [a, b]. The f is of bounded variation on [a, b] if and only if, f can be expressed as the difference of two increasing continuous function. [4]

10. State the theorem for integration by parts in R – S integral and hence evaluate: $\int_0^{\pi/2} x\,d(\cos 2x)$. [1+3]

11. State and prove second mean value theorem for R – S integral. [4]

OR

State and prove first function theorem of calculus. [1+3]

12. Prove that absolutely convergence of an infinite series $\sum_{n=1}^{\infty} a_n$ implies the convergence. Does the converse is true? Explain. [3+1]

13. Prove that the sequence $f_n(x) = x^n$ converges point wise but not uniformly on [0, 1]. [4]

14. Define improper integral with examples. Show that the integral $\int_a^{x} \frac{1}{x}\,dx$ diverges for $x > a > 0$.

OR

If the integral $\int_a^{\infty} |f(x)|\,dx$ converges, then $\int_a^{\infty} f(x)\,dx$ also converges.

Show that the integral $\int_{-\infty}^{-1} \frac{e^x}{x}\,dx$ converges. [2+2]

15. Let S be an open and connected subset of $\Re^n$ and let $f : S \to \Re^m$ be differentiable on S. If $f(c) = 0\ \forall\ c \in S$, then f is continuous on S. [4]

Tribhuvan University, 2072

Attempt ALL the questions

Group "A" 5×7=35

1. Let V be a finite dimensional vector space over the complex numbers of dimension ≥ 1 and let A : V → V be a linear map. Let P be its characteristic polynomial. Prove that P(A) = 0. [7]

2. Let V be a finite dimensional vector space over field K. Let W be a subspace of V. Let $W^{\perp}$ be a subspace of a dual space V^* consisting of all elements $\phi \in V^*$

which are orthogonal to W.

Prove that $\dim V = \dim W + \dim W^{\perp}$.

Also find the dual basis of {(3, 1), (2, 1)} of $\Re^2$. [5+2]

OR

What do mean by a Hermitian form? Give an example. Let A be a Hermitian matrix. Show that f is Hermitian matrix of C^n where f is defined by $f(X, Y) = X^T A\bar{Y}$. [2+2+3]

3. Give a non trivial example of a homomorphism and find its kernel. Let $\phi: G \to \bar{G}$ be a homomorphism and K be the kernel. Then prove that $\frac{G}{K}$ and Im ϕ are isomorphic and conversely if N is a normal subgroup of G, then there exists of homomorphism $h : G \to \frac{G}{N}$ with kernel N defined by $h(g) = Ng$ for all $g \in G$. [2+1+4]

OR

Define inner automorphism of a group G. Prove that the set of all inner automorphism of A(G) of a group G is normal subgroup of A(G), where A(G) is a usual meaning. [2+5]

4. What is an ideal of a ring R? Give an example. Let R be a commutative ring with unit element whose only ideas are {0} and R itself. Prove that R is a field. [2+5]

5. Define roots of a polynomial. Prove that a polynomial of degree n order a field can have at most n roots in any extension field. [1+6]

Group "B" 10×4=40

6. Define linearly independent set of vectors. Let V and W be two vectors spaces and $L : V \to W$ a linear map, let $w_1, w_2, \ldots\ldots, w_n$ be elements of W which are linearly independent and let $v_1, v_2, \ldots\ldots, v_n$ be elements of V such that $L(v_i) = w_i$ for $i = 1, 2, \ldots\ldots, n$; show that $v_1, v_2, \ldots\ldots, v_n$ are linearly independent. [1+3]

7. If $f: \Re^2 \to \Re^2$ is a linear map defined by $f(a, b) = (3a + 2b, -a + 4b)$. Find a matrix representation of f relative to the given basis. {(1, –1), (1, 1)}. [4]

OR

Let V and W be vector spaces over a field K and $F : V \to W$ be a linear map. Let β and β' be the bases for V and W respectively. Explain how do you obtain the matrix $M^{\beta}_{\beta'}(F)$ for F. [4]

8. What is scalar product on a vector space V?

For all $v, w \in V$ prove that $|\langle v, w\rangle| \leq \|v\|\|w\|$. [1+3]

9. Let V be a finite dimensional vector space over a complex number with a fixed positive define hermitian from $\langle . \rangle$. If A is an operator such that $\langle Av, v\rangle = 0$ for all $v \in V$. Prove that $A = 0$. [4]

OR

State and prove Sylvestor's theorem.

10. Find all the characteristic roots of the matrix. [4]

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & 6 & 4 \end{bmatrix}$$ [4]

11. Let $L : \Re^2 \to \Re^2$ be a linear map defined by
$L(x, y) = (2x + y, 3x - 5y)$. Show that L is invertible. [4]

12. Define equivalence relation. Let G be a group and H be a subgroup of G. For a, $b \in G$ define $a \equiv b \pmod H$ if $a\vec{b} \in H$, show that it is an equivalence relation. [1+3]

13. Define a homomorphism of groups and illustrate it with a non-trivial example. Let G be a finite group such that G = AB where A is a normal subgroup of G and B is a subgroup of G. Show that if $\frac{G}{A} \equiv B$, then $A \cap B = \{e\}$. [4]

OR

Define conjugacy class of an element a of a group G. Find conjugacy class of (13) is S_3 and also find N(13) in S_3. [1+1+2]

14. If J_p, the ring of integers mod p, then show that p is a prime number. [4]

OR

What do you mean by Euclidean ring? Prove that Euclidean ring possesses a unit element. [1+3]

15. If $p(x) \in F[x]$ and if K is an extension of F, prove that for any element $b \in K$, $p(x) = (x - b)\, q(x) + p(b)$ where $q(x) \in K[x]$ and $\deg q(x) = \deg p(x) - 1$. [3+1]

Tribhuvan University, 2072

Attempt ALL the questions

Group "A" $3 \times 10 = 30$

1. What is the system of coplanar forces? Prove that a system of coplanar forces acting at different points of a rigid body may be reduced to a single force, acting through an arbitrary point, and a couple. Find the general condition of equilibrium. [1+6+3]

2. Define angular velocity and acceleration of a particle moving in a plane curve at any instant. Find them. Also obtain the relation between angular and linear velocities. [2+4+4]

OR

Define radial and transverse velocities of a point.
If the radial and transverse velocities of a point are always proportional to each other and which holds for acceleration also, prove that its velocity will vary as some power of the radius vector. [2+8]

3. Find the M.I. of an isosceles triangle ABC of mass M about [5+5]
i. an axis through the vertex A perpendicular to the opposite side
ii. an axis through a perpendicular to the plane of the triangle.

Group "B" $9 \times 5 = 45$

4. Forces proportional to 1, 2, 3 and 4 act along the sides AB, BC, AD and DC respectively of a square ABCD, the length of whose sides is 2 feet. Find the magnitude and the line of action of their resultant. [5]

7. Solve: $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by the method of variation of parameters. [4]

8. Define partial differential equation. Find the partial differential equation of a plane making equal intercepts from the axes of x and y. [1+3]

OR

Solve: $(mz - ny)\, p + (nx - lz)\, q = ly - mx$. [4]

9. Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial t}$. [4]

OR

Solve by Charpit's method the equation.
$(p^2 + q^2)y = q$. [4]

10. Solve: $\frac{d^2x}{dt^2} + 4x + y = te^{3t}$; $\frac{d^2y}{dt^2} + y - 2x = \cos^2 t$. [4]

OR

Solve: $\frac{dx}{dt} + y = e^t$; $\frac{dy}{dt} - x = e^t$. [4]

11. Prove that the necessary and sufficient condition for a curve to be straight line is $k = 0$ at all its points. [4]

12. Evaluate $\int_s \vec{F} \cdot dr$, where $\vec{F} = x\vec{i} - xy\vec{j}$ from origin to the point (1, 1) along the parabola $y^2 = x$. [4]

13. Using Green's theorem to evaluate.
$\int_c (2x - y + 4)\, dx + (5y - 3x - 6)\, dy$ around a circle of radius 4 with centre at origin. [4]

14. Define Harmonic conjugate. Show that
(a) $v(x, y) = 3x^2y - y^3$ is harmonic
(b) Find the conjugate function $u(x, y)$. [1+1.5+1.5)

OR

Show that $f(z) = \bar{z}$ is not differential even though it is continuous. [4]

15. Find the Fourier series of
$|\sin x|$ in $-\pi \le x \le \pi$. [4]