17. Mathematics

11. Sample Surveys and Design of Experiments (Stat.331) (New+Old) 2068

Bachelor Level /III Year/ Science & Tech. + Humanities **Full Marks: 100**

Time: 3 hrs

Group "A"

1. (Compulsory) Attempt any SIX questions. 6x5=30

   a. Discuss the techniques' in controlling errors in experimental designs.

   b. Describe a procedure to estimate a missing value in randomized block design.

   c. Explain 2^3 factorial design with the help of an illustrative example.

   d. Discuss how data is analyzed in the single factor experimental design.

   e. Compare complete enumeration with sampling under different circumstances.

   f. In simple random sampling without replacement, which form of the sample variance is an unbiased estimate of population variance?

   g. Differentiate between sampling and non-sampling errors in sample surveys.

   h. Point out the importance of systematic sampling. Describe a method of selecting a systematic sample.

Group "B"

Attempt any FIVE questions. 5x7=35

2. Give the layout and analysis of Latin square design. Obtain its relative efficiency as compared to randomized block design.
3. Obtain the expectations of sum of squares for randomized block design.
4. What do you mean by analysis of covariance and how does it differ from

analysis of variance? Describe a procedure to carry out the analysis in completely randomized design.

5. A manufacturer of television sets is interested in the effect on tube conductivity of three different types of coating for colour picture tubes. The following data on conductivity Is obtained.

Coating Type 1	Coating Type 3	Coating Type 3
143	152	129
141	149	127
150	140	132
147	143	138
145	150	135

Test whether there is a significant difference in conductivity due coating type. [Tabulated value of $F_{0.05}(2, 12) = 3.89$]

6. Describe completely randomized design Obtain the least of effects square estimates and sum of squares for the design.

7. Distinguish between complete and partial confounding. Illustrate the layout and analysis of a partially confounded 2^3 design in two blocks and 'r' replicates.

Group "C"

Attempt any FIVE questions. 5x7=35

8. Let the population consists of units 2, 5. 3, 7, 9. If we draw simple random sample of size 3 without replacement show that sample mean is an unbiased estimate of population mean. Also verify that the standard error of the sample mean is equal to $\sqrt{\frac{a^2}{n}\frac{(N-n)}{(N-1)}}$ where a is population standard deviation, N is the size of the population and n is the size of the sample.

9. Show that the simple random sampling without replacement sample mean square is an unbiased estimate of population mean square.

10. What are the advantages of stratification in sample surveys? Describe the procedure of optimum allocation of sample sizes for different strata in stratified random sampling.

11. Obtain an unbiased estimate of population mean in. systematic sampling and also find the relative efficiency of systematic sampling with respect to simple random sampling without replacement.

12. Define regression method of estimation. Obtain the variance of the regression estimate.

13. What do you mean by probability proportional to size sampling? Describe the cumulative total method of selecting units in this sampling method.

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Attempt ALL the questions.

Group "A" 5x7=35

1. Define convergent double sequence. Let $\lim_{p,q \to \infty} f(p, q) = \ell$

If for each fixed p, the limit $\lim\limits_{q \to \infty} f(p,q)$ *exists*, then prove that the iterated limit $\lim\limits_{q \to \infty}\left(\lim\limits_{q \to \infty} f(p,q)\right)$ also exists and has the same value ℓ.

Investigate the existence of iterated limit and double limit of the double sequence given by

$f((p,q) = \dfrac{pq}{p^2+q^2}$ p,=1,2,.........., q= 1, 2....., [1+4+2]

2. Define an accumulation point of a set in $\Re^n$.

Prove Bolgano - Weixrstrass theorem for $\Re^n$, n > 1. [1+6]

OR

Define metric subspace. Let (S, d) be a metric subspace of the metric space (M, d) and X a subset of S, prove that X is open in S if and only if X = A $\cap$S for some set A which is open in M. [1+3+3]

3. Assume that g is differentiable at 'a' with total derivative, g'(a). Let b = g(a) and assume that *f* is differentiable at b with total derivative *f*'(b). Prove that the composite, function h =*f*og is differentiable at a with the total derivative h'(a) and is given by h'(a) = *f*'(b) o g'(a). [7]

4. Define upper and lower Stieltjes integrals. Assume that $\alpha \rightarrow$ on [a, b], prove that I(*f*, ∝)⁼ *I*(*f*, ∝) and show that this inequality holds by means of suitable example. [2+2+3]

5. State and prove Dirich lets test for the convergence of improper. integral of the first kind. Show that $\int_a^{\infty} \frac{\sin n}{x^p}\,dx$ converges if a > 0, p > 0. [1+4+2]

OR

Define an improper integral f $\int_a^{\infty} f(x)dx$. Interprets it geometrically. Let *f*, g be integrable over [a, t] for all t $\geq$ a and 0 $\leq$ *f(x)* $\leq$ g(x) for all x $\geq$ a, prove that if $\int_a^{\infty} g(x)dx$ converges, then $\int_a^{\infty} f(x)dx$. converges or if $\int_a^{\infty} f(x)dx$. diverges then $\int_a^{\infty} g(x)dx$ diverges. [1+3+3]

Group "B" **10x4=40**

6. Prove that absolutely convergence of an infinite series $\sum_{n=1}^{\infty} a_n$ la, implies the convergence. *Is* converge true? Give an example. [2+2]

7. Prove that the series $\Sigma(-1)^n$ (1–x) converges point wise but not uniformly on [0, 1]. [4]

OR

Let $\{f_n\}$be a sequence of functions defined on a set S. If $\{f_n\}$ statisfies the Cauchy's condition, then prove that $f_n \rightarrow f$ uniformly on S. [4]

8. Prove that the integral $\int_{x}^{-1}\frac{e^{x}}{x}dx$ converges. [4]

9. Define accumulation point of a subset of $\Re^n$. If x is an point of S, then prove that every open n-ball B(x) contains infinitely many points of S. [4]

10. State and prove Lindel of covering theorem. [4]

OR

Prove that the set $[-2, 2] \cup [3, 4]$ in $\Re$ is compact. [4]

11. Let F be any collection of sets in T and let $f: S \to T$ be a function, prove that

$$f^{-1}\left(\underset{A \in F}{\cup A}\right) = \underset{A \in F}{\cup}\left[f^{-1}(A)\right]$$ [4]

12. Define a directional derivative. Justify that a function can have a finite directional derivative $f'(c; u)$ of a function at c for every u but may not imply the continuity of that function at c, by taking

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^4} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

OR

Let S be an open and connected subset $\Re^n$ and let $f: S \to \Re^n$ be differentiable on S. If $f'(c) = 0 \ \forall c \in S$, then prove that f is continuous on S. [4]

13. Determine whether or not the function

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$$

is bounded variation on [0, 1]. [4]

14. Evaluate $\int_{0}^{\frac{x}{2}} xd(\cos 2x)$ by using integration by parts. [4]

15. State and prove first Mean Value Theorem for Riemann Stieltjes integral. [4]

OR

Show that the- second Mean Value- :Theorem for Riemann integral does not hold on [-1, 1] for $A^X) = g(x) = x^2$. [4]

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Attempt ALL the questions.

Group "A' **5x7=35**

1. State Serret - Frenet formulae.

For the curve $\vec{r} = (a\cos\theta, a\sin\theta, a\theta\cot\beta)$, find expressions for k and T. [2+5]

OR

Prove that $(x''')^2 + (y''')^2 + (z''')^2 = \frac{1}{p^2\sigma^2} + \frac{1+p^2}{p^4}$ [7]

OR

2. Solve: $\frac{d^3y}{dx^3} + 9y = \sec 3x$, by the variation of parameter method. [7]

OR

Find the first and second integrals of

$$x\frac{d^3y}{dx^3} + (x^2 - 3)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$$

Hence find the solution. [3+2+2]

3. State Green's theorem for a plane. Verify the theorem for $\int_C (xy + y^2)\,dx + x^2dy$, where C is closed curve of the region bounded by $y = x$ and $y = x^2$. [2+5]

4. Define anlytic function. Show that the function $f(z) = e^{-z^{-4}}$, $z \neq 0$, $f(0) = 0$ is not analytic at $z = 0$ although C.R. conditions are satisfied at that point. [2+5]

5. Define Fourier series and its coefficients.

Pove that $2\left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \ldots\ldots\right] = \pi - x, x \in [0, \pi]$. [2+5]

Group "C" **10x4=40**

6. Solve : $\frac{d^4y}{dx^4} + a^2\frac{d^2y}{dx^2} = 0$ [4]

7. Solve : $\frac{d^2y}{dx^2} + \cot x\frac{dy}{dx} + 4y\,\text{cosec}^2x = 0$ [4]

by change of independent variable method.

8. Solve : $\frac{dx}{y - ax} = \frac{dy}{x + yz} = \frac{dz}{x^2 + y^2}$ [4]

OR

Find the general solution of $2\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = x + y + 1$,

9. Solve by Charpit's method :

$(p + q)(px + qy) - 1 = 0$ [4]

OR

Find the PDE from $z = f(x + ay) + \phi(x - ay)$

10. Solve : $r - 7x + 12t = e^{x-y}$

OR

Solve by Mnge's method : $q^2r - 2pqs + p^2t = 0$

11. $\iint_C x^2\,dydz + y^2\,dzdx + 2z(xy - x - y)\,dxdy$, where S is the surface of the cube,

$0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. [4]

12. $\iint\limits_{C} [(x^2 + y^2)\vec{i} - 2xy\vec{j}]\, d\vec{r}$. where C is a rectangle in xy plane bounded by y = 0, y = b, x = 0, x = a. [4]

13. Find the harmonic conjugate of $u = x^3 - 3xy^2$ and the corresponding analytic function. [4]

OR

Show that an analytic function with constant modulus in a domain is constant.

14. Find the Fourier series of $f(x) = |x|$ in $-\pi \le x \le \pi$. [4]

15. Prove that : $\dddot{r} = k^1 n - k^2 t + k\tau b$
and $r'''' = (k'' - k^3 - kr^2)\, n - 2kk't + (2k't + kt')\, b$. [2+2]

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Attempt ALL the questions.

Group "A" **5x7=35**

1. Define dimension of a vector space and kernel of a linear map. If $L : \Re^3 \to \Re^3$ defined by $L(x, y, z) = (x - y, x + z, x + y + 3z)$, then show that L is linear map. Determine the kernel L and its dimension. [2+3+2]

2. Define dual space. Let V be a finite dimensional vector space over a field K, prove that the dual space V^* is also finite dimensional and $\dim V = \dim V^*$. [1+6]

3. Define inner automorphism of a group. If I(G) is the set of all inner automorphism of a group G, then prove that $I(G) \cong \frac{G}{X(G)}$ where Z(G) is the centre of G. [1+6]

OR

Let H and K be two subgroups of a group G.
Let HK = {hk/h∈, k∈K}. Prove that HK is a subgroup of G if and only if HK = KH. IN S_3, find two subgroups of S_3 such that HK ≠ KH. If 0 (H) $\sqrt{0\,(G)}$, 0 (K) > (0 (G)), then prove that that $H \cap K \neq \{e\}$. [3+2+2]

4. What do you mean by a Euclidean ring? Show that the integral domain [Z,+,.] is a Euclidean ring where Z is the set of integers. Let R be a Euclidean ring and a, b ∈ R, if b ≠ 0 is, not a unit in R, then show that d(a) < d(ab). [1+2+4]

OR

How do you define an, ideal of a ring? Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field. [1+6]

5. Let $f(X)$ E F[x] be of degree n 2≥1. Prove that there is an extension E of F of degree at most n! in which $f(x)$ has n roots. [7]

Group "B" **10x4=40**

6. Let V be a vector space. Let P : V → V be a linear map such that PoP = P. Let U be the image of P and W be the kernel of P. show that V = U⊕W. [4]

7. Let $V = \Re^3$ be a vector space. If two bases
$\beta = \{(1, 1, 0), (-1, 1, 1), (0, 1, 2)\}$ and
$\beta' = \{(2, 1, 1), (0, 0, 1), (-1, 1, 1)\}$, then find $M_{\beta}^{\beta'}$ (id). [4]

8. Let V be the subspace of $\Re^3$ generated by the two vectors A = (1, 1, 1) and B =

(1, −1, 2). If X = (x_1, x_2, x_3) and y = (y_1, y_2, y_3) are vectors in $\Re^3$, define their product to be {x, y} = x_1y_1 + $2x_2y_2$ + x_3y_3, then find an orthogonal basis of V with respect to this product. [4]

OR

What is scalar product on a vector space V ?

For all v, w, ∈ V, prove that |{v, w}| ≤ ‖ v ‖ ‖ w ‖. [1+3]

9. Let V be a finite dimcosion A vector space over the complex field with positive. Definite hermitian form. Let A be an operator. Prove that A is hermitian if and only if (Av, v) is real for all v ∈ V, [4]

10. Define minimal polynomial of a matrix A. Prove that the minimal polynomial of a matrix A exists and its unique. [1+3]

OR

Let V be a finite dimensional vector space over the field K and λ∈K. Let A : V → V be a linear map. Prove that λ is an eigen value of A if and only if A − λI is not invertible. [4]

11. What is fan basis? Let {u_1, u_2, ...u_n} be a fan basis for a linear map A : V → V. Prove that the matrix associated with A relative to this basis is an upper triangular matrix. [1+3]

12. Distinguish between order of a group and order of an element of a group. Let G be a group and o|G| = p^2, p a prime, prove that G is abelian. [1+3]

OR

Define conjugacy class of an element a ∈ G, where G is a group. Find conjugacy class of (12) in S_3 and also find N(12) in S_3.

13. Suppose that G is the internal direct product of N_1, N_2,.......N_{n0} prove that for i ≠ j, $N_i \cap N_j$ = (e) and if a ∈ N_i, b∈N_j then ab = ba. [4]

14. If a ring R is an integral domain, then prove that R[x] is also an integral domain, where R[x] is the ring of polynomials of R. [4]

15. Define an algebraic element of degree n over a field F. Prove that the element in which is algebraic over F form a subfield of K. [4]

OR

If p(x) ∈F[x] and if K is an extension of F, then prove that for any element b∈ K, p(x) = (x - b) q(x) + p(b) where q(x) ∈: K[x] and deg q(x) =deg[4] P(^{x}) - 1

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Attempt ALL the questions.

Group "A" **3x10=30**

1. What is the system of coplanar forces? Prove that a system of coplanar forces acting at different points of a rigid body may be always reduced to a single force through a given point and a couple. Find the necessary conditions for the equilibrium of the rigid body. [1+6+3]

2. Find the components of acceleration along the tangent and normal to the curve at any instant for a particle moving in a plane curve. Hence find them for a particle moving in a circle. [8+2]

OR

The velocities of a particle along and perpendicular to the radius from a fixed origin are γr and $\mu\theta$: find the path. Show that the acceleration along and perpendicular to the radius vector are

$$\gamma^2 r - \frac{\mu^2\theta^2}{r} \text{ and } \mu\theta\left(\lambda + \frac{\mu}{r}\right)$$

3. If the M.I. and P.I. of a body about three mutually perpendicular and intersecting lines are known then to determine M.I. of the body about any other line through the point of intersection. Hence define momental ellipsoid. [8+2]

Group "B" 9x5=45

4. Forces P, Q, R act along the lines

$$x = 0,\ y = 0 \text{ and } x\cos\theta - y\sin\theta = p,$$

axes being rectangular. Find the magnitude and the line of action of the resultant. [5]

OR

A uniform beam of length 2a, rests in equilibrium with one end resting against a smooth vertical wall and with a point of its length resting upon a smooth horizontal rod which is parallel to the wall, and at distance b from it. Show that, the inclination of the beam to the vertical is $\sin^{-1}(b/a)^{1/2}$.

5. State the principal of virtual work for a system of coplanar forces acting on a particle. A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely joined together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that its tension is $\frac{W}{\sqrt{3}}$ [1+4]

6. What are the catenary and uniform catenary? Obtain its equation in Cartesian form. [1+4]

7. Find the centre of gravity of the area included between the curve

$$y^2(2a - x) = x^3$$

and its asymptote.

OR [5]

Find the centre of gravity of the volume formed by the revolution of the portion of the parabola $y^2 = 4ax$, cut off by the ordinate $x = h$, about the axis of x.

8. Define S.H.M. Deduce the equation $x = \alpha con\sqrt{\mu t}$ with usual notations. [1+4]

9. A particle describes a curve (for which s and y vanish simultaneously) with uniform speed v. If the acceleration at any point s be $\frac{v^2 c}{s^2 + c^2}$ curve prove that cue is a catenary. [5]

10. A particle is projected with velocity V from the cusp of a smooth inverted cycloid vertex is $2r$ down the arc; show that the time of reaching the vertex 2 $\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{\sqrt{4ag}}{r}\right)$ [5]

11. Define central force-and central orbit. If the central orbit is an ellipse with centre of force at focus, find the law of force. [1+4]

OR

A particle moves under a central repulsive force= $\left(\frac{m\mu}{(distance)^2}\right)$ and is projected from an apse at a distance a with velocity V. Show that the equation to path is reosp θ = a where $p^2 = \frac{a^2v^2+\mu}{a^2v^2}$ [5]

12. Determine M.I. of a hollow sphere about a diameter: a, b being external and internal radii. [5]

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Attempt ALL the questions.

Group "A" **3x10=30**

1. A company during the festival season combines two factors A and B to form a megha gift pack which must weight 5 kg. At least 2 kgs. of A and not more than 4 kgs of B should be used. The net profit contribution to the company is Rs.5 per kg for A and Rs.6 per kg for B.
   (a) Formulate this problem as a linear programming.
   (b) Determine the quantities of factors A and B so that the total profit is maximised using graph indicating clearly the feasible region on a graph. [5+5]

2. Fill all basic feasible solutions for the system

$$2x_1 + x_2 - x_3 = 2$$
$$3x_1 + 2x_2 + x_3 = 3, \qquad x_1, x_2, x_3 \geq 0.$$

Indicate the basic variables and non-basic variables in the solutions obtained by you. Identify degenerate and non-degenerate solutions. Show that the intersection of any two convex sets of 91° is given a convex set. What happen their union? [3+2+5]

OR

Define convex polyhedron with an example. If the feasible region of an LPP is a convex polyhedron, then show that there exists an optimal solution to the LPP and at least one basic feasible solution must be optimal. [3+7]

3. Reduce the following game into their corresponding primal and dual linear programming problem: [10]

$$\begin{matrix} \text{Player B} \\ \text{Player A} \end{matrix} \begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \end{pmatrix}$$

Group "B" **9x5=45**

4. Use simplex method to solve the following

   Max $Z = 3x_1 + 2x_2 + 3x_3$

   Subject to the constraints

   $2x_1 + x_2 + x_3 \leq 2$; $\quad 3x_1 + 4x_2 + 2x_3 \geq 8$; $\quad x_1, x_2, x_3 \geq 0$. [5]

5. Show that the dual of the dual of an L.P.P. is the primal. [5]

6. if $X_{12} = 6$, $X_{23} = 2$, $X_{24} = 6$, $X_{31} = 4$, $X_{33} = 6$ is the degenerate basic feasible solution of the following transportation problem whose unit cost matrix is given below:

Destinations

	D_1	D_2	D_3	D_4	capacity
Origins 0_1	1	2	3	4	6
0_2	4	3	2	0	8
0_3	0	2	2	1	10
Requirements	4	6	8	6	

Check the given basic solution for optimality. If not, determine optimal schedule. [5]

7. Solve the following assignment problem. Machines:

Job	W	X	Y	Z
A	18	24	28	32
B	8	13	17	19
C	10	15	19	22

OR [5]

Solve the travelling salesman.. problem'-with the following cost matrix $[C_{ij}]$ where C_{ij} is the cost, travelling from city I to the city j [5]

To city

From city	1	2	3	4
1	∞	15	30	4
2	6	∞	4	1
3	10	15	∞	16
4	7	18	13	∞

8. Six jobs are perform first on machine X and then on machine Y. The time taken in hours by each job on each machine is given below:

Jobs	A	B	C	D	E	F
MachineX	4	8	3	6	7	5
MachineY	6	3	7	2	8	4

Determine the Qptimal sequence of jobs that minimize the total elapsed time to compete all jobs. [5]

9. The pay-off matrix of a game is as given below:

	Player B		
Player A	1	3	1
	0	–4	–3
	1	5	–1

Determine the number of saddle points and the corresponding optimal solutions. Find the best strategy for each player and the value of the game. Is the game fair? [2+2+1]

10. Using the method of Lagrangian multiplier, find the extreme value for the function $f(x, y) = x^2 + y^2$ subject to the constraint $x + 4y = 2$

OR [5]

Obtain the set of Kuhn-Tucker conditions for the following non linear programming problem

$$\text{Max } Z = 2x_1^2 + 2x_1x_2 - 7x_2^2$$

Subject to the constraints

$$2x_1 + 5x_2 < 98$$

$$X_1, X_2 > 0.$$

11. Determine the initial basic feasible solution of the transportation problem given in the above Question No.6 using the Least Cost Entry method. [5]

12. Define r^{th} factorial of x. Find the function whose first difference is $x^3 + 2x + 9$.

OR [5]

Reduce the difference equation

$$\Delta^2 y_x - 2\Delta y_x = 3$$

to the linear form (involving successive values of dependent variable). Also find its complete solution.

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Attempt ALL the questions.

Group "A" **3x10=30**

1. Define standard deviation. "Standard deviation is the best measure of dispersion." Give reasons to justify this statement.

A computer while calculating the mean and the standard deviation of 25 observations, obtained the following values.

Mean = 56. standard deviation = 2.

It was later discovered at the time of checking that he had wrongly copied down an observation as 64.

What is the mean and standard deviation if incorrect value is omitted?[1+3+6]

OR

What are raw moment and the central moment? What are the relations between the raw and the central moments? How do the moments use in the measure of skewness and kurtosis?

The standard deviation of a symmetrical distribution is 5. What must be the value of the fourth moment about the mean in order that the distribution is (a) lepto - kurtic (b) platy - kurtic? [2+2+2+4]

2. Define correlation coefficient between the two variables. Prove that the correlation coefficient between two variables lies between −1 and +1. What conclusion can be drawn if the correlation coefficient between two variables is 1?

In a partially destroyed laboratory record of an analysis of correlation data, the following results are legible.

Regression: $8x - 10y + 66 = 0$

and $40x - 18y = 214$

Find the correlation coefficient between x and y. [1+4+1+4]

3. Give the meaning of conditional probability of an event with an example. What will be the result if the events are independent?

A speaks the truth in 80% of the cases and B in 70% of the cases. In what percentage of the cases are they likely to contract each other in stating the same fact? [3+1+6]

Group "B" **9x5=45**

4. What are the graphs and diagrams? Distinguish between the graphs and diagrams. [2+3]

5. A variable takes the values a, ar, ar^2.....ar^{n-1} each with frequency unity. Find the arithmetic mean and the geometric mean. [5]

6. In two sets of variables x and y with 50 observations each, the following data were observed:

$$\overline{x = 10, \alpha = 3} \overline{y = 6, \alpha_y} = 2, r = 0.3$$

Find the regression equation of y on x. [5]

OR

The following are the marks obtained by a group of students in two papers. Calculate the rank correlation coefficient

Eco. 78 36: 82 25· 75 63

Stat. 84 51 81 69 52 62 [5]

7. If X is a random variable and a, b are constant. Prove that

(i) E (aX) = aE(X), (ii) Var(aX – b) a^2 Var(X). [2+3]

8. Is the function defined below

$$f(x) = -\begin{cases} x & 0 \le x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

is a density function?

Find $P\left(\frac{1}{2} \le x \le \frac{3}{2}\right)$ [3+2]

9. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers exactly 4 will be caught by the disease?

OR

Suppose the number of telephone calls on an operator received from 9.00 to 9.05 A.M. follows a Poisson distribution with mean 3. Find the probability that the operator will receive no calls in that time interval tomorrow. (e^{-3} 0.05) [5]

10. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation 2.5. Assuming the normality of the distribution, find the probability that a candidate selected at random will score above 15?

Given: $P(0 \le z \le 0.4) = 0.1554$. [5]

11. Fit a straight line trend by the method of least squáre for the following data: [5]

X:	1	3	4	6	8	9
Y:	1	2	4	4	5	7

12. What is F - distribution? What are the importance's of F - distribution in Statistics? [2+3]

OR

A random sample of 500 pineapples was taken from a large consignment and 65 were found to be bad. Show that the standard error of the proportion of bad one's in a sample of this size is 0.015. [5]

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Attempt ALL the questions.

Group "A' 5×7=35

1. Let V be a finite dimensional vector space with a positive definite scalar product. Let W be subspace of V and let $\{w_1, w_2, ... w_n\}$ be an orthogonal basis of W. If $W \neq V$, prove that there exist elements $w_{w+1}, w_{m+2}, ...w_n$, of V such that $\{w_1, w_2, ... w_n\}$ is an orthogonal basis of V. Find an orthonormal basis for the subspace of $\Re^3$ generated by the vectors (1,3,–1) and (2,1,1). [5+2]

2. Let V be a finite dimensional vector space over the complex number of dimension ≥ 1 and let $T: V \to V$ be a linear map. Let P be its characteristics polynomial, prove that $P(T) = 0$.

   OR [7]

   Let V be a finite dimensional vector space over the complex numbers with positive definite hermitian product and let dim V ≥ 1. Let $A : V \to V$ be a unitary operator. Prove that mere exists an orthogonal basis of V consisting of eigenvectors of A.

3. Define homomorphism of groups and illustrate it with a non-trivial example. Let G be a finite group such that G = AB where A is normal subgroup of G and B a subgroup of G. Show that if $\frac{G}{A} \cong B$ then $A \cap B = \{e\}$. [2+5]

   OR

   Let G be a finite abelian group and p divides 0(G), where p is a prime number, prove that there exists an element $a \neq e$ in G such that $a^p = e$. [7]

4. Define Euclidean ring with example. Let R be a Euclidean ring, prove that every element in R is either a unit in can be written as the product of a finite number of elements of R. [1+1+5]

5. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover [L:F] = [L:K][K:F]. [7]

Group "B" 10×4=40

6. Let $L: \Re^3 \to \Re^3$ be a linear map defined by
   $L(x, y, z) = (x - y, x + z, x, + y + 3z)$ show that L is invertible. [4]

7. Let $V = \Re^3$ be a vector space. If two bases $\beta = \{(1,1,0), (-1,1,1), (0,1,2)\}$ and $\beta' = \{(2,1,1), (0,0,1), (-1,1,1)\}$, then find $M_\beta^{\beta'}(id)$. [4]

8. Let V be the vector space of functions generated two functions $f(t) = t$ and $g(t) = t^2$ with, scalar product $\langle f, g\rangle = \int_0^1 f(t)\, g(t)\, dt$. Find an orhtonormal basis for V. [5]

   Or

   Define dual basis. Find he dual basis of {(1, 2), (–1, 3)} of $\Re^2$. [1+3]

9. Let f be the bilinear form on $\Re^2$ over $\Re^2$ over $\Re$ defined by $f\{(x_1, x_2), (x_2, y_2)\} = x_1y_1 + x_2y_2$. Find the matrix of f in the order basis $\{(1, 1), (0, 1)\}$ of $\Re^2$. [4]

OR

Let V be a finite dimensional vector space over the complex numbers with a fixed positive definite I form $\{\bullet, \bullet\}$. If T is an operator such that $\{Tv, v\} = 0$, for all $v \in V$ then prove that $T = O$.

10. Let V be a vector space over the field K and λ is an eigen-value of an operator $T : V \to V$. Let V_λ be the set of all eigenvectors of T associated with eigenvalue λ. Show that V_λ is a subspace of V. [4]

11. Define fan basis. Let $\{u_1, u_2, \ldots\ldots u_n\}$ be a fan basis for a linear map $T : V \to V$, prove that the matrix associated with T relative to this basis is an upper triangular matrix. [1+3]

12. If I(G) is the set of all inner automorphism of the set of all automorphism A(G) of a group G, then prove that I(G) is a normal subgroup of A(G). [4]

OR

What is normalizer of an element of a group? Prove that normalizer N(a), $a \in G$, is a subgroup of a group G. [1+3]

13. Suppose G is the internal direct product of $N_1, N_2, \ldots N_3$, prove that for $i \neq j$, $N_i \cap N_j = (e)$ if $a \in N_i$, $b \in N_j$ then $ab = ba$. [4]

14. If J_p, the ring of integers mod p, is a field, then prove that prime number.

OR [4]

If the primitive polynomial $f(x)$ can be factored as the product of two polynomials having rational coefficients, then prove that it can be factored as the product of two polynomials having integer coefficients.

15. Define algebraic element of degree n over a field F. Prove that the elements in K which is algebraic over F form a subfield of K. [1+3]

Tribhuvan University, 2070

Attempt ALL the questions.

Group "A" **3×10=30**

1. What are the catenary and a uniform catenary? Obtain the equations of the uniform catenary in intrinsic form and hence obtain its equation in Cartesian form. [2+8]

2. Define simple harmonic motion. Show that the period of oscillation of the SHM is independent of the amplitude. Also prove that if a particle describes a circle with constant angular velocity, the foot of perpendicular from it an any diameter executes SHM. [1+6+3]

OR

A particle moves in a straight line under an attraction towards a fixed point on the line varying inversely as the square of the distance from the fixed point. Discuss the nature of the motion. If the law of attraction be inverse distance show that the time of descent is $a\sqrt{\frac{\pi}{2\mu}}$ [6+4]

3. If the moments and products of inertia about all axes through centre of gravity of a body are given, then find those about ail parallel axes. Hence determine the M..L of a solid sphere about a tangent. [6+4]

Group "B" 9×5=45

4. Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x \cos \alpha - y \sin \alpha = p$, axes being rectangular. Find the magnitude of the resultant and equation of its line of action.

OR [5]

A beam whose centre of gravity divides it into two portions. a and b. is placed inside the a smooth sphere; show that if 9 be the inclination of the horizon in the position of equilibrium and 2α be the angle subtended by the beam at the centre of the sphere then $0 = \left(\frac{b-a}{b+a}\right)$ [5]

5. Enumerate the forces which may be omitted informing the equation of virtual work. [5]

6. Using intrinsic equation of catenary. obtain the equations (i) $y = c \sin \psi$ (ii) $y^2 - s^2 + c^2$, with usual notations.

OR [5]

A given length, 2s, of a uniform chain has to be hung between two points at the same level and the tension has not to exceed the weight of length b of the chain. Show that the greatest span is $(\sqrt{b^2 - s^2}) \log \left(\frac{b+s}{b-s}\right)$.

7. Fine the C. G. of the area bounded by the parabola $y^2 = 4ax$, the axis of x and the latus rectum. [5]

8. In a S.H.M. of amplitude a and period T, prove than $\int_0^T V^2\, dt \ \frac{2\pi^2 a^2}{T}$. [5]

9. A particle moves along a circle $r - 2\, a\cos\theta$ in such a way that its acceleration towards the origin is always zero.

Prove that $\frac{d^2\theta}{dt^2} = -2 \cot \theta \, , \theta^2$. [5]

OR

Prove that the tangential and normal acceleration of a particle describing a plane curve be constant throughout the motion, the angle y through which the direction of motion turns in time T is given by $v = A \log (1 + Bt)$. [5]

10. A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Find the velocity and reaction at any point of the circle. [5]

11. Define central force and central orbit. If the orbit is a cardiode $r = a(1 + \cos \theta)$, find the law of forces. [5]

12. Determine the M.I. of truncated cone about its axis, a, b being radii of ends.

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Attempt ALL the questions.

Group "A" **3×10=30**

1. A company produces two types of models M_1 and M_2. Each Mi model requires 4 hours of grinding and 2 hours of polishing where each M_2 requires 2 hours of grinding and 5 hours of polishing. The company has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polishers works for 60 hours a week. Profit on an M_1 model is Rs.3.00 and on M_2 model is Rs.4.00; whatever is produced in a week is sold in the market. How should the company allocate its production capacity to two types of models so that it may make the maximum profit in a week? Formulate this problem as a linear programming problem and hence solve graphically. [10]

2. Prove that the value of a game with pay off $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, without any saddle point, is $\frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$ and hence find the solution to the $\begin{pmatrix} 2 & 5 \\ 4 & -1 \end{pmatrix}$ [6+4]

3. Give mathematical formulation of transportation problem. Show that the condition $\Sigma a_i = \Sigma b_j$ (i.e. total supply = total demand) is both necessary and sufficient for the existence of the feasible solution of an m × n transportation problem. Do you agree the solution of the T.P. never unbounded? Justify your answer. [2+5+2]

Or

What is an unbalanced transportation problem? Solve the following cost minimizing transportation problem. [1+9]

Destinations

Sources		D_1	D_2	D_3	D_4	Supplies
	S_1	25	17	25	20	40
	S_2	15	10	18	15	30
	S_3	16	20	8	10	60
		40	40	50	10	

Requirements

Group "B" **9×5=45**

4. By using simplex algorithm, show that the LPP

$$\text{Max } Z = -x_1 + x_2$$

Subject to the constraints

$x_1 - x_2 \geq -1$

$-x_1 + 2x_2 \leq 4;$

$x_1 \geq 0, x_2 \geq 0$

admits of an alternative optimal solution. [5]

5. Find the dual of the following primal problem:

$\text{Min } Z = 3x_1 - 2x_2$

Subject to the constraints

$2x_1 + x_2 \leq 1$

$-x_1 + 3x_2 \geq 4;\ x_1 \geq 0, x_2 \geq 0.$

Also verify that the dual of the dual is primal. [5]

6. If the objective function on an LPP assumes its optimal value at more than one extreme point, then prove that it takes on the same value for every convex combination of those particular points. [5]

OR

Solve the following assignment problem:

	I	II	III	IV
A	12	18	22	26
B	12	7	11	13
C	4	9	13	16

7. Solve the travelling salesman problem given by the following data:
$C_{12} = 4, C_{13} = 7, C_{14} = 3, C_{15} = 4, C_{23} = 6, C_{24} = 3, C_{25} = 4, C_{34} = 7, C_{35} = 5, C_{45} = 7$
where $C_{ij} = C_{ji}$ and there is no route between cities i and j (i.e. $C_{ij} = \propto$) [5]

8. Find the minimal value of
$f(x, y, z) = x_2 + y_2 + z_2$ when $x + y + z = 3,\ x, y, z \geq 0.$ [5]

9. Find the best strategy for each player from the following rectangular game[5]

Player B

$$\text{Player A} \quad \begin{pmatrix} 2 & 3 & 11 \\ 7 & 5 & 2 \end{pmatrix}$$

Or

Use dominance principle to solve the following 3 × 3 game

B

$$A \quad \begin{pmatrix} 2 & 3 & 1/2 \\ 3/2 & 2 & 0 \\ 1/2 & 1 & 1 \end{pmatrix}$$

Or

10. Use Kuhn – Tucker condition to solve the following non-linear programming problem:

Max $Z = 36x - 4x^2 + 16y - 2y^2$

Subject to the constraints [5]

$$2x + y \leq 10,\ x, y \geq 0.$$

11. Solve the difference equation
$y_x = y_{x-1} + y_{x-2}$, $x \geq 2$ given that $y_0 = 1$ and $y_1 = 1$. [5]

Or

Express the polynomial $3x^3 + 2x^2 - 2x + 5$ in fractional notation taking difference interval 1 (unity) and hence find the first difference of the function. [3+2]

12. Define hyperplane in $\Re^n$. Prove that the hyperplane is a convex set. [1+4]

Tribhuvan University, 2070

Attempt ALL the questions.

Group "A" 3×10=30

1. Define standard deviation. Explain why standard deviation is considered as the most suitable measure of the dispersion.

Find the mean and standard deviation of the natural numbers, the frequency of each being unity, under what circumstances would you use A.M., G.M., H.M. the most suitable to describe the measure of central tendencies of a distribution. [1+3+2+4]

OR

Distinguish between central moment and raw moment of a distribution. Establish the relation

$\mu r = \mu_r^1 - rc_1\, \mu_1^1\, \mu_{r-1}^1 + rc_2\, \mu_r^{12}\, \mu_{r-2}^1 - rc_3\, \mu_1^{13}\, \mu_{r-3}^1 + \ldots\ldots\ldots + (-1)^r \mu_1^1$.

The first three moments of a distribution about the value 2 are 1, 16 and −40, show that the mean is 3, the variance 15. [2+3+5]

2. Define Karl Pearson's correlation coefficient between two variable x and y. Prove that the correlation coefficient between two variable is independent of the change of origin and scale.

Calculate the rank correlation coefficient between x and y

x:	3	5	8	4	7	10	2	1	6	9			
y:	6	4	9	8	1	2	3	10	5	7			[1+4+5]

3. What are mutually exclusive events? Give an experimental definition of probability. What do you mean by a sample space? If A, B, C be not three mutually exclusive event prove that

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$. [1+1+1+7]

Group "B" 9×5=45

4. Draw both less than and more than ogives from the following data and hence find the median mark.

Marks :	10-20	20-30	30-40	40-50	50-60	60-70	
No. of students :	5	7	12	18	15	3	[4+1]

5. Show that in a descrets series if deviation are small compared with mean m so that $\left(\frac{x}{m}\right)^3$ and higher powers of $\left(\frac{x}{m}\right)$ are neglected $G = m\left(1 - \frac{\sigma^2}{2m^2}\right)$ where m is the A.M. and a is the s. d. of the series. [5]

6. Find the mean deviation from the mean and standard deviation of the A.P.

$$a,\ a + d,\ a + 2d,\ \ldots\ldots\ldots\ldots,\ a + nd.$$ [2+3]

OR

In the two sets of variables x and y with 50 observations each, the following data were observed

$$\bar{x} = 10,\ \sigma_x = 3,\ \bar{y} = 6,\ \sigma_y = 2,\ r = 0.3$$

Find the regression equation of y on x.

7. If X is a random variable and a, b are constants prove that

(i) $E(aX + b) = aE(X) + b$

(ii) var (aX + b) = a^2 var X.

6. Verify that the function $f(x)$ is a d.f and derive the p.d.f

$$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 & \text{for } 0 \le x \le 1/2 \\ 1-3(1-x)^2 & \text{for } ½ \le x < 1 \\ 1 & \text{for } x \ge 1 \end{cases}$$ [5]

9. What is Poisson distribution? Find the mean of the Poisson distribution.

Or

If the mean and variance of a binomial distribution are 4 and 4/3 respectively. Find $P(X \ge 1)$. [5]

10. Fit an equation of the form $y = a + bx + cx^2$ to the following data : [4+1]

X:	3	4	2	1	5
Y:	33	39	28	25	28

11. The probability that a man aged 70 years will die within a year is 0.01125, what is the probability that of such men

(i) no body will die

(ii) 1 person will die. [1+2+2]

12. Show that the mean and variance of the geometric distribution

$$p(x) = px^x \quad x = 0, 1, 2 \ldots$$

are $\frac{p}{q}$ and $\frac{q}{p^2}$

Or

Describe briefly t-test and f-test. [5]

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Attempt ALL the questions.

Group "A" 5×7=35

1. What is exact differential equation of first order ?

Show that $(x^3 - x)\frac{d^3y}{dx^3} - (8x^2 - 3)\frac{d^2y}{dx^2} + 14x\frac{dy}{dx} + 4y = \frac{2}{x^3}$ is exact. Find the first and second integral. [1+2+2+2]

Or

By the method of variation of parameters, solve differential equation $\frac{d^2y}{dx^2} + (1 - \cot x)\frac{dy}{dx} - y\cot x = \sin^2 x$. [7]

2. Define curvatue and torsion for a curve at a point on the space curve. Hence state and prove Serret – Frenet formula. [2+5]

Or

Prove that $[t' \ t'' \ t'''] = k^3(k\tau' - k'\tau) = \kappa^5 \frac{d}{ds}\left(\frac{\tau}{k}\right)$; the symbols have their usual meanings. [7]

3. When a function is analytic at a point ? Write the Cauchy – Riemann equations in Cartesian and polar form. Find λ such that the function $f(z) = r^2$

$\cos 2\theta + ir^2 \sin \lambda\theta$ is analytic. [1+2+4]

4. State Green's theorem and use it to evaluate $\int (2x - y + 4)\, dx + (5y + 36 - 6)\, dy$ around a circle of radius 4 units with centre at origin. [1+6]

5. Define trigonometric series. State, when it becomes Fourier series. If a function is such that

$$f(x) = \begin{cases} -\frac{1}{2}, & -\pi < x < 0 \\ \frac{1}{2}, & 0 < x < \pi \end{cases}$$

then show that Fourier coefficients for $f(x)$ is $a_n = 0$, $b_n = 0$ or $\frac{2}{nr}$ according as n is even or odd.

Group "B" 10×4=40

6. Define osculating plane and determine its equation. [4]

7. Solve : $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3 = 0$ [4]

8. Solve : $\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 4x^2y = 0$ by removing the first derivatives. [4]

9. From a PDE by eliminating f from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. [4]

10. Find P.I. of $s + p - q = z + xy$. [4]

11. Solve : $p^2x + q^2y = z$ by Charpit's method. [4]

Or

Solve $z(qs - pt) = pq^2$ by Monge's method. [4]

12. Prove that $u = x^2 - y^2$, $V = \frac{y}{x^2 + y^2}$ both **satisfy Laplace's equation. Find the harmonic conjugate of** $v = \arg z$ **[2+2]**

13. **If $\vec{F} = r^2\vec{r}$ show that $\vec{F}$ is conservative field and scalar potential is $\phi = \frac{r^4}{4}$ + constant.** [4]

Or

Evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = y^2z^2\vec{i} + z^2x^2\vec{j} + x^2y^2\vec{k}$ and S is the surface of sphere $x^2 + y^2 + z^2 = 1$ about xy – plane.

14. Find the real and imaginary parts of e^z and coshz. [4]

Or

Define limit and continuity of a complex valued function. Discuss the limit of $\frac{|z|}{z}$ at $z = 0$.

15. Show that cosmx and cosnx are orthogonal on any interval of length 2π, provided $m^2 \neq n^2$. Write down the complex form of Fourier series and the formulas for its coefficients. [4]

Or

Define periodicity of a function. Develop the Fourier series for

$$f(x) = |x|, -\pi \le x \le \pi.$$

Tribhuvan University, 2070

Attempt ALL the questions

Group "A"

1. Define uniform convergence of a sequence of functions. State and prove Cauchy's condition of uniform convergence for a sequence. [1+6]

2. Define the improper integral of the first kind with an example.

   Evaluate, $\int_1^\infty \frac{dx}{x^2}$. Prove that if $0 \le f$ is integrable over [a, t] for all t > a and for p > 1, $\lim_{x\to\infty} x^p f(x) = L$, then $\int_a^\infty f(x)\,dx$ converges. [2+1+4]

   Or

   Assume that

   (i) f is integrable over [a, t] for all $t \ge a$ and $\exists M > 0$, $\forall t \ge a$

   $$\left|\int_a^t f(x)\,dx\right| \le M.$$

   (ii) g (x) is monotonically decreasing to 0 as x tends to ∞, then prove that $\int_a^\infty f(x)\,g(x)\,dx$ exists. Also show that $\int_{-\infty}^{\infty} \frac{dx}{x^4+4}$ converges. [5+2]

3. Prove that in Euclidean space $\Re^k$, every Cauchy sequence is convergent. Discuss the convergence of a real sequence (a_n) such that $|a_{n+2} - a_{n+1}| \le \frac{1}{2} |a_{n+1} - a_n|$ for all n > 1. [5 + 2]

4. Define a function of bounded variation on [a, b]. Show that the function f defined by

   $$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

   is bounded variation on [0, 1]. Also prove that if f and g are each of bounded variation ion [a, b] then so is their sum and $V_{f\pm g} \le V_f + V_g$. [1+2+4]

5. Let f, α be defined and abounded on [a, b]. What is a Riemann – Stieltjes sum of f with respect to α when f is said to be integral with respect to α ? And what is the Riemann – Stieltjes integral of f with respect to α ? Prove that if f in integrable with respect to α, then it satisfies Riemann's condition with respect to α on [a, b]. [3 + 4]

   Or

   Let $f \in R(\alpha)$ on [a, b] and assume that α has a continuous derivative α' on [a, b]. Prove that the Riemann integral $\int_a^b f(x)\,\alpha'(x)\,dx$ exists and $\int_a^b f(x)\,d\alpha(x) = \int_a^b f(x)\,\alpha'(x)\,dx$. [7]

Group "B" $10\times4=40$

6. Define point wise convergence of a sequence of function on a set. Prove that $\{x^n\}_{n=1}^{\infty}$ converges point wise but not uniformly on[0, 1]. [1 + 3]

7. Prove that a finite union of closed sets in $\Re^n$ is closed in $\Re^n$. Prove on disprove that any union of closed sets in $\Re^n$ is closed in $\Re^n$. [2+2]

Or

What is open set in $\Re^n$? Show that on an open n ball is an open set in $\Re^n$. [1 + 3]

8. Define an open covering of a set. Prove that a closed subset of a compact metric space is compact in the metric space. [1+3]

9. Define isolated point. Prove that every function is continuous at an isolated point. [1+3]

Or

Let $f: S \to T$ be a function where S and T are two sets. If $X \subseteq S$ and $Y \subseteq T$, then prove that
(i) $X = f^{-1}(Y) \Rightarrow f(X) \subseteq Y$ (ii) $Y = f(X) \Rightarrow X \subseteq f^{-1}(Y)$ [2+2]

10. Let S be a subset on $\Re^n$. If $f: S \to \Re^n$ is differentiable at c, then prove that f is continuous at c. [4]

11. Investigate the convergence or divergence of the integral $\int_a^b \frac{dx}{\sqrt{x}}$. [4]

12. Define total variation of a function on [a, b]. Prove that $V_f(a, b) = 0$ if and only if f is constant function on [a, b]. [1 + 3]

Or

Prove or disprove that the reciprocal of a function of bounded-variation on [a, b] is of bounded variation on [a, b]. [4]

13. Evaluate $\int_{\pi}^{2n} \sin d(\cos x)$, by changing into Riemann integral. [4]

14. Show that the second mean value theorem for Riemann integral does not hold on [−1, 1] for $f(x) = g(x) = x^2$. [4]

15. If $\{a_n\}$ has a finite limit superior $\cup$, then show that it is unique. [4]

Or

Define subseries of a series. If Σa_n converges absolutely, then prove that every subseries Σb_n also converges absolutely, moreover

$$\left|\sum_{n=1}^{\infty} b_n\right| \le \sum_{n=1}^{\infty} |b_n| \le \sum_{n=1}^{\infty} |a_n|$$ [1+3]

Tribhuvan University, 2071

Attempt ALL the questions.

Group "A" $5\times7=35$

1. Define curvature, torsion and screw curvature in a curve. Find the equation of the osculating plane at the point t on the curve $\vec{r} = (a\cos t, a\sin t, ct)$ [4+3]

OR

Find the curvature and torsion of the curve $\vec{r} = \vec{r}(t)$ and $\vec{r} = \vec{r}(s)$. [4+3]

2. Define Monge's method of solving second order partial differential equation $Rr + Ss + Tt + U(rt - s^2) = V$ where R, S, T, U, V are functions of p, q, x, y, and z. [7]

OR

Define second order PDE. Solve: $r - 7s + 12t = e^{x-y}$. [1+6]

3. State Divergence theorem. Evaluate

$$\iint_s (y^2z^2\vec{i} + z^2x^2\vec{j} + z^2y^2\vec{k}).\vec{n}\ ds$$

where s|s in the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy plane and bounded by this plane. [1+6]

4. Define analytic function. Prove that CR equations are satisfied for $f(z) = z^2$ but not for $f(z) = |z|^2$ when $z \neq 0$. [1+6]

5. Define Fourier series. Determine the Fourier coefficients. Hence find the Fourier series for the function/(x) defined as

$f(x) = x\ (-\pi \leq x \leq \pi)$ [1+2+4]

Group "B" 10×4=40

6. Solve: $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$, by variation of parameter method. [4]

7. Solve : $u = r\frac{d}{dr}\left(r\frac{du}{dr}\right) + ar^3$. [4]

8. Solve : $\frac{dx}{dt} + y = e^t; \frac{dy}{dt} - x = e^{-t}$ [4]

Or

Solve : $\frac{d^2x}{dt^2} + 4x + y = te^{3t}; \frac{d^2y}{dt^2} + y - 2x = \cos^2t$ [4]

9. Solve by Charpit's method : $q = px + p^2$. [4]

Or

Solve : $(9p^2z + q^2) = 4$ [4]

10. Solve : $r = a^2t$ by Monge's method. [4]

Or

$\frac{\partial z^2}{\partial x^2} = \frac{\partial z}{\partial t}$. [4]

11. Show that the principal normal's at consecutive points do not intersect unless $\tau = 0$. [4]

12. Evaluate $\int_c \vec{F}.d\vec{r}$, where $\vec{F} = x^2y^2\vec{i} + y\vec{j}$ and the curve C is $y^2 = 4x$ in the xy plane from (0, 0) to [4, 4]. [4]

13. State Green the theorem and evaluate.

$\oint_c (\cos x \sin y - xy)\,dx + \sin x \cos y\,dx$, by the theorem where C is the circle : $x^2 + y^2 = 1$. [4]

14. What do you understand by a harmonic function ? Show that if $f(z) = u + iv$, is analytic, then u and v are harmonic. [4]

Or

Define Harmonic conjugate. Show that

(a) $V(x, y) = 3x^2y - y^3$ is harmonic

(b) Find the conjugate function $u(x, y)$ [1+1.5 + 1.5]

15. Expand $f(x) = x^2$ for $-\pi \le x \le \pi$ in a Fourier series. [4]

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Attempt ALL the questions.

Group "A" 5×7=35

1. Define linearly dependent set of vectors. Let V be a vector space over the field K. Let $\{v_1, v_2, \ldots, v_m\}$ be a basis of V, let $w_1, w_2, \ldots, w_n$ be elements of V. If $n > m$, prove that $w_1, w_2, \ldots, w_n$ are linearly dependent. Show that if $v_1 = (1, 2)$, $v_2 = (1, -3)$ is a basis of $\Re^2$, then $w_1 = (4, -1)$, $w_2 = (3, 1)$ and $w_3 = (2, 5) \in \Re^2$? are linearly dependent vectors. [½ +5+1½]

OR

Let V and W be two subspaces of a finite dimensional, vector space over the field K, prove that $\dim(V + W) = \dim V + \dim W - \dim(U \cap W)$. [7]

2. "What do you mean by a hermitian form? Let V be a vector space of continuous complex valued 'functions on the interval $[-\pi, \pi]$, if $f, g \in V$ and $\{f, g\} = \int_{-\pi}^{\pi} f(t)\,\bar{g}(t)\,dt$, show that this is a hermitian form. Also show that f is hermitian form on C^n where f is defined by $f(x, y) = X' A \bar{Y}$. [1+2+4]

3. Give a non trivial example of a homomorphism and find its kernel. Let $L : G \to \bar{G}$ be a homomorphism and K be the kernel. Prove that $\frac{G}{K}$ and Im L are isomorphic and also prove that if N is a normal subgroup of a group G then there exists a homomorphism $h : G \to \frac{G}{N}$ with kernel N defined by $h(g) = Ng$ for all $g \in G$. [1+4+2]

OR

Define normalizer of an element a in a group G. Prove that the number of elements conjugate to a in G is the index of the normalizer of a in G i.e. $C = \frac{0(G)}{0(N(a))}$. Find the conjugacy class of (12) and normalizer N(12) in S_3 and verify above statement. [1+4+2]

4. Define Euclidean ring. Give an example of a Euclidea ring. Prove that the ideal $A = (a_0)$ is a maximal ideal c the Euclidean ring R if and only if a_0 is prime element of R. [1+1+5]

5. How do you define roots of a polynomial? Prove that polynomial of degree n over a field can have at most roots in any extension field. [1+6]

Group "B" 10×4=40

6. Define linear map. Let $F : \Re \to \Re^2$ be a linear mapping such that $F(x, y) = (3x - y, 4x + 2y)$. show that F has a inverse linear map. [1+3

7. Find the matrix representation of $T : \Re^3 \to \Re^3$ relative t the usual basis where $T(x, y, z) = (2x - 3y + 4z, 5x - y + 2z, 4x + 7y)$. [4]

8. Find the dual basis of the basis vectors

$B = \{(1, 2), (-1, 3)\}$ for $\Re^2$. [4]

OR

Let $v_1, v_2, \ldots, v_n$ be vectors which are mutually perpendicular and such that $\|v_i\| \neq 0$ for all i. Let v be an element of a vector space. V and let C_i, be the Fourier coefficient of v with respect to v_i. Let $a_1, a_2, \ldots, a_n$ be numbers, prove that

$$\left\| v - \sum_{k=1}^{n} a_k v_k \right\| \leq \left\| v - \sum_{k=1}^{n} a_k v_k \right\|$$

If $v_1 = (-1, 2)$, $v_2 - (2, 1) \in \Re^2$ and $v = (1, 0) \in \Re^2$ and $C_1 = \frac{1}{5}$, $C_2 = \frac{2}{5}$ are the Fourier coefficients of v with v_1 then verify above statement. [3+1]

9. Define quadratic form. What is the associated matrix of the quadratic form $f(x, y) = x^2 - 3xy + 4y^2$? [1+3]

Or

Determine the index of nullity and index of positivity for each form determined by the symmetric matrix $C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$ on $\Re^2$ by using Sylvester's theorem. [2+2]

10. Let V be a finite dimensional vector space over the complex number and assume dim $V \geq 1$. Let $A : V \to V$ be a linear map, prove that there exists a non zero eigen vector of A. What happens if vector space V is over $\Re$? [3+1]

11. What is fan basis? Let $\{v_1, v_2, \ldots, v_n\}$ be a fan basis for a linear map $T : V \to V$. Prove that the matrix associated with T relative to this basis is an upper triangular matrix. [1+3]

12. What do you mean by an inner automorphism of a group G? Prove that the set of all inner automorphisms I (G) is a subgroup of the set of all automorphism A(G) of G. [2+2]

13. Let ϕ be a homomorphism of a group G onto a group $\bar{G}$ with kernel K, prove that $\frac{G}{K} \cong \bar{G}$. [4]

14. Define ring homomorphism with an example. If R is a ring with unit element 1 and ϕ is a homomorphism of R onto R', then prove that $\phi(1)$ is a unit element of R'. [1+3]

OR

Prove that a necessary and sufficient condition that the element a in the Euclidean ring be a unit is that $d(a) = d(1)$. [4]

15. If $f(x) \in F[x]$ is irreducible, prove that the following:

(a) If the characteristic of F is 0, f(x) has no multiple roots,

(b) If the characteristic of F is $p \neq 0$, f (x) has a multiple roots only if it is of the form $f(x) = g(x^p)$. [4]

OR

If F is a field of characteristic $p \neq 0$, then prove that the polynomial $x^{p^n} - x \in F[x]$ for $n \geq 1$, has distinct roots. [4]

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Attempt ALL the questions.

Group "A" $3\times10=30$

1. Define system of coplanar forces.

   If six forces of the relative magnitudes 1, 2, 3, 4, 5 and 6 act along the sides of a regular hexagon, taken in order. Show that the single equivalent force is of the relative magnitude 6 and that its line of action is parallel to the force 5 at a distance from the centre of the hexagon which is 3.5 times the distance of the side from the centre.

   OR

   A uniform beam of length 2a, rests in equilibrium with one end resting against a smooth vertical wall and with a point of its length resting upon a smooth, horizontal rod which is parallel to the wall, and at a distance b from it. Show that the inclination of the beam to the vertical is $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$

2. Define angular velocity and angular acceleration. Obtain the expression for tangential and normal velocities.

3. Show that the MI of a semi-circular lamina of mass M and radius a about the tangent parallel to the bounding diameter is $Ma^2\left(\frac{5}{4}-\frac{8}{3\pi}\right)$

Group "B" $9\times5=45$

4. Forces P, 2P, 3P acts along the sides of a triangle formed by the lines $x = 0$, $y = 0$ and $3x + 4y = 5$. Find the magnitude of the resultant and equation of the line of action.

5. Two equal uniform rods AB and AC, each of length 2b, are freely joined at A and rest on a smooth vertical circle of radius a show that, if 2θ be the angle between them then $b\sin^3\theta = a\cos\theta$.

   OR

   Four uniform rods are freely jointed at their extremities and form a parallelogram ABCD which is suspended by the joint A and is kept . in shape by a string AC. Prove that the tension of the string is equal to half of the whole weight.

6. Define the term a vertex, axis, directrix, span and a sag of catenary. Further obtain the relation between x and ψ

7. Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$, the axis of x and the latus rectum.

8. A point describes uniformly a given straight line. Show that its angular velocity about a fixed point varies inversely as the square of its distance from the fixed point.

   OR

   A point moves in a curve, so that its tangential and normal accelerations are equal and tangent rotates with constant angular velocity. Prove that the equation of the path is of the form

   $S = Ae^{\psi} + B$

9. A particle moves in a straight line from a distance a towards the centre of the force, the force varying inversely as the cube of the distance. Show that the

time of descent to the centre is $\frac{a^2}{\sqrt{\mu}}$.

10. If the orbit is a cardiod $r = a(1 + \cos\theta)$, find the law of force.

OR

A particle acted upon by a central attractive force $\frac{\mu}{r^3}$ is projected with velocity $\frac{\sqrt{\mu}}{a}$ at an angle $\frac{\pi}{4}$ with its initial distance a from the centre of force, prove that the orbit is $r = ae^{-\theta}$.

11. A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the archaeology. Show that the time of $2\sqrt{\frac{a}{g}} \tan^{-1}\left(\frac{\sqrt{4ag}}{v}\right)$

12. Find the M.I of a hollow sphere of mass M about a diameter whose external and internal radii are a & b.

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Attempt ALL the questions.

Group "A" **3×10=30**

1. Define correlation coefficients between two variables and types of correlation. Prove that the correlation coefficient between two variables lies -1 to 1. What conclusion can be drawn if the correlation coefficient between the two variables is (a) 0 and (b) 1? Calculate the covariance and the coefficient of correlation between x and y, when
$n = 10, \Sigma x = 60, \Sigma y = 60, \Sigma x^2 = 400, \Sigma y^2 = 580$ and $\Sigma xy = 305$. [2+1+3+4]

OR

Describe regression and its types. If the data is

x (age of husband)	25	22	28	35	20	22	40	20	18
y (age of wife)	18	15	20	22	14	16	21	15	14

Find the regression coefficients and hence the equations of the two lines of regressions. Estimate the age of husband, when the age of wife is 19. [2+6+2]

2. Define skewness and its types. Calculate the Karl Pearson's coefficient of skewness of the data

Mid value of the income	150	250	350	450	550	650	750	850
No. of workers	80	105	120	165	100	90	60	40

Interpret the result. [3+6+1]

3. Define Baye's theorem. Illustrate it by an example. Two dice are thrown, what is the probability that the sum is greater than eight ? [2+3+5]

Group "B" **9×5=45**

4. Define types of data. Give an example of continuous and discrete type of data. [2+3]

5. Define measure of central tendency. Find the geometric mean of the

following data:

x :	100	150	200	250	300	350
f :	5	7	20	15	8	5

6. Find the line of regression of x on y for the data : [5]

x	6	2	10	4	8
y	9	11	5	8	7

OR

Calculate the rank correlation coefficient of the data :

x	1	2	3	4	5	6	7	8	9	10
y	3	8	1	7	10	2	9	4	6	5

7. Define law of total probability for two sets.
Describe the case when the two sets are
(i) mutually exclusive
(ii) not mutually exclusive [1+4]

8. If X is a random variable and a, b are constants, prove that
(i) $E(ax) = aE(X)$ (ii) $var(aX - b) = a^2 \, var(X)$ [2+3]

9. Find K for the probability density function

$$f(x) = \begin{cases} kx, & 0 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $P(1 \le x \le 2)$. [3+2]

10. What is binomial distribution ? Fit a binomial distribution to the following data : [1+4]

x	0	1	2	3	4
f	28	62	46	10	4

Or

If the mean is 6 and the standard deviation is $\sqrt{2}$, write out the all terms of the binomial distribution. [5]

11. What is Poisson distribution ? If a random variable has a Poisson distribution such that [1+4]
$P(X = 1) = P(X = 2)$, find $P(X = 4)$ [1+4]

12. What is t distribution ? What are the importance of the distribution ?

OR [2+3]

A manufacturer intends that his electric light bulbs have a life of 1000 hours. He tests a sample of 20 bulbs, drawn at random from a batch and discovers that the mean life of the sample bulbs is 990 hours with a standard deviation of 22 hours. Does this signify that the batch is not up to the standard ? [5]

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Attempt ALL the questions.

Group "A' **3×10=30**

1. A farmer requires 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B, and C respectively, per jar, and a dry product contains 1, 2 and 4 units of A, B, and C respectively per carton. I the cost of the liquid product is Rs.3 per jar and

the cost of the dry product is Rs.2^2 per carton, how many of each should be purchased to minimize the cost in linear programming and hence solve it graphically, assuming that the left-over materials cannot be used. [10]

2. Prove that the system

$x_1 + 2x_2 + x_3 = 6;$ $4x_1 + 3x_2 + x_4 = 12;$

$x_1 \geq 0, x_2 \geq 0; x_3 \geq 0, x_4 \geq 0$ has a basic feasible solutions

$x_1 = 0, x_2 = 0; x_3 = 6, x_4 = 12$. If possible find other set of basic feasible solutions. [10]

3. Show that the number of basic variables in a transportation problem of m origin and n destinations is at most m + n – 1. Explain the lowest cost entry method for obtaining an initial basic solution of the following transportation problem.

		Destinations			
		I	II	III	IV
Warehouse	1	21	16	25	13
	2	17	18	14	23
	3	32	27	18	41
Requirement		16	10	12	15

Also the total transportation cost according to the initial solution.

Or [6+3+1]

By Vogel approximation method, solve the following transformation problem:

O \ D	14	8	23
17	13	15	16
12	7	11	2
16	19	20	9

Group "B" 9×5=45

4. Use simplex method, solve the following:

Max $Z = 4x_1 + 10x_2$ subject to the constraints;

$2x_1 + x_2 \leq 50;\ 2x_1 + 5x_2 \leq 100;\ 2x_1 + 3x_2 \leq 90;\ X_1, x_2 \geq 0.$ [5]

5. Show that the dual of the dual of an L.P.P. is the primal [5]

OR

Find the dual minimum of the following:

Max $Z = 2x_1 + 3x_2 + x_3$ subject to the constraints; $4x_1 + 3x_2 + x_3 = 6;\ x_1 + 2x_2 + 5x_3 = 4;$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$ [5]

6. In a 2 × 3 transportation, draw a diagram which shows that the capacities of the sources and destination arid transfer matrix as well as the cost matrix. [5]

7. An office wants to buy two types of pens P_1 and P_2 from two contractors C_1 and C_2. The offers by C_1 are Rs.5 and Rs. 4 for P_1 and P_2 respectively while the offer by C_2 are Rs.5.40 and Rs.3.75 for them. Determine the model of assignments of the pens to the contractors by the office so that the cost of the office will be minimum. [5]

8. A children's game is as follows. Each of players R and C says 'stone' or 'disk' or 'paper', denoted by S, D, P respectively. If one says S and other says D, then the former wins two rupees from the latter. Similarly D beats P, and P