

## 5. Mathematics Education

(a) Geometry (Math. Ed. 321)

Exam 2067

Time: 3 hrs.

Full Marks: 100

Attempt all the questions.

### Group "B" [8x7=56]

1. Define an incidence geometry. Verify that the four-point geometry is an incidence geometry.
2. Prove that a point is on the perpendicular bisector of a line segment if and only if it is equidistant from the end points of the line segment.

OR

State and prove AAA similarity-theorem condition for triangles.

3. Define Saccharin quadrilateral and prove that two Saccheri quadrilaterals in hyperbolic geometry are congruent if their summits and summit angles are congruent.
4. If two non-coplanar triangles are line perspective. Prove that the triangles are also point perspective.

OR

Explain the following:

- a. Traversability of network
  - b. Genus of a surface
  - c. Moebius strip
5. Find the equation of the plane passing through the intersection of the plane  $x + y + z = 5$  and  $2x + 3y + 4z - 5 = 0$  and passing through the origin.
  6. Find the perpendicular distance of the point  $(3, -1, 11)$  from the line  $\frac{x}{2} + \frac{y-2}{3} + \frac{z-3}{4}$ . Also obtain the equation of the perpendicular.
  7. Find the equation of a cone whose vertex is  $(a, B, Y)$  and base is  $y^2 = 4ax, z = 0$ .

OR

Show that the tangent planes at the extremities of any diameter of any ellipsoid are parallel.

8. Show that the pair of tangent drawn from the point  $(-6, 9)$  to the parabola  $y^2 = 24x$  are at right angles.

### Group "C" 2x12=24

9. (a) If  $P$  and  $q$  are the intercepts made by the tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  on major and minor axes respectively. Prove that  
(b) Find the equations of tangent to the hyperbola  $3x^2 - y^2 = 3$  Which is parallel to the line  $y = 2x + 4$ .

OR

Prove that the general equation of second degree in  $x$  and  $y$  represents a conic section.

10. (a) Define homothetic (dilation). If  $c$  is between  $A$  and  $B$  and let  $a$  be a homothety then prove that  $a(c)$  is between  $a(A)$  and  $a(B)$ .  
(b) Prove that the inverse of a line not through the center  $O$  of inversion is a circle through the center  $O$ .

### Group "A" 20

Attempt all the questions. Tick (✓) the best answers.

1. Which one is not an axiom of Fanon's Geometry?
  - a. There exist at least one line
  - b. There are exactly three points on every line
  - c. There are exactly three lines on every point
  - d. Not all points are on the same line
2. Which of the following is false?
  - a. Two lines may or may not intersect
  - b. Two distinct lines never intersect
  - c. Two distinct lines may intersect at one point

- d. Two distinct lines may not intersect
3. 'In two angles of a triangle are congruent, then the sides opposite those angles are congruent.' It is a statement of  
 a. converse of isosceles triangle theorem  
 b. inverse of isosceles triangle theorem  
 c. isosceles triangle theorem  
 d. Hinge theorem
4. Which one of the following statements is false?  
 a. The perpendicular bisector of a chord of a circle contains a diameter of the circle.  
 b. A secant to a circle is a line that contains exactly two points of the circle.  
 c. Diameter bisects every chord of the circle  
 d. A tangent to a circle is a line that contains exactly one points of the circle.
5. The reflection image of the point  $(-3, 4)$  on the line  $y = -x$  is  
 a.  $(3, -4)$       b.  $(-4, -3)$       c.  $(-4, 3)$       d.  $(-3, -4)$
6. Which of the following is not an isometric transformation?  
 a. Reflection      b. Glide reflection      c. Translation      d. Dilation
7. 'Similar triangles are congruent' is a fact of  
 a. Euclidean geometry      b. Hyperbolic geometry  
 c. Elliptic geometry      d. Projective geometry
8. Which of the following is the dual of the projective axiom 'there exists at least one line'  
 a. There exists at least one point  
 b. Each point is contained by at least three lines  
 c. Two distinct lines determine a unique point  
 d. Two distinct points determine a unique line
9. Topologically equivalent surfaces are called  
 a. Homomorphic      b. Orientable  
 c. Dual of each other      d. Unicursal
10. What does the equation  $x^2 + y^2 - 2x = 0$  become when it is transformed to parallel axes through the point  $(1, -1)$ ?  
 a)  $x^2 + y^2 - y = 0$       b)  $x^2 + y^2 - 2x = 0$   
 c)  $x^2 + y^2 - 2y = 0$       c)  $2x^2 + 2y^2 - y = 0$
11. What is the length of the latus rectum of the parabola  $y^2 = 4ax$ ?  
 a)  $\frac{a}{2}$       b) a      c) 2a      d) 4a
12. What are the coordinates of the foci of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $b > a > 0$ )?  
 a)  $(a, \pm b)$       b)  $(0, \pm be)$       c)  $(\pm a, 0)$       d)  $(\pm ae, 0)$
13. The director circle of the hyperbola  $\frac{x^2}{9} - \frac{y^2}{4} = 1$  is given by  
 a)  $x^2 + y^2 = 5$       b)  $x^2 + y^2 = -5$   
 c)  $x^2 + y^2 = 13$       d)  $x^2 + y^2 = 1$
14. The general equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents ellipse if  
 .....  
 a)  $h^2 = ab$       b)  $h^2 < ab$       c)  $h^2 \leq ab$       d)  $h^2 > ab$
15. The Cartesian coordinates of the cylindrical coordinates  $1, \frac{\pi}{2}, 1$  are given by  
 a)  $(1, 0, 1)$       b)  $(1, 1, 0)$       c)  $0, 1, 1)$       d)  $(1, 1, 1)$
16. What is the length of the perpendicular from the point  $(1, 2, 3)$  on the plane  $x + y + z - 3 = 0$ ?  
 a)  $3\sqrt{3}$       b) 3      c) 9      d)  $\sqrt{3}$
17. The straight line  $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$  lies in the plane  $ax + by + cz + d = 0$  if  
 a)  $al + bm + zn = 0$       b)  $l^2 + m^2 + n^2 = p$   
 c)  $lx_1 + my_1 + nz_1 = 0$       d)  $\frac{x_1}{l} = \frac{y_1}{m} = \frac{z_1}{n}$

18. The plane  $lx + my + nz = p$  may touch the sphere  $x^2 + y^2 + z^2 = r^2$  if  
 a)  $l^2 + m^2 + n^2 = p^2$  b)  $r^2(l^2 + m^2 + n^2) = p^2$   
 c)  $r(l^2 + m^2 + n^2) = p^2$  d)  $r^2(l^2 + m^2 + n^2) = p$
19. What is the equation of a cone with vertex at  $(0, 0, 0)$  and passing through the curve  $ax^2 + by^2 + cz^2 = 1$ ,  $lx + my + nz = p$ ?  
 a)  $p(ax^2 + by^2 + cz^2) = (lx + my + nz)^2$   
 b)  $p^2(ax^2 + by^2 + cz^2) = (lx + my + nz)$   
 c)  $p^2(ax^2 + by^2 + cz^2) = (lx + my + nz)^2$   
 d)  $(ax^2 + by^2 + cz^2) = p^2(lx + my + nz)^2$
20. What is the equation of the tangent plane at  $(\alpha, \beta, \gamma)$  to the conicoid  $ax^2 + by^2 + cz^2 = 1$ ?  
 a)  $ax\alpha + b\beta y + c\gamma z = 0$  b)  $ax\alpha + b\beta y + c\gamma z + 1 = 0$   
 c)  $\frac{\alpha x}{a} + \frac{\beta y}{b} + \frac{\gamma z}{c} = 1$  d)  $ax\alpha + b\beta y + c\gamma z = 1$

### EXAM 2068

Attempt ALL the questions.

#### Group 'B' [8×7=56]

1. Prove that the each point of Fano's geometry has exactly three lines on each point.
2. Given any APQR and any line segment  $\overline{AB}$ , there exists a  $\triangle ABC$  having  $\overline{AB}$  as one of its sides such that  $\triangle ABC$  is similar to  $\triangle PQR$  prove it.

OR

Prove that a point is on the bisector of an angle if and only if it is equidistant from the sides of the angle.

3. State hyperbolic parallel postulate and prove that the summit angles of a Saccheri quadrilateral are equal and acute.
4. In the real projective plane, show that every two points A and B determine a unique line.

OR

What are Euler's discoveries about Network? Also prove that the numbers of odd vertices in a network is always even.

5. Find the equation of the plane passing through the points  $(1, 1, 1)$ ,  $(3, -1, 2)$  and  $(-3, 5, -4)$ .
6. Find the angle between the lines  $x - 2y + z = 0 = x + y - z$  and  $x + 2y + z = 0 = 8x + 12y + 5z$ .
7. Find the equation of reciprocal cone of the cone  $ax^2 + by^2 + cz^2 = 0$ .

OR

Show that the plane  $x + 2y + 3z = 2$  touches the conicoid.

$x^2 - 2y^2 + 3z^2 = 2$ . Find also the point of contact.

8. Prove that the locus of the middle points of the chords of the parabola  $y^2 = 4ax$  which passes through the focus is the parabola  $y^2 = 2a(x+a)$ .

9. a) Find the latus rectum, eccentricity, foci and equation of directrices of ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$

- b) Show that the line  $x \cos \alpha + y \sin \alpha = p$  will be a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  if  $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$ .

OR

Find the condition that the general equation of second degree  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represent different conic sections. Prove that the conic represented by the equation  $5x^2 - 2xy + 5y^2 + 2x - 10y = 0$  is an ellipse.

10. a) State the isosceles triangle theorem and prove this theorem by the method of transformation.
- b) Explain the inverse transformation of a point. Find the inverse image of the point  $(6, 10)$  with respect to  $x^2 + y^2 = 36$



Group 'A' [20]

Attempt ALL the questions. Tick the best answers.

- Which of the following is true?
  - Fano's geometry contains exactly six lines.
  - Each point of the four-pointed geometry has exactly three lines on it.
  - Four pointed geometry has exactly six lines
  - All of the above
- The sum of the angles of the triangle in elliptic geometry is
  - equal to  $180^\circ$ .
  - less than  $180^\circ$
  - more than  $180^\circ$
  - All of the above
- Which of the following statement is equivalent to the Euclidean parallel postulate?
  - If a line intersects one of two parallel lines then it intersects the other
  - If a line is perpendicular to one of two parallel lines then it is perpendicular to the other
  - the opposite sides of the parallelogram are equal
  - All of the above
- Which of the following is not a fact of Euclidean geometry?
  - Two chords of circle are equidistant from its center.
  - an angle inscribed in semi-circle is a right angle
  - Angles inscribed in the congruent arcs are congruent
  - Minor arcs of congruent chords of a circle are equal
- Which of the following is false?
  - Dilation preserves angle measure
  - Dilation preserves angle congruence of triangle
  - Dilation preserves collinearity
  - Dilation preserves betweenness
- The inverse image of the points  $(2, -5)$  with respect to the circle  $x^2 + y^2 = 1$  is
  - $\left(\frac{-2}{29} \frac{5}{29}\right)$
  - $\left(\frac{29}{2} \frac{29}{5}\right)$
  - $\left(\frac{2}{29} \frac{-5}{29}\right)$
  - $\left(\frac{29}{2} \frac{-29}{5}\right)$
- Two distinct lines perpendicular to the same line intersect in
  - Euclidean geometry.
  - Hyperbolic geometry
  - Elliptic geometry
  - Both (a) and (c)
- Which of the followings is the axioms of real projective plane?
  - There exists at least one line
  - Each line contains at least three points
  - Two distinct lines determine a unique point
  - All of the above
- What is the genus of a surface with Euler characteristics of surface 10?
  - 5
  - 6
  - 7
  - 4
- What does the equation  $x^2 - 2y^2 - 2x + 4y = 0$  become when it is transformed to the parallel axes through the point  $(1, -1)$ ?
  - $x^2 - 2y^2 + 4 - 7 = 0$
  - $x^2 - 2y^2 + 8x - 7 = 0$
  - $2x^2 - 2y^2 + 8y - 7 = 0$
  - $x^2 - 2y^2 + 8y - 9 = 0$
- What is the directrix of the parabola  $y^2 = 4ax$ ?
  - $y + a = 0$
  - $y - a = 0$
  - $x + a = 0$
  - $x - a = 0$
- What are the coordinates of the foci of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b > 0)$ ?
  - $(0, 0)$
  - $(\pm a, 0)$
  - $(\pm ae, e)$
  - $(\pm ae, 0)$
- The length of the latus rectum of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 
  - $\frac{2b}{a}$
  - $\frac{2b^2}{a}$
  - $\frac{2a}{b}$
  - $\frac{2a^2}{b}$

14. The general equation of  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents a parabola if  
 a)  $h^2 = ab$  b)  $h^2 < ab$   
 c)  $h^2 \leq b$  d)  $h^2 > ab$
15. Which of the following coordinates give the Cartesian coordinates of the spherical coordinates  $(4, \pi/2, \pi/3)$ ?  
 a)  $(\sqrt{2}, 6, 2\sqrt{2})$  b)  $(\sqrt{2}, \sqrt{6}, \sqrt{2})$   
 c)  $(\sqrt{2}, \sqrt{6}, 2\sqrt{2})$  d)  $(2, \sqrt{6}, \sqrt{2})$
16. The angle between the pairs of planes  $2x - y + z = 6$  and  $x + y + 2z = 7$   
 a)  $\pi/3$  b)  $\pi/4$  c)  $\pi/2$  d)  $\pi$
17. What is the point of intersection of the line  $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{-2}$  and the plane  $3x + 4y + 3z = 5$ ?  
 a)  $(-1, -3, -1)$  b)  $(-1, 3, -1)$   
 c)  $(1, -3, 1)$  d)  $(1, 3, -1)$
18. What is the radius of the sphere represented by the equation  $x^2 + y^2 + z^2 + 6x - 8y + 4z = 0$   
 a) 5 b) 6 c) 8 d) 4
19. What is the condition that the cone  
 a)  $a + b + c = 0$  b)  $a = b = c$   
 c)  $u^2 + v^2 + w^2 = 0$  d)  $u^2 + v^2 + w^2 \neq 0$
20. Which of the following represents the equation of the elliptic paraboloid?  
 a)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{2z}{c}$  b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{2z}{c}$   
 c)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  d)  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

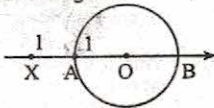
Exam 2069

Group 'A'

[20

Attempt ALL the questions. Tick the best answers.

1. Which of the following is not an incidence axiom?  
 a) For every line there exists at least two distinct points on it.  
 b) There exists at least one line  
 c) There exists at least three distinct points.  
 d) Not all points lie on the same line
2. Which of the following is not the Hilbert's postulate of connection?  
 a) Two distinct points determine one and only one straight line  
 b) through any two distinct points there passed not more than one line  
 c) on every line there exist at least two distinct points  
 d) every line contains an infinite number of points.
3. Which of the following statement is hyperbolic geometry is true?  
 a) The summit and the base of Saccheri quadrilateral are parallel  
 b) The diagonals of Saccheri quadrilateral are not congruent  
 c) Saccheri quadrilateral has three of its angles right angles.  
 d) The summit angles of a Saccheri quadrilateral are unequal
4. Summit angles of Saccheri quadrilateral in elliptic geometry are  
 a) congruent and acute b) congruent but not acute  
 c) acute but not congruent d) neither congruent nor acute
5. Which of the following is the power of the point X as shown in the figure?  
 a) 1 b) 2  
 c) 3 d) 4
6. Which of the following statement is false?  
 a) A reflection is an isometry  
 b) A translation is an isometry  
 c) Collinearity is invariant under an isometry  
 d) Betweenness is not invariant under an isometry
7. The inverse image of a point  $(3, 4)$  with respect to the circle  $x^2 + y^2 = 16$  is



a)  $\left(\frac{48}{25}, \frac{64}{25}\right)$

b)  $\left(\frac{48}{5}, \frac{64}{5}\right)$

c)  $\left(\frac{16}{3}, \frac{16}{4}\right)$

d)  $\left(\frac{25}{48}, \frac{25}{64}\right)$

8. A complete quadrilateral has  
 a) four vertices and six sides  
 c) four sides and six vertices  
 b) four vertices and four sides  
 d) six sides and six vertices
9. What is the dual of a cube?  
 a) Tetrahedron  
 c) Isosahedron  
 b) Dodecahedron  
 d) Octahedron
10. What does the equation  $2x^2 + y^2 - 4x + 4y = 0$  become when it is transformed to parallel axes through the points  $(1, -2)$ ?  
 a)  $2x^2 + y^2 - 6 = 0$   
 c)  $2x^2 + 2y^2 = 3$   
 b)  $x^2 + 2y^2 - 6 = 0$   
 d)  $x^2 + y^2 = 3$
11. What is the equation of the directrix of the parabola  
 a)  $x + a = 0$   
 c)  $x - a = 0$   
 b)  $y + a = 0$   
 d)  $y - a = 0$
12. What is the equation of the polar of the points  $P(x_1, y_1)$  with respect to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ?  
 a)  $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$   
 c)  $xx_1 + yy_1 = 1$   
 b)  $\frac{xx_1}{a} + \frac{yy_1}{b} = 1$   
 d)  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$
13. What is the condition that the line  $y = mx + c$  touches the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ?  
 a)  $C = \sqrt{b^2 - a^2m^2}$   
 c)  $C = \pm \sqrt{a^2m^2 - b^2}$   
 b)  $C = b^2 - a^2m^2$   
 d)  $C = \pm (a^2m^2 - b^2)$
14. The general equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents hyperbola if  
 a)  $h^2 = ab$   
 c)  $h^2 < ab$   
 b)  $h^2 > ab$   
 d)  $h^2 \geq ab$
15. The cylindrical coordinates of  $(a, 1, 1)$  are given by  
 a)  $(1, \frac{\pi}{2}, 1)$   
 c)  $(1, \pi, 1)$   
 b)  $(0, \frac{\pi}{2}, 1)$   
 d)  $(0, \pi, 1)$
16. two planes  $a_1x + b_1y + c_1z + d_1 = 0$  and  $a_2x + b_2y + c_2z + d_2 = 0$  are perpendicular if  
 a)  $a_1a_2 + b_1b_2 + c_1c_2 = 1$   
 c)  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$   
 b)  $a_1a_2 + b_1b_2 + c_1c_2 = 0$   
 d)  $a_1a_2 + b_1b_2 + c_1c_2 + d_1d_2 = 0$
17. The angles between the line  $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$  and the plane  $ax + by + cz = 0$  is given by  
 a)  $\sin \theta = \frac{a\ell + bm + cn}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma \ell^2}}$   
 c)  $\sin \theta = \frac{a\ell + bm + cn}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma \ell^2}}$   
 b)  $\cos \theta = \frac{a\ell + bm + cn}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma \ell^2}}$   
 d)  $\cos \theta = \frac{a\ell + bm + cn}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma \ell^2}}$
18. What is the center of the sphere represented by  $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ ?  
 a)  $(-1, -2, 3)$   
 c)  $(-1, 2, -3)$   
 b)  $(1, 2, 3)$   
 d)  $(1, -2, 3)$
19. If Z axis be the axis of the right circular cone, then the equation of the cone is given by  
 a)  $z^2 + x^2 = y^2 \tan^2 \theta$   
 c)  $x^2 + y^2 = z^2 \tan^2 \theta$   
 b)  $y^2 + z^2 = x^2 \tan^2 \theta$   
 d)  $(x^2 + y^2) \tan^2 \theta = z^2$



20. The equation of an imaginary ellipsoid is given by

a)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$

b)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

c)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$

d)  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1 = 0$

Attempt ALL the questions.

**Group 'B'**

[8×7=56]

1. Given an example of an abstract axiomatic system. Give a model for your example.
2. Use Hilbert's axioms to prove that the base of an isosceles triangle are congruent.

OR

Write equivalent statement for Euclidean parallel postulate and prove that they are equivalent.

3. Explain the defect of a triangle. Also show that two triangles are equivalent if they have the same defect.
4. If two coplanar triangles are point perspective with the same non-coplanar triangle, prove that they are line perspective with each other.

OR

Explain the traversability of a network. Prove that a connected network with two odd vertices can be travelled in a single path whose initial and the terminal vertices are the two odd vertices of the network.

5. Find the equation of the plane passing through the points (2, 2, 1) and (9, 3, 6) which is perpendicular to the plane  $2x + 6y + 6z = 9$ .
6. Find the point where the line joining (2, -3, 1) and (3, -4, -5) cuts the plane  $2x + y + z = 7$ .
7. Show that the plane  $x + 2y + 3z = 2$  touches the conicoid  $x^2 - 2y^2 + 3z^2 = 2$ . Find also the point of contact.

OR

Find the equation of the enveloping cone of the sphere  $x^2 = y^2 + z^2 - 2x + 4z = 1$  with vertex at (1, 1, 1).

8. Find the equation of the common tangent to the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ .
- Group 'C'** [2×12=24]
9. a) The foci of a hyperbola coincides with the foci of the ellipse  $16x^2 + 25y^2 = 400$ . Find the equation of the hyperbola if its eccentricity is 3.

9. b) Prove that the equation of the tangents to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  which makes equal intercepts on the coordinate axes are  $x + y \pm \sqrt{a^2 + b^2}$ .

OR

Prove that the general equation  $ax^2 + 2hxy + by^2 + 5gx + 2fy + c = 0$  always represents a conic section.

10. a) Define translation and prove that it is an isometry.
10. b) Define an isometric transformation. If  $\alpha$  is an isometric transformation on the plane  $\pi$  and  $\ell$  is a line on  $\pi$ . If  $\alpha(A) = A$  and  $\alpha(B) = B$ , where  $a$  and  $B$  are points of  $\ell$ , then show that  $\alpha(C) = C$  for all points of  $\ell$ .

**Exam 2070**

Time: 3 hrs.

Full Marks: 100

Attempt all the questions.

**Group "B"**

[8×7=56]

1. What are the axioms for incidence geometry? Prove that for each point there exist at least two lines containing it.
2. If a line parallel to one side of a triangle intersects the other two sides in two different points, prove that this line divides the two sides into segments that are proportional.
3. Define Saccheri quadrilateral. Prove that the summit angles of a Saccheri quadrilateral, in elliptic geometry, are congruent and obtuse.

OR

If  $\Delta ABC$  is partitioned into a pair of component triangles by a Cevian line, the defect of  $\Delta ABC$  is equal to the sum of the defects of the component triangles, prove it.

- Define a network. Prove that the total number of odd vertices of any network is always even.
- Find the equation of the plane through the point  $(\alpha, \beta, \gamma)$  which is parallel to the plane  $ax + by + cz + d = 0$ .
- Find the distance of the point  $(3, -4, 5)$  from the plane  $2x + 5y - 6z = 16$  measured along a line with direction cosines proportional to  $2, -1, -2$

OR

Find the shortest distance between the lines  $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$  and  $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$

- Find the equation of a sphere which has its centre at origin and touches the line  $2(x+1) = 2 - y = z + 3$ .

OR

Show that the plane  $z=0$  cuts the enveloping cone of the sphere  $x^2 + y^2 + z^2 = 26$  which has its vertex at  $(5, 3, 3)$  is a rectangular hyperbola.

- Find the locus of the point from which there mutually perpendicular tangent lines can be drawn to the paraboloid  $ax^2 + by^2 = 2z$ .

Group "C" [2×12=24]

- (a) Prove that a homothety maps triangles onto similar triangles.
- (b) If  $I_{o,r}$  is an inversion and  $\lambda$  is any line that does not contain  $O$ , then prove that the image of  $\lambda$  under  $I_{o,r}$  is a circle that contains  $O$ .
- (a) Show that the straight line  $x \cos \alpha + y \sin \alpha = p$  may be a tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  if  $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$ .
- (b) Show that the feet of the normal that can be drawn from the point  $(h, k)$  to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  lie on the curve  $b^2 x(k-y) + a^2 y(x-h) = 0$ .

OR

Prove that the equation  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$  represents a conic section.

Group "A" [20]

Attempt all the questions. Tick() the best answers.

- Not all points lie on the same line' is an axiom of
  - Four-point geometry
  - Incidence geometry
  - Young's geometry
  - Fano's geometry
- Which of the following is not the Hilbert's postulate of connection?
  - two distinct points determine one and only one line
  - on every line, there exist at least two distinct points
  - through any three points not on the same line, there is one and only one plane
  - every line contains an infinite number of points
- The intersection point of the medians of a triangle is called
  - centroid
  - excentre
  - incentre
  - orthocentre
- What is the reflection image of the point  $(-3, 4)$  on the line  $y = -x$ ?
  - $(4, -3)$
  - $(-3, 4)$
  - $(-4, 3)$
  - $(-3, -4)$
- Which of the following is not an isometric transformation?
  - Reflection
  - Rotation
  - Translation
  - Homothety
- Two distinct lines intersect at two points' is a fact of
  - Euclidean geometry
  - hyperbolic geometry
  - elliptic geometry
  - projective geometry
- The fourth angle of Lambert quadrilateral is acute in
  - hyperbolic geometry
  - elliptic geometry
  - Euclidean geometry
  - both (a) and (b)



8. If the co-ordinates of a point in Cartesian co-ordinates is (2,3, 4) then the value of r is cylindrical co-ordinate is  
 (a)  $\sqrt{11}$  (b)  $\sqrt{12}$   
 (c)  $\sqrt{13}$  (d)  $\sqrt{14}$
9. If a surface of genus P is partitioned into F regions by V vertices joined E arcs, then which of the following is true?  
 (a)  $V-E+F=2$  (b)  $V-E+F=2P$   
 (c)  $V-E+F=2P-2$  (d)  $V-E+F=2-2P$
10. What are the coordinates of the foci of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ( $a > b > 0$ )?  
 (a) (0,0) (b)  $(\pm a, 0)$   
 (c)  $(\pm ae, a)$  (d)  $(\pm ae, 0)$
11. What is the equation of the tangent at  $(x_1, y_1)$  to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ?  
 (a)  $xx_1 - yy_1 = a^2 b^2$  (b)  $xx_1 + yy_1 = a^2 b^2$   
 (c)  $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$  (d)  $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$
12. Two diameters  $y = m_1 x$  and  $y = m_2 x$  of a hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  are conjugate to each other if  
 (a)  $m_1 m_2 = b^2 a^2$  (b)  $m_1 m_2 = \frac{b^2}{a^2}$   
 (c)  $m_1 m_2 = b^2 + a^2$  (d)  $m_1 + m_2 = \frac{b^2}{a^2}$
13. The line  $y = mx + c$  touches the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  if  
 (a)  $a^2 m^2 + b^2 = c^2$  (b)  $b^2 m^2 + a^2 = c^2$   
 (c)  $\sqrt{a^2 m^2 + b^2} = c$  (d)  $\sqrt{b^2 m^2 + a^2} = c$
14. What does the equation  $12x^2 - 23xy + 10y^2 - 25x + 26y - 14 = 0$  represent?  
 (a) Parabola (b) Hyperbola  
 (c) Ellipse (d) Circle
15. Two planes  $a_1 x + b_1 y + c_1 z + d_1 = 0$  and  $a_2 x + b_2 y + c_2 z + d_2 = 0$  are perpendicular if  
 (a)  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 1$  (b)  $a_1 a_2 + b_1 b_2 + c_1 c_2 + d_1 d_2 = 0$   
 (c)  $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$  (d)  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
16. The straight line  $\frac{x-x_1}{1} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$   
 (a)  $ax + by + cz + d = 0$  if (b)  $e + m + n = a + b + c$   
 (c)  $ae^2 + bm^2 + cn^2 = 0$  (d)  $e + m + n = 0$
17. What is the length of the perpendicular from the point (1, 2, 3) on the plane  $x + y + z - 3 = 0$ ?  
 (a)  $3\sqrt{3}$  (b)  $\sqrt{3}$   
 (c) 3 (d) 9
18. The sphere represented by  $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$   
 (a)  $u^2 + v^2 + w^2 - d < 0$  (b)  $u^2 + v^2 + w^2 + d = 0$   
 (c)  $u^2 + v^2 + w^2 - d < 0$  (d)  $u^2 + v^2 + w^2 - d > 0$
19. What is the condition that the cone  $ax^2 + by^2 + cz^2 + wfyz + 2gzx + 2hxy = 0$  has three mutually perpendicular generators?  
 (a)  $a + b + c = 0$  (b)  $u^2 + v^2 + w^2 = 0$   
 (c)  $a + b + c = u^2 + v^2 + w^2$  (d)  $au + bv + cw = 0$
20. Any plane which bisects a system of parallel chords is called?  
 (a) principal plane (b) diametral plane  
 (c) conjugate diametral plane (d) tangent plane

Attempt the all questions.

1. Define ellipse and find its standard equation.

OR

Show that the line  $lx + my + n = 0$  may be tangent to the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  if  $l^2a^2 - m^2b^2 = n^2$ .

2. Prove that the lines  $x = \frac{y-2}{2} = \frac{z+3}{3}$  and  $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$  are coplanar. Also find the equation of the plane in which they lie.
3. Find the equation of the sphere which touches the sphere  $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$  at  $(1, 2, -2)$  and passes through origin.

OR

If  $x = \frac{y}{2} = \frac{z}{3}$  represents one of the sets of three mutually perpendicular generators of the cone  $5yz - 8zx - 3xy = 0$ , find the equations to the other two.

4. Find the equations to the tangent planes to the conicoid  $7x^2 - 3y^2 - z^2 + 21 = 0$  which pass through the line  $7x - 6y + 9 = 0, z = 3$ .
5. The sum of the measures of any two sides of a triangle is greater than the measure of the third side. Prove it.
6. A point is on the bisector of an angle if and only if it is equidistance from the sides of the angle. prove it.
7. Define homothety and prove that betweenness is invariant under a homothety.

OR

Prove that for any polyhedron,  $V - E + F = 2$ , where  $V, E$  &  $F$  are the number of vertices, edges and faces respectively.

8. In a projective plane, every two distinct points  $A$  and  $B$  determine a unique line. Prove it.

Group "C"

2×12=24

9. Prove that the equation  $ax^2 + 2hyx + by^2 + 2gx + 2fy + c = 0$  always represents a conic section.
10. (a) Prove that parallel line are not everywhere equidistant.  
(b) Define defect of triangle. Prove that defect of every triangle is positive.

OR

10. (a) Prove that the Euclidean parallel postulate is equivalent to the statement: The angle sum of a triangle is  $180^\circ$ .  
(b) Prove that the three medians of a triangle are concurrent at a point.

Group "A"

20

Attempt ALL the questions. Tick (✓) the best answers.

1. Which of the following is not an axiom of four – point geometry?  
a. there exist exactly four points  
b. any two distinct points have exactly one line on both of them  
c. each line is on exactly two points  
d. not all points are on the same line
2. Which of the following is not the substitute of the Euclid's fifth postulate?  
a. two distinct points determine one and only one line  
b. two parallel lines are equidistant  
c. a circle exist through any three non-collinear points  
d. there exist two similar and non-congruent triangles
3. The quadrilateral having a pair of congruent opposite sides both perpendiculars to the third side is called  
a. a square  
b. a Lambert quadrilateral  
c. a Saccheri quadrilateral  
d. rectangle
4. If two angles of a triangle are congruent then the sides opposite the congruent angles are congruent, is called the

- a. Isosceles triangle theorem  
 b. Hinge theorem  
 c. Inverse of isosceles triangle theorem  
 d. Converse of isosceles triangle theorem
5. If an isometric transformation fixes, three non-collinear points then it is  
 a. rotation  
 b. identity transformation  
 c. reflection  
 d. translation
6. Which of the following is false?  
 a. Homothety preserves angle measure  
 b. Homothety preserves congruence of triangle  
 c. Homothety preserves collinearity  
 d. Homothety preserves betweenness
7. The difference between two right angles and the angle sum of a triangle is called  
 a. defect of the triangle  
 b. area of the triangle  
 c. perimeter of the triangle  
 d. altitude of the triangle
8. Which of the following is the dual element of a line?  
 a. space  
 b. plane  
 c. line  
 d. point
9. If a network has 4 vertices and 7 arcs, then how many inner regions does it have?  
 a. 2  
 b. 3  
 c. 4  
 d. 5
10. If by any change of axes, without change of origin, the expression  $ax^2 + 2hxy + by^2$  becomes  $AX^2 + 2HXY + BY^2$  then which of the following is true?  
 a.  $a + b = A + B$   
 b.  $ab - h^2 = AB - H^2$   
 c.  $AB = a + b$   
 d. both a and b
11. A conic section with eccentricity 0 (zero) is  
 a. a circle  
 b. a hyperbola  
 c. an ellipse  
 d. a parabola
12. The auxiliary circle of the ellipse  $9x^2 + 4y^2 = 36$  is  
 a.  $x^2 + y^2 = 36$   
 b.  $x^2 + y^2 = 9$   
 c.  $x^2 + y^2 = 4$   
 d.  $x^2 + y^2 = 6$
13. Which of the following is the equation to the asymptotes of the hyperbola  $9x^2 - 16y^2 = 144$ ?  
 a.  $4y = 3x$   
 b.  $4y = -3x$   
 c. both a and b  
 d.  $3y = \pm 4x$
14. The cylindrical co-ordinates of the Cartesian co-ordinates (1, 1, 3) are  
 a.  $(r_2, \frac{\pi}{4}, 3)$   
 b.  $(\sqrt{11}, \frac{\pi}{4}, 3)$   
 c.  $(\sqrt{11}, \frac{\pi}{4}, \frac{\pi}{2})$   
 d.  $(1, \frac{\pi}{4}, 3)$
15. The length of perpendicular from (0, 0, 0) to the plane  $2x - 3y + 6z = 14$  is  
 a. 0  
 b. 1  
 c. 2  
 d. 3
16. The value of K for lines  $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$  and  $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$  to be perpendicular to each other is  
 a.  $-\frac{1}{3}$   
 b.  $-\frac{1}{2}$   
 c.  $\frac{10}{7}$   
 d.  $-\frac{10}{7}$
17. The equation of tangent plane to the sphere  $x^2 + y^2 + z^2 = 9$  at (1, -2, 2) is  
 a.  $x - 2y + 2z = 9$   
 b.  $x + 2y + 2z = 9$   
 c.  $x - 2y + 2z = 3$   
 d.  $x + 2y + 2z = 3$
18. The condition for the cone  $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$  to have three mutually perpendicular generators is  
 a.  $f + g + h = 0$   
 b.  $a + b + c = 0$   
 c.  $f^2 + g^2 + h^2 = 0$   
 d.  $a^2 + b^2 + c^2 = 0$
19. What does the equation  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  represent?  
 a. ellipsoid  
 b. hyperboloid  
 c. imaginary ellipsoid  
 d. hyperboloid of one sheet
20. Equation to the polar plane of the point (2, -3, 4) with respect to the conicoid  $x^2 + 2y^2 + z^2 = 4$  is  
 a.  $2x - 3y + 4z = 0$   
 b.  $2x - 3y + 4z = 2$   
 c.  $x - 6y + 4z = 2$   
 d.  $x - 6y + 4z = 4$



Time: 3 hrs.

Attempt All the questions.

[8×7=56]

## Group "B"

- Define hyperbola and find its equation in standard form.
- Find the equation of the director circle to the hyperbola  $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$ .
- Find the distance of the point (3, -4, 5) from the plane  $2x + 5y - 6z = 16$  measured along the line with direction cosines proportional to (2, -1, -2)

OR

Find the length of shortest distance and the equation of shortest distance between

the lines  $\frac{x}{2} = \frac{y}{-3} = z$  and  $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ 

- Prove that the equation  $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$  represents a cone if  $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$ .

OR

Define central conicoid and find the condition that the plane  $ex + my + nz = p$  should touch the conicoid  $ax^2 + by^2 + cz^2 = 1$ .

- A point is on the perpendicular bisector of a line segment if and only if it is equidistant from the end points of the line segment. Prove this.

OR

Describe why mathematician questioned about Euclid's 5<sup>th</sup> postulate. State the substitutes of 5<sup>th</sup> postulate.

- Prove that medians of a triangle are concurrent at a point.
- Prove, by using transformation, the diagonals of a square are congruent.
- In a projective plane, prove that every two distinct points determine a unique line.

## Group "C"

[2×12=24]

- Prove that the general equation of second degree in x and y always represents conic section.
- (a) Define equivalent polygons. If two triangles are equivalent, then prove that they have the same defect.  
(b) In elliptic geometry, prove that the summit angles of a Saccheri quadrilateral are congruent and obtuse.

OR

- Prove that every triangle has a positive defect.
- In any network the total number of odd vertices is always even. Prove this.

## Group "A"

20

Attempt All the questions. Tick (✓) the best answers.

- The axiom: "There exist at least one point on any two distinct lines, "belongs to
  - Fano's geometry
  - Four point geometry
  - Young's geometry
  - Incidence geometry
- Which of the following is true?
  - There exist exactly four points in Fano's geometry
  - There are exactly three points on every line in Fano's geometry
  - Not all points lie on the same line in Young's geometry
  - both a and c
- Which of the following is not the property of axiomatic system?
  - completeness
  - independence
  - approximate
  - consistency
- "Through a point not on a given line there is exactly one line parallel to the given line." is an axiom of
  - Euclidean geometry
  - Elliptic geometry
  - Hyperbolic geometry
  - Topology
- Which of the following is false?
  - two lines may or may not intersect
  - two lines may intersect at a point

- c. two distinct lines never intersect  
 c. two distinct lines may intersect at a point
6. The angle sum of a triangle in hyperbolic geometry is  
 a.  $180^\circ$       b. less than  $180^\circ$       c. More than  $180^\circ$       d.  $270^\circ$
7. "Two distinct lines intersect at two points" is true in  
 a. Elliptic geometry      b. Hyperbolic geometry  
 c. Topology      d. Euclidean geometry
8. What is the image  $A'$  of the point  $A(3, 4)$  under the translation  $\text{top}(A) = A'$ , where  $O = (0, 0)$  and  $P = (2, -1)$ ?  
 a.  $(-1, 5)$       b.  $(5, 3)$       c.  $(-1, -5)$       d.  $(6, 4)$
9. The inverse of the point  $(3, 4)$  with respect to the circle  $x^2 + y^2 = 1$  is  
 a.  $(3, 4)$       b.  $(\frac{3}{4}, \frac{4}{5})$       c.  $(\frac{9}{25}, \frac{16}{25})$       d.  $(\frac{3}{25}, \frac{4}{25})$
10. Which of the following is invariant under projective transformation?  
 a. collinearity      b. perpendicularity  
 c. similarity      d. congruence
11. What does the equation  $x^2 + y^2 = 25$  become under the translation of origin to  $(1, 2)$ ?  
 a.  $x^2 + y^2 + 2x + 4y = 25$       b.  $x^2 + y^2 + 2x + 4y = 20$   
 c.  $x^2 + y^2 = 5$       d.  $x^2 + y^2 = 25$
12. What is the length of latus rectum of the ellipse  $9x^2 + 4y^2 = 36$ ?  
 a.  $18/4$       b.  $18/2$       c.  $8/3$       d.  $8/9$
13. The position of the point  $(3, 4)$  on the plane of the ellipse  $4x^2 + 2y^2 = 4$  is  
 a. outside the ellipse      b. inside the ellipse  
 c. on the ellipse      d. both b and c
14. If  $y = m_1x$  and  $y = m_2x$  be two conjugate diameters of the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$  then which of the following is true?  
 a.  $m_1 = m_2$       b.  $\frac{m_1}{m_2} = \frac{b^2}{a^2}$   
 c.  $m_1 \cdot m_2 = \frac{a^2}{b^2}$       d.  $m_1 \cdot m_2 = \frac{b^2}{a^2}$
15. What does the equation  $5x^2 - 2xy + 5y^2 - 12 = 0$  represent?  
 a. an ellipse      b. a hyperbola  
 c. a circle      d. a parabola
16. If  $\alpha, \beta, \gamma$  be the angles made by a straight line with  $x, y$  and  $z$  - axis respectively, then what is the value of  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$ ?  
 a. 0      b. 1      c. 2      d. 3
17. The Cartesian co-ordinates of the point whose spherical co-ordinates are  $(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2})$  is  
 a.  $(1, 1, 1)$       b.  $(1, 0, 1)$       c.  $(0, 1, 1)$       d.  $(1, \frac{\pi}{2}, 1)$
18. Which of the following is the length of perpendicular from  $(1, 1, 2)$  to the plane  $2x - y + 21 = 6z$ ?  
 a.  $6/9$       b.  $6/5$       c.  $-1/9$       d.  $1/3$
19. The angle between the line  $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{11}$  and the plane  $2x - 3y + z = 3$  is  
 a.  $90^\circ$       b.  $0^\circ$       c.  $60^\circ$       d.  $30^\circ$
20. The radius of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 6z + 3 = 0$  is  
 a.  $\sqrt{17}$       b.  $\sqrt{3}$       c.  $\sqrt{-1}$       d.  $\sqrt{11}$

Time: 3 hrs

Full Marks: 100

Attempt ALL the questions.

Group 'B'

(8×7=56)

1. What is completeness axiom? Prove that the set  $Q$  of rational numbers is not complete.
2. Define open set. Prove that
  - a) every open interval is an open set and
  - b) every open set is a union of open interval

OR

Define compact set and prove every closed and bounded subset of real number is compact.

3. Define the limit of a sequence and prove that every bounded sequence has at least one limit point. Also find the limit point of the sequence  $(1 + (-1)^n)$ .
4. A sequence  $\langle u_n \rangle$  defined by  $u_n = a^n$  is convergent if  $-1 < a \leq 1$ ; prove it.

OR

Define subsequence of a sequence and prove that every sequence has a monotonic subsequence.

5. If  $\lim_{x \rightarrow a} g(x) = m$ , prove that  $\lim_{x \rightarrow a} |g(x)| = |m|$ .

Show that by suitable example that the converse is not true.

6. A function defined on a set  $\mathfrak{R}$  of real numbers is continuous on  $\mathfrak{R}$  if and only if for each open set  $G$  in  $\mathfrak{R}$   $f^{-1}(G)$  is open in  $\mathfrak{R}$ , prove this.

OR

If  $f$  is continuous on closed interval  $[a, b]$ , prove that it attains its supremum and infimum on  $[a, b]$ .

7. State and prove Cauchy's Mean Value Theorem.
8. If  $f$  possess continuous derivatives of every order in  $(a, a + h)$  and Taylor's remainder  $R_n \rightarrow 0$  as  $n \rightarrow \infty$  prove that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \dots + \frac{h^n}{n!} f^{(n)}(a) + \dots$$

Group 'C'

(2×12=24)

9. If  $\sum u_n$  and  $\sum v_n$  be positive terms series such that  $u_n < k v_n \forall n \in \mathbb{N}$ , where  $k$  is a fixed positive integer, prove that  $\sum u_n$  converges if  $\sum v_n$  converges and  $\sum v_n$  diverges if  $\sum u_n$  diverges.

Also show that the series  $1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$  is convergent.

OR

Explain the convergence of an infinite product. Prove that an infinite product  $\prod u_n$  converges if and only if to each  $\varepsilon > 0$  there exists  $m \in \mathbb{N}$  such that

$$\left| \prod_{r=1}^p u_{n+r} - 1 \right| < \varepsilon \forall n \geq m \text{ and } p \in \mathbb{N}.$$

10. a) Define upper and lower Darboux sums of  $f$  on  $[a, b]$ .

If  $P \in \mathcal{P}[a, b]$  prove that  $U(P, f)$  and  $L(P, f)$  are bounded and  $L(P, f) \leq U(P, f)$ .

b) If  $\int_a^b f(x) dx$  exists and there exists a function  $F$  such that  $F'(x) = f(x) \forall x \in [a,$

$b]$ , prove that

$$\int_a^b f(x) dx = F(b) - F(a).$$

a



Attempt ALL the questions. Tick ( $\checkmark$ ) the best answers.

- For any real number  $a$ , which of the following is true?
  - $|a| = \max\{a, -a\}$
  - $-|a| = \min\{a, -a\}$
  - $-|a| \leq a \leq |a|$
  - all of the above
- Which of the following set contains its supremum
  - $\{x : x \geq a\}$
  - $\{x : x \leq a\}$
  - $\{x : a < x < b\}$
  - $\{x : a \leq x < b\}$
- Which of the following statement is false?
  - $\mathcal{N}$  is a neighbourhood of each of its points.
  - Finite set not a neighbourhood of any points.
  - Interior of a set  $S$  is not a subset of  $S$
  - Interior of any set is open set
- A set  $S$  is said to be dense in itself if
  - every point of  $S$  is limit point of  $S$
  - some points of  $S$  are limit points of  $S$
  - $S$  is an open interval
  - $S$  is discrete set
- Which of the following set is compact?
  - $\{1, 2, 3, 4, \dots\}$
  - $\dots, 3, -2, -1, 0, 1, 2, 3, \dots$
  - $\{x + x \geq 1\}$
  - $\{x : 1 \leq x \leq 3\}$
- Which one of the following sequences oscillated finite?
  - $\langle (-1)^n \rangle$
  - $\langle (-1)^n + 1 \rangle$
  - $\langle (-2)^n \rangle$
  - $\langle 2x - n^2 \rangle$
- Which of the following is not a divergent sequence?
  - $\langle 1, 2, 3, 4, \dots, n, \dots \rangle$
  - $\langle (1, 1) + \frac{1}{2}(\cdot) + \frac{1}{2} + \frac{1}{3}(\cdot) \dots \rangle$
  - $\langle (1, \frac{1}{2}(\cdot), \frac{1}{3}(\cdot), \dots, \frac{1}{n}(\cdot), \dots) \rangle$
  - $\langle 1^2, 2^2, 3^2, \dots, n^n, \dots \rangle$
- A sequence  $\langle u_n \rangle$  is said to be monotonically increasing if
  - $u_n \leq u_{n+1}$
  - $u_n < u_{n+1}$
  - $u_n \geq u_{n+1}$
  - $u_n > u_{n+1}$
- The sequence  $\langle u_n \rangle$  defined by  $u_n = a \forall n \in \mathbb{N}$  is (c, 1) summable and its value is
  - 0
  - $a$
  - $2a$
  - $\frac{a}{2}$
- The geometric series  $1 + r + r^2 + r^3 + \dots$  is oscillatory if
  - $r = 0$
  - $r < 1$
  - $r = 1$
  - $r = -1$
- If  $\sum u_n \in S^*$  and  $\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = l$  (finite or infinite) then  $\sum u_n$  is convergent when
  - $l \geq 1$
  - $l \leq 1$
  - $l > 1$
  - $l < 1$
- The series  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  is
  - conditionally convergent
  - absolutely convergent
  - divergent
  - non-convergent
- A function  $f$  is said to have a discontinuity from left at  $x = a$  if
  - $f(a-0) = f(a) \neq f(a+0)$
  - $f(a-0) \neq f(a) = f(a+0)$
  - $f(a-0) = f(a+0)$
  - $f(a-0) = f(a) = f(a+0)$
- A function  $f(x)$  satisfied Lipschitz condition of  $l$  if for all  $x_1, x_2 \in I$  there exists some constant  $k > 0$  such that
  - $|f(x_1) - f(x_2)| < k|x_1 - x_2|$
  - $|f(x_1) - f(x_2)| > k|x_1 - x_2|$
  - $|f(x_1) - f(x_2)| \geq k|x_1 - x_2|$
  - $|f(x_1) - f(x_2)| \leq k|x_1 - x_2|$
- If  $f(x) = \frac{|x-4|}{x-4}$ , what is the value of  $\lim_{x \rightarrow 4} f(x)$  ?

- a) 1                      b)  $\infty$                       c) 0                      d) does not exist
16. A continuous function  $f$  on  $[a, b]$  is derivable on  $(a, b)$ . It is increasing on  $(a, b)$  if  
 a)  $f'(x) \leq 0$                       b)  $f'(x) > 0$   
 c)  $f'(x) \geq 0$                       d)  $f'(x) \leq 0$
17. What is the value of  $C$  in Lagrange's mean value theorem for  $f(x) = x^n$  in  $[25, 3]$ ?  
 a)  $\frac{5}{2}$                       b)  $\frac{5}{4}$                       c)  $\frac{1}{2}$                       d)  $\frac{3}{2}$
18. If  $P_1, P_2 \in \mathcal{P}[a, b]$ , which of the following is true?  
 a)  $L(P_1) < U(P_1)$     b)  $L(P_1) \leq U(P_1)$   
 c)  $L(P_1) \leq L(P_2)$     d)  $L(P_2) \leq L(P_1)$
19. If  $F'(x) = f(x)$ , the value of  $\int_a^b f dx$  is  
 a)  $f(a) - f(b)$                       b)  $F(a) - F(b)$   
 c)  $F(b) - F(a)$                       d)  $f(b) - f(a)$
20. If  $f, g \in \mathcal{R}[a, b]$ , which of the following need not be true?  
 a)  $f + g \in \mathcal{R}[a, b]$     b)  $f - g \in \mathcal{R}[a, b]$   
 c)  $f \cdot g \in \mathcal{R}[a, b]$                       d)  $\frac{f}{g} \in \mathcal{R}[a, b]$

EXAM 2068

Attempt ALL the questions.

Group 'B' [8×7=56]

1. Define bounded set with example. Prove that the set of positive real number  $\mathcal{R}^+$  is bounded below and the set of negative real number  $\mathcal{R}^-$  is bounded above
2. Define neighbourhood of a point  $x$ . If any one of the set of set  $M$  and  $N$  is a neighbourhood of  $x$ , prove that  
 a)  $M \cup N$  is a neighbourhood of  $x$   
 b)  $M \cap N$  is a neighbourhood of  $x$

OR

Prove that

- a) every finite subset of  $\mathcal{R}$  is compact  
 b) The set of all real numbers  $\mathcal{R}$  is not compact
3. Define convergence of a sequence. Prove that the sequence cannot converge to more than one limit.
4. If  $(F_n)$  be a sequence of a closed interval such that  $F_{n+1} \subset F_n$  and  $\lim_{n \rightarrow \infty} (\text{length of } F_n) = 0$  then prove that  $\bigcap_{n=1}^{\infty} F_n$  consists of exactly one point.

OR

Define subsequence of a sequence with example. If

$\{u_{2n}\} \rightarrow l$  and  $\{u_{2n-1}\} \rightarrow l$  (finite or infinite), prove that  $(u_n) \rightarrow l$ .

5. Discuss different types of discontinuities of a function with examples.
6. If  $f$  is continuous on  $[a, b]$  and  $f(a) \neq f(b)$  prove that for every  $K$  lying between  $f(a)$  and  $f(b)$  there exists  $c \in (a, b)$  such that  $f(c) = K$ .

OR

If  $f$  is continuous on  $[a, b]$  prove that  $f$  is uniformly continuous on  $[a, b]$

7. If  $f$  is continuous on  $[a, b]$  and derivable on  $(a, b)$  then prove that there exists a point  $c \in (a, b)$  such that  $f(b) - f(a) = (b - a) f'(c)$ . Verify this theorem for the function  $f(x) = x(x-1)(x-2)$  in  $[0, \frac{1}{2}]$
8. If  $f$  is differentiable at a point  $C$ , prove that it is continuous at  $C$ . Show by an example that continuous functions at any point may not be differentiable at that point.

9. If  $\sum u_n \in \mathbb{R}^+$  and  $\sqrt[n]{u_n} = l$ , finite or infinite prove that
- $\sum u_n$  converges for  $l < 1$
  - $\sum u_n$  diverges for  $l > 1$  and
  - test fails for  $l = 1$ .

If  $x > 0$ , then test the convergence of  $Z \sum \frac{(1+\frac{1}{2})^n}{x^n}$

OR

Define absolute convergence and conditional convergence of a series.

- Prove that absolutely convergent series is convergent but converse is not true.
  - If  $\sum u_n$  be an absolutely convergent series then prove that the series of its positive terms and the series of its negative terms are both convergent.
10. a) If  $f$  be bounded function of the interval  $[a, b]$  then prove that for  $\epsilon > 0 \exists \delta \geq 0$  such that  $3\delta > 0$  such that  $\mu(P, f) < \epsilon \forall P \in \rho[a, b]$  with  $\mu(P) \leq \delta$ .
- b) If the function  $f$  and  $g$  are continuous on  $[a, b]$  and have bounded and continuous derivatives on  $(a, b)$ , prove that
- $$\int_a^b fg' = f(b)g(b) - f(a)g(a) - \int_a^b fg$$

Group 'A'

[20]

Attempt ALL the questions. Tick the best answers.

- Which of the following set is an example of ordered field?
  - set of integers
  - set of non-positive integers
  - set of whole numbers
  - set of real numbers
- Which of the following statement is true?
  - $\mathbb{R}^+$  is bounded below but unbounded above
  - $\mathbb{R}^+$  is unbounded below but unbounded above
  - $\mathbb{R}^+$  is neither bounded above nor bounded below
  - $\mathbb{R}^+$  is bounded below but unbounded above
- For every set  $A$ ,  $A$  is
  - open
  - closed
  - neither open nor closed
  - sometimes open sometimes closed
- Which of the following set is compact?
  - $\{x: x \geq 1\}$
  - $\{x: 1 < x < 2\}$
  - $\{x: 1 \leq x \leq 2\}$
  - $\{x: x \leq 1\}$
- Which of the following set is neighbourhood of  $x$ ?
  - $\{x: a < x < b\}$
  - the open interval  $(x-c, x+c)$
  - the set  $\mathbb{R}$  of real numbers
  - all of the above
- The sequence  $((-1)^n)$  is
  - convergent
  - divergent
  - oscillatory
  - none of the above
- Which of the following is true statement?
  - every convergent sequence is a Cauchy sequence
  - every Cauchy sequence is convergent
  - every Cauchy sequence is bounded
  - all of the above
- Which of the following is a monotonic sequence?



a)  $(0, 1, 0, 1)$

b)  $\langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots \rangle$

c)  $\langle 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots \rangle$

d)  $\{1, -1, 1, \dots\}$

9. The limit of the sequence  $\langle (-1)^n (2 + \frac{1}{n}) \rangle$  is

a) -2

b) -1

c) 1

d) 2

10. The series  $\sum \frac{1}{n}$  is

a) oscillatory

b) convergent

c) divergent

d) none of the above

11. An absolutely convergent series is always

a) conditionally convergent

b) oscillatory

c) divergent

d) convergent

12. The geometric series  $1 + r + r^2 + \dots$  is convergent if

a)  $r > 1$

b)  $r$

c)  $r1$

d)  $r < 1$

13. If  $f$  be a function defined on  $91 - (0)$  by  $f(x) = \left| \frac{x}{x} \right|$  what is the value of  $\lim_{x \rightarrow 0} \frac{a}{x} (x)$ ?

a) 1

b) -1

c) 0

d) does not exist

14. If  $f$  is continuous on  $[a, b]$ , which of the following is not true?a)  $f$  is bounded on  $[a, b]$ b)  $f[a, b] = [\inf f, \sup f]$ c)  $f$  is uniformly continuous on  $[a, b]$ 

d) none of the above

15. If  $f$  be defined in the neighbourhood of  $a$  and  $f(a, -0) = f(a) \pm 6f(a+0)$  then  $f$  is said to have discontinuity of the type

a) removable discontinuity

b) discontinuity from right

c) discontinuity from the left

d) discontinuity of the second kind

16. The function  $f(s)$  defined by  $f(x) = |x|$  at  $x = 0$  is

a) continuous but not differentiable

b) not continuous but differentiable

c) neither continuous nor differentiable

d) continuous and differentiable both

17. The value of  $c$  in Rolle's theorem for the function  $f(x) = x(x-1)$  on  $\{a, 1\}$  is

a) 1

b)  $1/2$

c)  $1/4$

d)  $a/4$

18. If  $P_1, P_2 \in \mathcal{P}$  and  $P_1 \subset P_2$ , which of the following is true?

a)  $\cup(P_2) \subset \cup(P_1)$

b)  $\cup(P_1) \subset \cup(P_2)$

c)  $L(P_1) < L(P_2)$

d)  $L(P_1) \leq L(P_2)$

19. The lower Riemann integral of  $f$  on  $(a, b)$  is given by

a)  $\int_a^b f = \sup \cup (P, f)$

b)  $\int_a^b f = \cup (P, f)$

c)  $\int_a^b f = \sup L(P, f)$

d)  $\int_a^b f \inf L(P, f)$

20. If  $f \in \mathcal{R}[a, b]$  and  $M, m$  are upper and lower bounds of  $f$  on  $[a, b]$ , then for  $b \geq a$ , which of the following is true?

a)  $M(b-a) \geq \int_a^b f \geq m(b-a)$

b)  $M(b-a) \leq \int_a^b f \leq m(b-a)$

$$c) m(b-a) > \int_a^b f > M(b-a)$$

$$d) m(b-a) < \int_a^b f < M(b-a)$$

Exam 2069

Attempt ALL the questions.

Group 'B'

[8×7=56]

- If  $x > 0$  prove that for any  $y \in \mathbb{N}$  there exists  $n \in \mathbb{N}$  such that  $nx > y$ . Also show that the set of natural numbers  $\mathbb{N}$  is unbounded above.
- Define limit point. Prove that every bounded infinite set of real numbers has at least one limit point.

OR

- Define compact set. Prove that every compact subset of  $\mathbb{R}$  is closed.
- Define Cauchy sequence. Prove that every convergent sequence is a Cauchy sequence.
- If a sequence is divergent then prove that its reciprocal sequence is a Cauchy sequence.
- If a sequence is divergent then prove that its reciprocal sequence (if exists) convergent to 0. Illustrate the result with an example.

OR

Prove that a sequence  $(a^n)$  is convergent iff  $-1 < a \leq 1$ .

- Show that the series  $\sum \frac{1}{n^p}$  converges or diverges according as  $p > 1$  or  $p \leq 1$ .
- If  $\sum y_n$  is convergent with the sum  $s$  then it is  $(C, 1)$  summable with the Cesaro sum  $s$ . Prove it.

OR

For  $n > 0$ , test the convergence of the series

$$\frac{2.4}{3.5} + \frac{2.4.6}{3.5.7} + \frac{2.4.6.8}{3.5.7.9}$$

- If  $\lim_{x \rightarrow a} f(x) = \ell$  and  $\lim_{x \rightarrow a} g(x) = m$  then prove that  $\lim_{x \rightarrow a} (fg)(x) = \ell m$ .
- If  $f(n)$  exists and it bounded some interval  $I$  then prove that  $f$  is uniformly continuous on  $I$ .

Group 'C'

[2×12=24]

- Define uniformly continuous function and prove the following:  
a) If  $f$  is uniformly continuous on an interval  $I$  then it is continuous on  $[a, b]$ .

OR

- If  $f$  is continuous on a closed interval  $[a, b]$  then it is uniformly continuous on  $[a, b]$ .

OR

Define derivability of a function. If  $f$  is derivable on  $[a, b]$  and  $f(a) \neq f(b)$  then prove that for each  $K$  lying between  $f(a)$  and  $f(b)$  there exists  $c \in (a, b)$  such that  $f'(c) = K$ .

- State the necessary and sufficient condition for the integrability of a function. Also, prove the following.  
a)  $f \in \mathcal{R}[a, b]$  if  $f$  is continuous on  $[a, b]$   
b)  $f \in \mathcal{R}[a, b]$  if  $f$  is monotonic on  $[a, b]$ .

Group 'A'

[20]

Attempt ALL the questions. Tick the best answers.

- If  $x \in \mathbb{R}$  then which of the following is true?  
a)  $|x| = \max\{x, -x\}$   
b)  $-|z| = \min\{x, -x\}$   
c)  $-|x| \leq x \leq |x|$   
d) all of the above
- For any set  $S, S^c$  is  
a) open  
b) closed  
c) neither open nor closed  
d) both (a) and (b)
- The set  $\{x : a < x \leq b\}$  is the derived set of  
a)  $\{x : a < x \leq b\}$   
b)  $\{x : a \leq x, b\}$   
c)  $\{x : a < x < b\}$   
d) all of the above

4. Which of the following set is not a closed set?  
 a)  $\phi$  b)  $\mathbb{R}$   
 c)  $\mathbb{Q}$  d)  $[a, b]$
5. Which of the following statement is correct?  
 a) every bounded sequence has at least one limit point  
 b) the set of limit points of a bounded sequence is bounded  
 c) the set of limit points of a sequence is bounded  
 d) all of the above
6. Which of the following sequence has no limit point?  
 a)  $u_n = (-1)^n \forall n \in \mathbb{N}$  b)  $u_n = 1 + (-1)^n \forall n \in \mathbb{N}$   
 c)  $u_n = 2n \forall n \in \mathbb{N}$  d)  $u_n = \frac{(-1)^n}{n!} \forall n \in \mathbb{N}$
7. The infinite series  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  is  
 a) convergent b) divergent  
 c) oscillatory d) non-convergent
8. If  $\lim u_n = 2$  then the series  $\sum u_n$  .....  
 a) converges to 2 b) diverges to  $+\infty$   
 c) diverges to  $-\infty$  d) converges to a finite number other than 2
9. If the infinite product  $\prod u_n$  ( $u_n \neq \forall n \in \mathbb{N}$ ) is convergent then which of the following is true?  
 a)  $\lim u_n = 0$  b)  $\lim u_n = 1$   
 c)  $\lim u_n = \ell$  where  $0 \leq \ell < 1$  d)  $\lim u_n = \infty$
10. If the function  $f$  is monotonically non-decreasing in  $(a, b)$  than for  $a < x < y < b$ , which of the following is not true?  
 a)  $f(x+0) \leq f(y-0)$  b)  $f(x-0) \leq f(x+0)$   
 c)  $f(y+0) \leq f(y-0)$  d)  $f(y-0) \leq f(y+0)$
11. Which of the following is true for the function  $x^2$  on  $(0, 1)$ ?  
 a) It attains its suprema on  $(0, 1)$  b) It attains its infima on  $(0, 1)$   
 c) It attains its suprema and infima on  $(0, 1)$   
 d) It attains its neither suprema nor infima on  $(0, 1)$
12. Which of the following is the image of an interval under a continuous function?  
 a) an interval b) a singleton set  
 c) an interval or a singleton set d) an infinite set
13. Which of the following is a condition for the uniformly continuous function on an interval  $I$ ?  
 a)  $f$  is continuous on  $I$  b)  $f$  satisfies Lipschitz's condition on  $I$   
 c)  $f$  is continuous and bounded on  $I$  d)  $f$  attains its suprema and infima on  $I$
14. If the functions  $f$  and  $g$  are continuous on  $[a, b]$ , derivable on  $(a, b)$  and  $f'(n) \neq 0 \forall n \in (a, b)$  such that  
 a)  $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$  b)  $\frac{g(b) - g(a)}{f(b) - f(a)} = \frac{g'(c)}{f'(c)}$   
 c)  $\frac{f(a) - f(b)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$  d)  $\frac{g(a) - g(b)}{f(b) - f(a)} = \frac{g'(c)}{f'(c)}$
15. If  $f$  is continuous on  $[a, b]$  and derivable on  $(a, b)$  then which of the following is true for the function  $f$  to be increasing on  $(a, b)$ ?  
 a)  $f'(n) > \forall n \in [a, b]$  b)  $f'(n) \geq 0, \forall n \in [a, b]$   
 c)  $f'(n) \leq 0 \forall n \in [a, b]$  d)  $f'(n) < 0, \forall n \in [a, b]$
16. Which of the following is the value of  $\lim_{x \rightarrow 0} \sin x \log x$ ?  
 a) 1 b) 0  
 c)  $\infty$  d) the value does not exist
17. Which of the following function is integrable on  $[a, b]$ ?  
 a) continuous function on  $[a, b]$  b) monotonic function  $[a, b]$   
 c) bounded function on a finite set of points of discontinuity on  $[a, b]$   
 d) all of the above



18. If  $f$  maps  $[a, b]$  onto  $[f(a), f(b)]$  with positive continuous derivatives and  $g \in \mathcal{R}[f(a), f(b)]$  then which of the following is true?
- a)  $\int_a^{g(b)} g(a) = \int_b^a (fg)f$       b)  $\int_a^{f(b)} g = \int_a^b (gf)f$   
 c)  $\int_a^{g(b)} g = \int_a^b (f \circ g)g'$       d)  $\int_a^{f(b)} f = \int_a^b (fg)f'$
19. Which of the following is not true?
- a)  $f \in \mathcal{R}[a, b] \Rightarrow |f| \in \mathcal{R}[a, b]$       b)  $|f| \in \mathcal{R}[a, b] \Rightarrow f \in \mathcal{R}[a, b]$   
 c)  $f, g \in \mathcal{R}[a, b] \Rightarrow f + g \in \mathcal{R}[a, b]$       d)  $f, g \in \mathcal{R}[a, b] \Rightarrow fg \in \mathcal{R}[a, b]$
20. If  $K$  is constant function on  $[a, b]$  which of the following is the value of  $\int_a^b K$ ?
- a)  $K(b-a)$       b)  $K(a-b)$   
 c)  $a-b$       d)  $\frac{b^2}{2}(b-a)$

Exam. 2070

Time: 3 hrs.

Full Marks: 100

Attempt all the questions.

- Define rational number. Prove that there exist infinitely many rational numbers between any two distinct real numbers.
- Define the neighbourhood of a point and prove that open interval  $(a, b)$  is a neighbourhood of each of its points.

OR

Define an open set. Prove that the union of an arbitrary family of open sets is open.

- Discuss the roundedness of a sequence. Prove that every bounded sequence has at least one limit point.
- Let  $\langle u_n \rangle$  be a sequence s.t.  $\lim u_n = e$

Prove that  $\lim \frac{u_1 + u_2 + \dots + u_n}{n} = 1$ .

OR

State Cauchy's condition for the convergence of a sequence and show that the sequence  $\langle 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{n} \rangle$  diverges to  $+\infty$ .

- If  $\lim u_n + e \neq 0$  then  $\sum u_n$  converges to  $+\infty$  or  $-\infty$  according as  $e > 0$  or  $e < 0$  and is finite or infinite. Prove it.
- If  $\sum u_n$  is convergent and  $\langle v_n \rangle$  is a monotonic non-increasing bounded sequence, prove that  $\sum u_n v_n$  converges.

OR

If  $0 < u_n < 1 \forall n \in \mathbb{N}$  and  $\sum u_n$  is convergent. prove that  $\prod(1+u_n)$  is convergent.

- Discuss different types of discontinuities with examples.
- Define extreme value of a function. If  $f$  has an extreme value at  $c$  and  $f'(c)$  exists, prove that  $f'(c) = 0$ .

Group "C"

[2\*12=24]

- If  $f, g'$  of  $f, g$  are continuous on  $[1-h, a+h]$  and derivable on  $(a-h, a+h)$ , prove that  

$$\frac{f(a+h)-2f(a)+f(a-h)}{g(a+h)-2g(a)+g(a-h)} = \frac{f'(\xi)}{g'(\xi)}$$
 for some  $\xi \in (a-h, a+h)$   
 provided  $g(a+h)-2g(a)+g(a-h) \neq 0 \forall \xi \in (a-h, a+h)$ .

OR

State and prove the intermediate value theorem.

- If  $g \in \mathcal{R}[a, b]$  and  $f$  is monotonic and non-negative on  $[a, b]$  then prove that for some  $\xi$  or  $\eta \in [a, b]$   

$$\int_a^b fg = f(a) \int_a^x g \text{ or } f(b) \int_x^b g$$
 according as  $f$  is monotonically non-increasing or non-decreasing on  $[a, b]$ .

Attempt all the questions. Tick ( ) the best answers.

- Which of the following set is bounded?
  - {1, 2, 3, 4}
  - the set of natural number  $\mathbb{N}$
  - the set of real numbers  $\mathbb{R}$
  - $\{x: x \in \mathbb{R} \text{ and } n < 1\}$
- Which of the following statements is true?
  - the union of an arbitrary family of closed sets is closed
  - the intersection of an arbitrary family of closed sets is closed
  - the intersection of an arbitrary family of open sets is open
  - the interior of a set  $S$  is the 3 smallest open set contained in  $S$
- Which of the following is the set  $\{x: 0 \leq x \leq 1\}$ ?
  - an open set
  - a closed set
  - neither open nor closed set
  - none of the above
- Which of the following set is compact?
  - set of natural numbers  $\mathbb{N}$
  - set of whole numbers  $\mathbb{N}$
  - $\{x: x \geq 1\}$
  - $\{s: 1 \leq x \leq 3\}$
- Which of the following is a correct statement?
  - The null set  $\phi$  has no limit points
  - A finite set has no limit points
  - The set of all limit points of  $\mathbb{Q}$  is  $\mathbb{R}$
  - All of the above
- Which of the following sequence is a null sequence?
  - $\langle 1/n \rangle$
  - $\langle n \rangle$
  - $\langle 1 + (-1)^n \rangle$
  - $\langle 2 + 1/n \rangle$
- If  $\langle u_n \rangle$  be a sequence define by  $u_n = \frac{2n^2+1}{2n^2-1} \forall n \in \mathbb{N}$  then which of the following is equal to  $\lim_{n \rightarrow \infty} u_n$ ?
  - 0
  - 1
  - 2
  - $\infty$
- If  $\sum u_n \in \mathbb{S}^+$  and  $\lim_{n \rightarrow \infty} \left( \frac{u_n}{u_{n+1}} - 1 \right) = 1$  then the series  $\sum u_n$  is convergent if
  - $e \geq 1$
  - $e > 1$
  - $e < 1$
  - $e \leq 1$
- Which of the following series is conditionally convergent series?
  - $1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$
  - $1 - \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$
  - $1 - 2 + 3 - 4 + \dots$
  - $\sum (-1)^n$
- If the infinite product  $\prod (1 + u_n)$  is convergent, the value of  $\lim_{n \rightarrow \infty} u_n$  is
  - 0
  - 1
  - a number lying between 0 and 1
  - a number greater than 1
- If the function  $f(x) = \frac{1}{x}$  is defined on  $(0, 1)$  then which of the following is true
  - bounded
  - unbounded
  - continuous and bounded
  - not continuous
- Which of the following discontinuity does the function  $f(x) = \sin \frac{1}{x}$  have at  $x = 0$ ?
  - removable
  - first kind
  - second kind
  - infinite
- If the function  $f$  is monotonically non-increasing in  $(a, b)$  and  $x, y \in (a, b)$  with  $x < y$  then which of the following is true?
  - $f(x+0) \leq f(y-0)$
  - $f(x-0) \leq f(x+0)$
  - $f(x+0) \geq f(y-0)$
  - $f(y-0) \leq f(y+0)$
- Which of the following is true?
  - A function  $f$  is uniformly continuous on an interval  $I$  if it is continuous  $I$

- (b) A function  $f$  is continuous on an interval  $I$  if it is uniformly continuous and one-one on  $I$
- (c) The function  $f$  is uniformly continuous on an interval  $I$  if it is continuous and one-one on  $I$
- (d) If  $f$  is continuous on an interval  $I$  then it is strictly monotonic on  $I$
15. Which of the following is true for all  $x \in (0, \frac{\pi}{2})$ ?
- (a)  $\sin x > x > \tan x$  (b)  $x > \tan x > \sin x$   
 (c)  $\tan x > x > \sin x$  (d)  $\tan x > \sin x > x$
16. Which of the following is true for the function  $f(x) = |x|$  at  $x = 0$ ?
- (a) continuous and differentiable  
 (b) continuous but not differentiable  
 (c) differentiable but not continuous  
 (d) neither continuous nor differentiable
17. Which of the following is not an indeterminate form?
- (a)  $0/0$  (b)  $0 \times \infty$   
 (c)  $1^\infty$  (d)  $\infty \times \infty$
18. If  $P_1, P_2 \in \rho [a, b]$  with  $P_1, P_2$  then which of the following is true?
- (a)  $U(P_1) \leq U(P_2)$  (b)  $U(P_1) \leq L(P_2)$   
 (c)  $L(P_1) < L(P_2)$  (d)  $U(P_2) \leq L(P_1)$
19. If the function  $f$  is defined by  
 $f(x) = 1$  when  $x$  is rational  
 $= -1$  when  $x$  is irrational
- then which of the following is the value of  $\int_a^b f$ ?
- (a)  $b-a$  (b)  $a-b$   
 (c)  $\pm(b-a)$  (d)  $\int_a^b f$  does not exist
20. If  $f(x) = x \forall x \in [0, 3]$  and  $P = \{0, 1, 2, 3\}$  be a partition of  $[0, 3]$  then which of the following is the value of  $L(P, f)$ ?
- (a) 2 (b) 3  
 (c) 4 (d) 6

Exam 2071  
 Group "B"

8×7=56

Attempt the all questions.

- Define supremum and infimum with examples. Prove that supremum and infimum of a set, if exist, are unique.
- Define closed set. Prove that a set  $A$  is closed if and only if  $\mathfrak{R} - A$  is open.  
 OR  
 Prove that every compact subset of  $\mathfrak{R}$  is closed.
- Prove that a sequence  $\langle u_n \rangle$  defined by  $u_n = a^n$  is convergent if  $-1 < a \leq 1$  and then  $\lim a^n = 0$  when  $-1 < a < 1$  and  $\lim a^n = 1$  when  $a = 1$ .
- Discuss monotonic sequence with examples. Prove that a monotonic sequence  $\langle u_n \rangle$  is convergent if and only if it is bounded.  
 OR  
 Prove that every sequence has a monotonic subsequence.
- A function defined on  $\mathfrak{R}$  is continuous on  $\mathfrak{R}$  if and only if for each open set  $G$  in  $\mathfrak{R}$ ,  $f^{-1}(G)$  is open in  $\mathfrak{R}$ , prove it.  
 OR  
 If  $f$  is continuous on  $[a, b]$ , prove that  $f$  attains supremum and infimum on  $[a, b]$ .
- If  $f$  is uniformly continuous on  $[a, b]$ , prove that it is continuous on  $[a, b]$ . Also show that the function  $f(x) = x^2$  is uniformly continuous on  $[-1, 1]$ .
- State and prove Cauchy's mean value theorem.
- If  $f$  possesses continuous derivatives of every order in  $[a, a+h]$  and Taylor's remainder  $\mathfrak{R}_n \rightarrow 0$  as  $n \rightarrow \infty$ , prove that  $f(a+h) = f(a) + \frac{h^2}{2!} f'(a) + \dots + \frac{h^2}{n!} f^{(n)}(a) + \dots$



9. Prove that the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges or diverges according as  $p > 1$  or  $\leq 1$ . Apply it to test the convergence of the series
- $$1 + \frac{1}{3} + \frac{1}{4} + \frac{1}{3} + \frac{1}{4} + \dots$$
- 2    3    4

OR

Prove the following

- (a) If  $\sum u_n$  be absolutely convergent series then the series of its positive terms and the series of its negative terms are both convergent.
- (b) If  $\sum u_n$  be conditionally convergent then the series of its positive and series of its negative terms are both divergent.
10. Prove the following:
- (a) A bounded function  $f \in \mathcal{R}[a, b]$  if and only if for every  $\epsilon > 0$   $\exists \delta > 0$  such that
- $$U(p, f) - L(p, f) < \epsilon \quad \forall \mu(P) \leq \delta$$
- (b) If  $\int_a^b f(x) dx$  exists and there exists a function  $F$  such that  $F'(x) = f(x) \in [a, b]$  Prove that  $\int_a^b f(x) dx = F(b) - F(a)$ .

## Group "A"

20

Attempt ALL the questions. Tick (✓) the best answers.

- For any  $\epsilon > 0$ , if  $|b - a| < \epsilon$ , which of the following is true?
  - $b = a$
  - $|a - b| < \epsilon$
  - $b < a + \epsilon$
  - all of the above
- Which of the following set contains its supremum?
  - $\{x : a < x \leq b\}$
  - $\{x : a \leq x < b\}$
  - $\{x : a < x < b\}$
  - $\{x : x > a\}$
- Which of the following statement is false?
  - every finite set is closed
  - null set open
  - interior of a set is open
  - union of open sets is not open
- Which of the following is not connected set?
  - $\phi$
  - $\{a\}$
  - $\{x : a < b\}$
  - the set  $\mathbb{N}$  of natural numbers
- The set  $\{x : a \leq x \leq b\}$  is the derived set of
  - $\{x : a < x < b\}$
  - $\{x : a \leq x < b\}$
  - $\{x : a \leq x \leq b\}$
  - all of the above
- Which of the following sequence oscillates infinitely?
  - $\langle a^n \rangle$  for  $a = 1$
  - $\langle \dots \rangle$
  - $\langle 1 + (-1)^n \rangle$
  - $\langle \dots \rangle$
- The sequence  $\langle 1, \frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots \rangle$  is
  - convergent
  - divergent
  - finite oscillatory
  - infinite oscillatory
- Which of the following statement is false?
  - Every sequence has a convergent subsequence.
  - Every sequence has a monotonic subsequence.
  - Every convergent sequence is a Cauchy sequence.
  - Every Cauchy sequence is convergent.
- If  $\langle u_n \rangle$  be a sequence define by  $u_n = a^n$ , then it is divergent when
  - $-1 < a < 1$
  - $a > 1$
  - $a = -1$
  - $a = 1$
- For what value of  $P$ , the series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is divergent?
  - $P > 1$
  - $p \geq 1$
  - $p < 1$
  - $p \leq 1$

11. The geometric series  $1 + r + r^2 + \dots$  converges to  $\frac{1}{1-r}$  if  
 a.  $a < -1$       b.  $r > 1$       c.  $-1 < r < 1$       d.  $-1 \leq r \leq 1$
12. Which of the following statement is true?  
 a. series of positive terms is always convergent  
 b. Series of negative terms is always convergent  
 c. Every absolutely convergent series converges  
 d. Every convergent series converges absolutely
13. What is the value of  $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$ ?  
 a. 0      b. e      c.  $\infty$       d.  $\pi$
14. If  $f(x) = \frac{\sin x}{x}$  for  $x \neq 0$  and  $f(0) = 0$ , the discontinuity of  $f(x)$  at 0 is  
 a. removable      b. discontinuity of first kind  
 c. oscillatory      d. discontinuity of second kind
15. If  $f$  is continuous on  $[a, b]$ , which of the following is not true?  
 a.  $f([a, b])$  is a closed set  
 b.  $f([a, b])$  is bounded  
 c.  $f$  attains supremum and infimum on  $[a, b]$   
 d.  $f$  is not uniformly continuous on  $[a, b]$
16. A function of continuous on  $[a, b]$  and derivable on  $(a, b)$  is increasing on  $[a, b]$  if  
 a.  $f'(x) \geq 0$       b.  $f(x) \leq 0$       c.  $f(x) \geq 0$       d.  $f(x) \leq 0$
17. The function  $f(x) = |x|$ , at  $x = 0$  is  
 a. continuous and differentiable  
 b. continuous but not differentiable  
 c. not continuous but differentiable  
 d. neither continuous nor differentiable
18. The function  $f(x)$ , defined in the neighbourhood of  $c$  has maximum value at  $c$  if  
 a.  $f'(c) = 0$  and  $f''(c) < 0$       b.  $f'(c) \neq 0$  and  $f''(c) < 0$   
 c.  $f'(c) < 0$  and  $f''(c) > 0$       d.  $f'(c) = 0$  and  $f''(c) < 0$
19. A bounded function  $f$  is Riemann integrable on  $[a, b]$  if and only if for  $\epsilon > 0 \exists \delta > 0$  such that  
 a.  $U(p) - L(p) < \epsilon \forall \|p\| \leq \delta$   
 b.  $U(p) - L(p) \leq \epsilon \forall \|p\| \leq \delta$   
 c.  $U(p) - L(p) > \epsilon \forall \|p\| \leq \delta$   
 d.  $U(p) - L(p) \geq \epsilon \forall \|p\| \leq \delta$
20. If  $F(x) = \int_a^x f(x) dx$ , which of the following is equal to  $\int_a^b f(x) dx$ ?  
 a.  $F(a) - F(b)$       b.  $F(b) - F(a)$       c.  $f(b) - f(a)$       d.  $f(a) - f(b)$

Exam 2072

Time: 3 hrs.

Full Marks: 100

Attempt All the questions.

Group "B"

8×7=56

- Define boundedness of sets with suitable examples. Prove that the set of negative real numbers is bounded above but unbounded below.
- Prove the following:
  - A set  $B$  is open if and only if  $B = B^{\circ}$ , and
  - Every open interval  $I$  is an open set.

OR

Prove that every bounded infinite set  $S$  of real numbers has at least one limit point.

- Prove that a sequence is convergent if and only if it is a Cauchy sequence.
- Define subsequence of a sequence  $\langle u_n \rangle$ . Prove that  $e$  is a limit point of a sequence  $\langle u_n \rangle$  if and only if there exists a subsequence  $\langle u_{n_k} \rangle$  of  $\langle u_n \rangle$  that converges to  $e$ .

OR

Show that the function sequence  $\langle f_n(x) \rangle$  defined on  $[0, 1]$  by  $f_n(x) = x^n$  converges point wise but not uniformly on  $[0, 1]$ .

5. If Show by a suitable example that its converse is not true.  
OR  
If  $f$  is continuous on  $[a, b]$ , prove that it is uniformly continuous on  $[a, b]$
6. If  $f$  is derivable at  $g(x)$  and  $g$  is derivable at  $x$ , prove that  $f \circ g$  is derivable at  $x$  and  $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$ .
7. If  $f$  and  $g$  are continuous on  $[a, b]$  with  $g'(x) \neq 0 \quad x \in (a, b)$ , prove that there exists  $c \in (a, b)$  such that  

$$\frac{f(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$
8. Define supremum and infimum with examples. Prove that supremum and infimum of a set, if exist, are unique.

Group "C"

2×12=24

9. If  $\sum u_n \in S^+$  and  $\sqrt[n]{u_n} = e$ , finite or infinite, prove that  $\sum u_n$  converges or diverges according as  $e < 1$  and  $e > 1$  and the test fails when  $e = 1$ . Use it to show that the series  $\sum \frac{n}{(n+1)^{n^2}}$  is convergent

OR

Prove that an infinite product  $\prod u_n$  converges if and only if to each  $\epsilon > 0 \exists m \in \mathbb{N}$  such

$$\left| \prod_{r=1}^n U_{n+r} - 1 \right| < \epsilon \quad n \geq m \text{ and } \phi \in \mathbb{N}$$

10. Prove the following:
- (a) If  $f$  is continuous on  $[a, b]$ , then  $f \in \mathcal{R}[a, b]$ ,
- (b) If  $f, g \in \mathcal{R}[a, b]$  and  $f \leq g$  on  $[a, b]$  then  $\int_a^b f \geq \int_a^b g$ .

Group "A"

20

Attempt All the questions. Tick ( $\checkmark$ ) the best answers.

1. What is the least upper bound of the set  $\left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$ ?
- a. 0                      b. 1                      c. 1/2                      d. +1/2
2. Which of the following is false?
- a. The set  $\mathcal{R}^+$  is bounded above      b. The set  $\mathcal{R}^-$  is bounded below  
c. The set  $\mathcal{R}$  is bounded              d. all of the above
3. A set  $A$  is closed if
- a.  $A' \subset A$                       b.  $A = \bar{A}$                       c.  $\mathcal{R} - A$  is open                      d. all of the above
4. Which of the following set has only one limit point?
- a. The set  $\left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$                       b. The set  $\{x : 1 < x < 5\}$   
c. The set  $Q$  of rational numbers      d. The set of real numbers
5. A set  $N$  is called a neighborhood of a point  $x$  if for any  $\epsilon > 0$ :
- a.  $N \subset (x - \epsilon, x + \epsilon)$       b.  $N \subset [x - \epsilon, x + \epsilon]$   
c.  $(x - \epsilon, x + \epsilon) \subset N$       d.  $[x - \epsilon, x + \epsilon] \subset N$
6. How many limit points are there of the sequence  $\langle 1 + (-1)^n \rangle$ ?
- a. 2                      b. 1                      c. 3                      d. 0
7. A sequence  $\langle u_n \rangle$  is monotonic increasing if
- a.  $u_{n+1} \geq u_n \forall n \in \mathbb{N}$                       b.  $u_n \geq u_{n+1} \forall n \in \mathbb{N}$   
c.  $u_{n+1} > u_n \forall n \in \mathbb{N}$       d.  $u_n > u_{n+1} \forall n \in \mathbb{N}$
8. Which of the following statement is false?
- a. every sequence has a monotonic subsequence.  
b. every bounded sequence has a convergent subsequence.



- c. some convergent sequences have divergent subsequences.  
 d. every Cauchy sequence is convergent.
9. The sequence  $\langle u_n \rangle$  defined by  $u_n = a \forall n \in \mathbb{N}$  is  $(c, 1)$  summable and its value is  
 a. 0                      b. a                      c. 2a                      d.  $\frac{a}{2}$
10. If  $\sum u_n \in \mathbb{S}^+$  and  $\lim \sqrt[n]{u_n} = e$ , finite or infinite, when  $\sum u_n$  is divergent  
 a.  $e \geq 1$                       b.  $e \leq 1$                       c.  $e > 1$                       d.  $e < 1$
11. Which of the following is series is not convergent?  
 a.  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$                       b.  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$   
 c.  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$                       d.  $1 + \frac{1}{2} + \frac{1}{3} + \dots$
12. Which of the following statement is true?  
 a. Absolutely convergent series is always convergent  
 b. Absolutely convergent series is sometimes convergent  
 c. Convergent series converges absolutely  
 d. Series of positive terms is always convergent
13. A function  $f$  is said to approach  $+\infty$  as  $x$  tends to  $a$  from above if for every  $k > 0$  there exists  $\delta > 0$  such that  
 a.  $a < x < a + \delta \Rightarrow f(x) > k$                       b.  $a - \delta < x < a \Rightarrow f(x) > k$   
 c.  $a < x < a + \delta \Rightarrow f(x) < k$                       d.  $a - \delta < x < a \Rightarrow f(x) < k$
14. If  $\lim_{x \rightarrow a} f(x)$  exists but the limit is not equal to  $f(a)$ , the discontinuity of this type is called  
 a. removable                      b. oscillatory                      c. discontinuity of first kind  
 d. discontinuity of second kind
15. A function  $f: S \rightarrow \mathbb{R}$  satisfies Lipschitz condition if for all  $x, y \in S$ ,  $\exists k > 0$  such that  
 a.  $|f(x) - f(y)| \geq k|x - y|$                       b.  $|f(x) - f(y)| > k|x - y|$   
 c.  $|f(x) - f(y)| < k|x - y|$                       d.  $|f(x) - f(y)| \leq k|x - y|$
16. The value of  $C$  of Rolle's theorem for the function  $f(x) = x(x-1)$  in the interval  $[0, 1]$  is  
 a.  $\frac{1}{4}$                       b.  $\frac{1}{2}$                       c.  $\frac{3}{4}$                       d. 1.
17. The necessary condition for  $f(x)$  to have extreme value at  $C$  is  
 a.  $f'(c) = 0$                       b.  $f(c) = 0$                       c.  $f''(c) = 0$                       d.  $f'(c) = 0$
18. At which point,  $f(x) = |x + 2|$  is not differentiable?  
 a.  $x = 2$                       b.  $x = -2$                       c.  $x = 1$                       d.  $x = -1$
19. If  $f$  is bounded on  $[a, b]$ , then to every  $\epsilon > 0$  there exists  $\delta > 0$  for every  $P \in \mathcal{P}[a, b]$  such that  
 a.  $U(P, f) < \int_a^b f + \epsilon$                       b.  $U(P, f) > f - \epsilon$   
 c.  $L(P, f) < \int_a^b f + \epsilon$                       d.  $L(P, \epsilon) > \int_{-a}^b f + \epsilon$
20. If  $f \in \mathcal{R}[a, b]$ , which of the following is true?  
 a.  $\left| \int_a^b f \right| = \left| \int_a^b f \right|$                       b.  $\int_a^b |f| = \left| \int_a^b f \right|$   
 c.  $\left| \int_a^b f \right| \leq \int_a^b |f|$                       d.  $\int_a^b |f| \leq \left| \int_a^b f \right|$

Time: 3 hrs

Full Marks: 80

Attempt ALL the questions.

Group 'B'

(6×7=42)

1. What are the major goals of math education? Describe them briefly.  
OR  
What are the issues raised against the teacher training in Nepal?
2. What are educational implication of Piaget's theory for both the teachers and curriculum developers?  
OR  
Explain how you would use the knowledge of the contrast and variation theorem in teaching mathematics as advocated by Burner.
3. Describe three approaches to teaching arithmetic that were practices from 1900 to 1960.  
OR  
Identify four basic features of meaningful arithmetic.
4. Explain Hoffer's taxonomy with examples.
5. What are the physical criteria that should be kept in mind while selecting an instructional material? Explain them briefly.
6. Illustrate diagrammatically that addition of two algebraic expressions:  
 $4x^2 + 3x - 2$  and  $x^2 - x + 5$

Group 'C'

(2×12=24)

7. What are the basic problems of instructions? Justify how they are interlinked and complementary to each other.
8. Illustrate with examples to teach section formula for finding the coordinates.  
OR  
Illustrate with examples to teach Pick's formula for finding the area of a figure on geo-board.

Group 'A'

Attempt ALL the questions. Tick (✓) the best answers.

14

1. Which one of the following is the characteristics of pre-operation stage?  
a) Transductive logic                      b) Inductive logic  
c) Deductive logic                      d) None
2. Which one of the following is the prime source of knowledge according to behaviorism?  
a) Sense                      b) Intellect                      c) Action                      d) Speech
3. Which one of the following movement prevails during eighties?  
a) New maths                      b) Back-to-basic  
c) NCTM                      d) Problem solving
4. Which one of the following graphs is kept in grade X compulsory math curriculum?  
a) Pie chart                      b) Histogram  
c) Ogive                      d) None of the above
5. Which one of the following type of question does not have precise scoring key?  
a) Objective                      b) Short answer  
c) Long answer                      d) None of the above
6. A test designed to determine the degree of mastery of specific objectives is termed as  
a) standard test                      b) non-reference test  
c) criterion-reference test                      d) all
7. Which of the following does not fall under the heading remediation method?  
a) Student projects                      b) Field trips  
c) Peer group tutoring                      d) Expert coach
8. Which of the following scoring is considered to be the better grading system?  
a) Numeral grading                      b) Point grading

- c) Percent grading  
d) Letter grading
9. Which one of the following is the most important benefit of a lesson plan?  
a) Sequential lesson  
b) Selection of appropriate material  
c) Save us from distraction  
d) Self confidence and responsibility
10. What is the first activity to begin a lesson according to Gagne?  
a) Arousal of interest  
b) Presentation of materials  
c) Selection of lesson  
d) Feedback
11. Which one of the following is a chief characteristics of module?  
a) It has built-in-self study avenues  
b) It has component similar to lesson plan  
c) It has detail description of the activity  
d) It has task extension component different from lesson plan.
12. Which one of the following example is not put forward in order to justify the need of lesson plan for the teacher?  
a) An actor without script  
b) A musician without a score  
c) A speaker without an outline  
d) A teacher without patience
13. Supervision is necessary for the new teacher because they need  
a) orientation for new program  
b) maintenance of old knowledge  
c) correction at appropriate time  
d) all of the above
14. The best strategy for the better implementation of the result of supervision is  
a) provide appropriate teaching materials  
b) provide appropriate textbook  
c) provide opportunity for video lessons  
d) provide presentation of model class

**EXAM 2068**

Attempt ALL the questions. Tick the best answers.

**Group 'A'**

[14]

1. Microscopic nature of mathematics is to  
a) solve the mathematical problems related to Other  
b) correct own weakness for the development if mathematics itself  
c) relate with outer world only  
d) concern practical problems only
2. New mathematics movement was concerned to  
a) content  
b) Method  
c) Evaluation system  
d) materials
3. Pre-operational period of Child development is known as  
a) logical age  
b) gang age  
c) intuitive age  
d) ego-centric age
4. Which is the highest level of understanding of Geometry according to Van Flick?  
a) Van Hiele  
b) formal deduction  
c) rigor level  
d) abstraction level
5. Which of the following approaches was focused during 1900-1920 form?  
a) New Math  
b) Drill  
c) Social  
d) Meaningful
6. Who advocated for expository method?  
a) Piaget  
b) Burner  
c) Daine  
d) Ausbel
7. Which of the following is the model of open question?  
a) What is the average depth of the river if its depth was taken from five places  
2m, 6m, 7.5m, and 3.5m  
b) find the area of a pond where radius is 5m and circumference is 22m  
c) how much area would be covered by 35M  
d) All of the above
8. Which one is the important criteria to select the instructional materials?  
a) physical criteria  
b) pedagogical criteria  
c) mental criteria  
d) both a and b
9. What is the main purpose of annual plan?  
a) to build up teachers confidence  
b) to finish the course in time



- c) to see the holidays  
d) to detect teacher
- Diagnostic test will be used when
    - the completion of course is required
    - the achievement level of students is demanded
    - way out of overcoming the hurdles of students is wanted
    - action to improve teaching learning situations
  - The best strategy for tic better implementation of the result of supervision is to
    - provide appropriate teaching materials
    - provide appropriate video-class
    - provide opportunities of model class
    - provide presentation of model class
  - Clinometers is an instructional material mostly useful in
    - trigonometry
    - probability
    - equation
    - menstruation
  - FLAC records the class-morn interaction on the time interval of
    - 5 seconds
    - 1 minute
    - 3 minutes
    - 4 minutes
  - What is the basic concept for the teaching of heights and distances from the following?
    - trigonometry
    - ratio
    - symmetry
    - none of the above

**Group 'B'**

[6×7=42]

- Differentiate between mathematics and mathematics education with example.
- Prepare a specific objective for each level of cognitive domain according to Bloom from the area geometry of secondary level.
- Describe the major factors of changing curriculum in mathematics.

OR

What types of assessment are supposed to be effective in student's evaluation in 21th century?

- Classroom diversity may Create mathematical anxiety in students 'Justify or refute your answer on this statement.
- What is the role of instructional material in teaching learning activities? Explain with examples.

OR

Why do teachers need a lesson plan? Prepare a lesson plan teaching any concept.

- Illustrate diagrammatically the addition of two algebraic expressions  $x^2 + 2x - 2$  and  $2x^2 - x^4$ .

OR

**Group 'C'**

[2×12=24]

- How does intellectual development occur according to Vygotskys social theory? Explain with its classroom implications.
- How do you show the area of circle's by the method of demonstrating manipulative materials? Provide two models.

OR

Illustrate five meaningful justifications for the teaching of  $-x=-+$   
Justify which one do you like most while teaching

Exam 2069

Group "A"

14

Attempt all the questions. Tick (✓) the best answers.

- Which one of the following is the meaning of reversibility?
  - Negation
  - Inversion
  - Reciprocity
  - all of the above
- Which one of the following is the prime source of knowledge according to cognitivism?

- (a) Sense (b) Reason  
(c) Action (d) Speech
- Which one of the following movement prevails during sixties?  
(a) New maths (b) Back-to-basic  
(c) NCTM (d) problem solving
  - Which one of the following construction is taught in grade X?  
(a) Construction of triangle (b) Construction of trapezium  
(c) Construction of triangle of equal area  
(d) Construction of circle of equal area
  - Which one of the following is not the implication of item analysis?  
(a) Item difficulty level (b) Discrimination index  
(c) Power of distracter (d) Scoring key
  - A test that has statistical information available and may be used to interpret individual score is termed as  
(a) standard test (b) norm-reference test  
(c) criterion-reference test (d) all of the above
  - Which one of the following is the full form of CAS?  
(a) Continuous alternative system  
(b) Continuous assessing system  
(c) Continuous assessment system  
(d) Casual assessment system
  - Which one of the following is said to be better grading system?  
(a) Numeral grading (b) Point grading  
(c) Percent grading (d) Letter grading
  - Which one of the following is not put forward in order to justify the need of lesson plan for teacher?  
(a) An alter without script (b) A teacher without patience  
(c) A musician without a score (d) A speaker without an outline
  - Which one of the following is a chief characteristic of module?  
(a) It has component similar to lesson plan  
(b) It has task extension component different from lesson plan  
(c) It has detail description of the activities  
(d) It has built-self study avenues
  - Which one of the following is not a vital component of lesson plan?  
(a) Objective (b) Material  
(c) Evaluation (d) Homework
  - ACI measures  
(a) holistic scoring (b) qualitative scoring  
(c) analytic scoring (d) subjective scoring
  - Which one of the following is not the aim of supervision?  
(a) Improvement of instruction (b) Improvement of total teaching  
(c) Providing leadership to teachers  
(d) Improvement of curriculum
  - Which one of the following is a representation of  $xy+3=0$   
(a) Ellipse (b) Parabola  
(c) Hyperbola (d) Parallel lines

**Group "B"**

6×7=42

- Justify with examples how new-math movement accelerate both math and math education.

OR

Explain the relationship among absolutism and other philosophies for the foundation of mathematics.

- Justify with example how you use the strategy of reversibility advocated by Piaget while teaching math to a child of concrete operational period.

OR

Describe the factors that influence the transition from one stage to another according to Piaget.

- Describe different international reform movements in math education

OR

Distinguish between New-math and meaningful approach in teaching mathematics.

4. Explain Krathwohl's taxonomy with examples.
5. What are the pedagogical criteria that are kept in mind while selecting an instructional material? Explain them briefly.
6. Illustrate diagrammatically over natural numbers.

Group "C"

2×12=24

7. List four different methods of teaching mathematics. Justify with examples how they are different from each others.
8. Illustrate diagrammatically how you teach basic geometric construction addition, subtraction, multiplication and division through straight edge and compass.

OR

Illustrate diagrammatically how you teach to show the median point for a group data.

Exam. 2070

Time: 3 hrs.

Full Marks: 80

Attempt all the questions.

Group "B"

[6×7=42]

1. Describe math education with its educational philosophies.

OR

What are the problems of teacher training in Nepal? Describe them briefly.

2. Define learning theory and differentiate behaviorist theories from cognitive theories of learning.
3. Describe Vygotsky's theory of intellectual development with example.

OR

What are the implications of Ansubel's learning theory in classroom teaching? Describe with example.

4. Define behavioural objectives with example. Prepare a behavioural objective of each level of cognitive domain.
5. State the problems of instruction in mathematics. Give reasonable suggestions for minimising any two of them.

OR

What are the criteria for selecting manipulative materials? Describe them with example.

6. Which method do you prefer in teaching the sum of the cubes of first n-natural numbers? Prove that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

Group "C"

[2×12=24]

7. State the types of instructional planning's with their importance. Prepare a lesson plan for the teaching of area of circle.
8. State different types of functions and explain them with suitable mapping diagrams. Also give examples to each of them.

OR

Define reliability of a test. Describe various methods for determining reliability of a test.

Group "A"

[14]

Attempt all the questions. Tick (✓) the best answers.

1. Which of the following is not the nature of mathematic?  
(a) Hypothetic (b) Inductive  
(c) Deductive (d) Applied
2. Which of the following is the important pedagogical implication of Bruner's theory for teaching mathematics?  
(a) inductive approach (b) discovery approach  
(c) guided discovery approach (d) inquiry approach
3. Which of the following is the highest level of learning according to Gagne?  
(a) chain learning (b) multiple discrimination learning  
(c) rule of principle learning (d) problem solving learning



4. Van Hiele's theory of learning is mainly related to the teaching of ....
  - (a) arithmetic
  - (b) geometry
  - (c) algebra
  - (d) statistics
5. Which of the following is the first step in framing a mathematics curriculum?
  - (a) selection of contents
  - (b) selection learning experiences
  - (c) organization of the contents
  - (d) formulation of objectives
6. Which of the following is not the criteria for evaluating a mathematics textbook?
  - (a) relation with objectives
  - (b) language
  - (c) writer
  - (d) examples
7. Which of the following is important in developing understanding of new materials on mathematics teaching?
  - (a) explanation of new terminologies, symbols and basic concepts
  - (b) active participation of students in teaching - learning activities
  - (c) guided discovery approach in teaching
  - (d) all of the above
8. Which of the following is the main purpose of maintenance programme in teaching mathematics?
  - (a) to carry on review work
  - (b) to plan drill
  - (c) to prevent forgetting
  - (d) to provide laboratory work
9. Which of the following represent the steps of guided discovery method?
  - (a) presentation, exploration, verification and generalization
  - (b) presentation of the problem, exploration under guidance and generalization
  - (c) presentation of the problem, exploration under guidance, verification and generalization
  - (d) all of the above
10. Which method is effective for teaching verbal problems in mathematics?
  - (a) discussion
  - (b) problem solving
  - (c) lecture
  - (d) discovery
11. Which of the following is not the criteria of selecting manipulative materials?
  - (a) attractive
  - (b) simplicity
  - (c) expensive
  - (d) durability
12. Which is the main component of lesson plan?
  - (a) topic
  - (b) objectives
  - (c) activities
  - (d) evaluation
13. Which of the following is the main purpose of formative evaluation?
  - (a) to improve teaching learning condition
  - (b) to validate the level of students
  - (c) to identify the weaknesses of students
  - (d) all of the above
14. Which of the following is not the instrument for evaluation of teaching?
  - (a) activity categories instrument
  - (b) Flander's interaction analysis categories
  - (c) computer assisted instruction
  - (d) general class observation form

Exam 2071

**Group "A"**

14

Attempt ALL the questions. Tick (✓) the best answers.

1. Which of the following is not related to mathematics education?
  - a. curriculum
  - b. teacher's training
  - c. supervision
  - d. teacher's salary
2. Which of the following is the prime source of knowledge according to constructivism?
  - a. sense
  - b. reason
  - c. action
  - d. interaction
3. The tendency of a child to fit every new experience into pre-existing mental structure is called
  - a. accommodation
  - b. assimilation
  - c. transfer
  - d. generalization
4. Which of the following is not a pair of inverse operation?

- a. since and cosine  
c. addition and subtraction
- b. division and multiplication  
d. square and square root
5. Which of the following is the implication of Gagne's theory in math teaching?  
a. new concepts should be taught after evaluating necessary pre-requisites  
b. process is important than product  
c. permanency of learned materials through repeated practice  
d. all of the above
6. What is the pedagogical implication of Anselm's theory in math teaching?  
a. expository method  
c. discovery method
- b. problem solving method  
d. rote method
7. Which of the following is included in the curriculum of both compulsory and optional mathematics of secondary level?  
a. vectors  
c. matrices
- b. trigonometry  
d. function and relation
8. Which of the following is the basis for evaluation of curriculum?  
a. relevancy of objectives  
c. appropriateness of content
- b. adequacy of teaching method  
d. all of the above
9. The transfer of learning essential element in  
a. generalization of mathematical concepts  
b. classification of mathematical concepts  
c. application of mathematical concepts  
d. explanation of mathematical concepts
10. The basic requirement for a maintenance program in math teaching is  
a. to carry on review work  
b. to prevent forgetting  
c. to help the student to learn new materials  
d. to plan drill
11. Which of the following test item falls under knowledge level?  
a. state any three conditions for the congruence of two triangles  
b. solve the equation:  $x^2 - 4x + 3 = 0$  by factorization  
c. solve the equation:  $x - 2y = 10$  and  $x^2 - y^2 = 10$  by graphic method  
d. both b and c
12. Which of the following factor is not helpful to the improvement of mathematics instruction?  
a. self improvement on the part of the teacher  
b. in service training  
c. over confidence on the part of the teacher  
d. all of the above
13. The role of mathematics supervisor is to  
a. improve the classroom behaviour during instruction  
b. guide in selecting and organizing the materials of instruction  
c. enrich the background of the teacher  
d. all of the above
14. Which one of the following techniques is helpful to the improvement in instruction through supervision?  
a. encouragement of teachers to seek self improvement  
b. implementation of ACI  
c. regular organization of conference  
d. implementation of FLAC

**Group "B"**

6×7=42

Attempt the all questions.

1. Define mathematics education. What are the goals of mathematics education? Explain any two of them.
2. Define behavioural objectives with examples. Prepare 1-1 behavioural objectives of each level of cognitive domain.

OR

Explain the Krathwool's taxonomy with examples.



- Which method do you prefer to teach the sum of first n-natural numbers? Explain and derive the sum.
- Explain the process of curriculum development in Nepal.

OR

- Give a critical appraisal of present compulsory mathematics textbook of grade X.
- State the basic problems of mathematics instruction and explain any two of them with examples.
- What are the basic criteria of selecting instructional materials? Explain briefly.

OR

Describe the importance of lesson plan. Prepare a lesson plan for teaching trigonometric ratios in grade IX.

**Group "C"**

2×12=24

- Explain briefly the different stages of intellectual development according to Piaget. What are the educational implications of Piaget's theory? Explain with examples.
- Illustrate with example to teach area of circle using manipulative materials.

OR

Explain different activities for finding the formula for area of a triangle.

Exam 2072

Full Marks: 80

Time: 3 hrs.

Attempt All the questions.

**Group "B"**

$$6 \times 7 = 42$$

- Compare and contrast between two subjects' mathematics and mathematics education with suitable example.

OR

Write down the broad goals of mathematics education and explain them briefly with suitable examples.

- Compare and contrast between the stage of Jean Piaget and Van Hiele's learning theory with examples.
- What are the levels of cognitive domain according to Bloom's Taxonomy? Explain them briefly with suitable examples.
- What are the causes of curriculum change? List and describe them briefly.

OR

What are the new concepts introduced in curriculum as a result of new math movement? Explain briefly.

- List seven mathematical concepts/principles that can be taught easily to the students through moving model.

OR

List seven mathematical concepts/principles that can be taught easily to the students through tangram.

- Prepare a lesson plan to teach the concept mean and median.

..... Group "C"

2×12=24

- Which method do you prefer in teaching the sum of square of the first natural numbers?
- List some of the basic constructions through pencil compass and devise activities to let student construct them.

OR

Devise activities to teach basic formula

$$\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

**Group "A"**

14

Attempt All the questions. Tick (✓) the best answers.

- Which one of the following is the highest level of learning according to Gagne?
  - chain learning
  - multiple discrimination learning
  - rule learning
  - problem solving learning
- Which one of the following becomes a cross cutting issue throughout the educational process?
  - teaching
  - learning
  - supervision
  - testing



3. Generally standard test on intelligence test has a reliability more than  
 a. 0.60                      b. 0.70                      c. 0.80                      d. 0.90
4. Which one of the following is not the cause of failure of supervision?  
 a. when supervisor work without planning  
 b. when supervisor has not sufficient related training  
 c. when teachers are oriented about the objectives of supervision  
 d. when old trained teachers are not mould into the spirit of new curriculum
5. Which of the following number is used for "havoc" in FIAC Scoring Scale?  
 a. 8                              b. 7                              c. 9                              d. 10
6. Which one of the following falls under intrinsic motivation?  
 a. expectation of good grade                      b. anxiety of failure  
 c. competition                                      d. joy of discovery
7. Which one of the following is the main purpose of formative evaluation?  
 a. to improve teaching learning condition  
 b. to validate the level of student  
 c. to identify the weakness of students  
 d. all of the above

8. In  $(\sqrt[n]{x})^m$  which one is meant for the exponent?

a. x                              b. m                              c. n

d.  $\frac{m}{n}$

9. Which matrix shows the route map given below?

a. ABC

$$\begin{bmatrix} A & 1 & 1 & 1 \\ B & 0 & 1 & 1 \\ C & 0 & 0 & 1 \end{bmatrix}$$

b. ABC

$$\begin{bmatrix} A & 1 & 0 & 1 \\ B & 0 & 1 & 0 \\ C & 1 & 1 & 1 \end{bmatrix}$$

c. ABC

$$\begin{bmatrix} A & 0 & 1 & 1 \\ B & 1 & 0 & 1 \\ C & 1 & 1 & 0 \end{bmatrix}$$

d. ABC

$$\begin{bmatrix} A & 1 & 1 & 1 \\ B & 1 & 0 & 1 \\ C & 0 & 0 & 1 \end{bmatrix}$$

10. Which one of the following matrix is stretch in X?

a.  $\begin{bmatrix} K & 0 \\ 0 & 1 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 0 \\ 0 & K \end{bmatrix}$

c.  $\begin{bmatrix} 1 & K \\ 0 & 1 \end{bmatrix}$

d.  $\begin{bmatrix} -1 & 0 \\ 0 & k \end{bmatrix}$

11. Which one of the following is not a type of reflection?

a. point reflection                      b. line reflection

c. plane reflection

d. solid reflection

12. Which one of the following method is credited to Vygotsky?

a. problem solving method

b. guided discovery method

c. reciprocal teaching method

d. expository method

13. Which one of the following is the main purpose of maintenance programme in teaching mathematics?

a. to carry on review work

b. to plan drill

c. to prevent forgetting

d. to provide laboratory work

14. Which one of the following is not the criterion for selecting manipulative materials?

a. attractive

b. simplicity

c. expensive

d. durability

