Group 'B'

Unit 12: Statics

1. Find the resultant and the angle subtended by it with P when the forces P and Q act at right angle. 2[Q.N.12(a), 2072'C']

   $\left(\text{Ans: } \sqrt{P^2+Q^2},\ \tan^{-1}\frac{Q}{P}\right)$

2. Two mean carry a weight 50N supported by two strings; one string is inclined at 30° to the vertical and other at 60°, find the tension of each string. 4

   (Ans: 25N, $25\sqrt{3}$ N) [Q.N.13(a), 2072'C']

3. Show that the resultant of two equal forces bisects the angle between them. 2[Q.N.12(a), 2072'D']

4. State and prove Lami's theorem. 4 [Q.N.13(a), 2072'D']

5. A body of weight 68 N is suspended by two strings of length 8m and 15m respectively, and the other ends of the strings are attached to two fixed points in a horizontal line 17m apart, find the tensions of the strings. [Q.N.13(a)(Or), 2072'D']

   (Ans: 32N, 60N)

6. Two forces P and 2P acting at a point have the resultant $\sqrt{3}$ P. Find the angle between the two given forces. 2[Q.N.12(a), 2072'E']

   (Ans: 120°)

7. A body of weight 65N is suspended by two strings of lengths 5 and 12m attached to two points in the same horizontal line whose distance apart is 13m; find the tensions of the string. 4 [Q.N.13(a), 2072'E']

   (Ans: 25N, 60N)

8. Three forces acting on a particle are in equilibrium; The angles between the first and second is 90° and that between the second and third is 120°, find the ratios of the forces. 2 [Q.N. 12(a), Set 'C' 2071]

   (Ans: $\sqrt{3}$: 1 : 2)

9. The resultant of two forces p and Q acting at an angle α is equal to $(2m+1)\sqrt{P^2+Q^2}$. When they act at an angle $(90°-\alpha)$ the resultant is $(2m-1)\sqrt{P^2+Q^2}$.

   Prove that : $\tan\alpha = \frac{m-1}{m+1}$. 4 [Q.N. 13(a), Set 'C' 2071]

10. Forces equal to 7p, 5p and 8p acting on a particle are in equilibrium. Find the angle between latter pair of forces. 2 [Q.N. 12(a), Set 'D' 2071]

    (Ans: $\alpha = 120°$)

11. The resultant of two forces P and Q is equal is $\sqrt{3}$ Q and makes an angle of 30° with the direction of P; Show that P is either equal to Q or is double of Q. 4 [Q.N. 13(a), Set 'D' 2071]

12. State and prove Lami's theorem. 4 [Q.N. 13(a) OR, Set 'D' 2071]

13. At what angle do the forces equal to P + Q and P – Q act so that the resultant may be $\sqrt{P^2+Q^2}$? 2 [Q.N. 12(a), 2070 'C']

    $\left(\text{Ans: } \cos^{-1}\left\{\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}\right)$

14. Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2N respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and direction of the resultant. 4 [Q.N. 13(a), 2070 'C']

    [Ans: 10N, along the diagonal which is inclined at 60° with 2N.]

15. State and prove Lami's theorem. [Q.N. 13(a)(OR), 2070 'D']

16. Two forces acting at an angle of 45° have a resultant equal to $\sqrt{10}$ N; if one of the forces be $\sqrt{2}$ N, find the other force. 2 [Q.N. 12(a), 2070 'D']
(Ans: 2N)

17. A body of weight 65N is suspended by two strings of lengths 5 and 12 m attached to two points in the same horizontal line whose distance apart is 13m; find the tensions of the string.
(Ans: 60 kg wt, 25 kg wt.) 4 [Q.N. 13(a), 2070 'D']

18. Show that the resultant of two equal forces bisects the angle between them.
2 [Q.N. 12(a), Supp. 2069]

19. Two forces whose magnitudes are p and $p\sqrt{2}$ N act on a particle in direction inclined at an angle 135 ∫ to each other, find the magnitude and the direction of the resultant.
(Ans: PN at right angled with first component) [Q.N. 12(a), Set 'A' 2069]

20. The resultant of two forces p and Q acting at an angle α is equal to $(2m + 1)\sqrt{P^2 + Q^2}$ When they act at an angle $(90° - \alpha)$ the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$.

21. Prove that : $\tan\alpha = \frac{m-1}{m+1}$. 4 [Q.N. 13(a), Set 'A' 2069]

22. If a force P be resolved into two forces making angles 45° and 15° with its direction; show that the latter force is $\frac{\sqrt{6}}{3}$ p. [Q.N. 12(a), Set 'B' 2069]

23. A body of weight 65 N is suspended by two strings of lengths 5m and 12m attached to two points in the same horizontal line whose distance apart is 13m, find the tension of the strings. [Q.N. 13(a), Set 'B' 2069]
(Ans: 25 kg. wt.)

24. Find the resultant of two forces P and Q acting at a point when the angle between them is α. [Q.N. 13(a)(OR), Set 'B' 2069]

25. Find the resultant of two forces P and Q when the angle between them is α
[Q.N. 13(a), 2068]

$\left(\text{Ans: } R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha},\ \tan\theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

26. A body of weight 65 N is suspended by two strings of length 5m. and 12 m. attached to two points in the same horizontal line whose distance apart is 13 m; find the tension of the strings. [Q.N.13(a)(Or),2068]
(Ans: 60N, 25N)

27. Two forces acting at an angle of 45° have a resultant equal to $\sqrt{10}$ N; if one of the forces be $\sqrt{2}$N, find the other force. [Q.N. 5(a), 2068]
(Ans: 2N)

28. Find the resultant of two forces equal to 3N and 6N respectively such that their diagonal is perpendicular to the first force. [Q.N. 5(a), 2067]
(Ans: $3\sqrt{3}$ N ⊥r to the force 3N)

29. If a force P be resolved into two forces making angles of 45° and 15° with its direction; show that the latter force of $\sqrt{\frac{2}{3}}$ P. [Q.N. 13(a), 2067]

30. State and prove converse of the Triangle of forces. [Q.N. 13 (a (Or)), 2067]

31. The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P. Prove that Q = R. [Q.N. 5(a), 2066]

32. Forces 2, $\sqrt{3}$ 5, $\sqrt{3}$, 2 Newtons respectively act at one of the angular points of a regular hexagon towards the five other points. Find the magnitude and direction of the resultant. [Q.N. 13(a), 2066]

(Ans: 10N, along the diagonal which is inclined at 60° with 2N)

33. State and prove the theorem on triangle of forces for three forces acting at a point. **[Q.N. 13(a)Or, 2066]**

34. The sum of two forces is 18 and the resultant whose direction is perpendicular to the smaller of the two forces is 12, find the magnitude of the forces. **[Q.N.5(b), 2065]**

(Ans: 5 and 13)

35. Two forces P & Q acting parallel to the length and base of an inclined plane respectively, would each of them singly support a weight W on the plane, prove that : $\frac{1}{P^2}-\frac{1}{Q^2}=\frac{1}{W^2}$ **[Q.N.13(a), 2065]**

36. State and prove : $\lambda-\mu$ theorem. **[Q.N.13. (a), or), 2065]**

37. Forces equal to 7P, 5P and 8P acting on a particle are in equilibrium. Find the angle between the latter pair of forces. **[Q.N. 5(b), 2064]**

(Ans: 120°)

38. The resultant of two forces P & Q acting at an angle α is equal to $(2m+1)\sqrt{P^2+Q^2}$, when they act at an angle $90°-\alpha$, the resultant is $(2m-1)\sqrt{P^2+Q^2}$. Prove that $\tan\alpha=\frac{m-1}{m+1}$. **[Q.N. 13(a), 2064]**

39. State and prove Lami's theorem. **[Q.N. 13(a)Or, 2064]**

40. At what angle of forces equal to (P + Q) newton and (P – Q) newton act so that the resultant may be $\sqrt{P^2+Q^2}$ newton ? **[Q.N. 5(b), 2063]**

$\left(\text{Ans: } \cos^{-1}\left\{-\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}\right)$

41. Forces of 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2 newtons respectively act at one of the angular points of a regular hexagon towards the five other angular points. Find the magnitude and direction of the resultant. **[Q.N. 13(a), 2063]**

(Ans.: $5\sqrt{3}$ and 60°)

42. Find the resultant of two forces P and Q acting at a point. **[Q.N. 13(a) Or, 2063]**

$\left(\textit{Ans.: Magnitude} = \sqrt{P^2+Q^2+2PQ\cos\alpha} \textit{ and direction} = \frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$

43. Two forces whose magnitudes are P and $P\sqrt{2}$ act on a particle in directions inclined at an angle of 135° to each other; find the magnitude and direction of the resultant. **[Q.N. 6(b), 2062]**

(Ans.: P Newton, 90° with P)

44. State and prove converse of triangle of forces. **[Q.N. 13(a), 2062]**

45. If a force P be resolved into two forces making angles of 45° and 15° with its directions. Show that the latter force is $\frac{\sqrt{6}}{3}$ p. **[Q.N. 5(a), 2061]**

46. The resultant of two forces P & Q is equal to $\sqrt{3}$ Q and making an angle of 30° with the direction of P. Show that P is either equal to Q or is double of Q. **[Q.N. 13(a), 2061]**

47. A uniform plane lamina in the form of a rhombus, one of whose angle is 120°, is supported by two forces applied at the centre in the directions of the diagonals so that one side of the rhombus is horizontal; show that if P and Q be the forces and P be the greater then $P^2 = 3Q^2$. **[Q.N..13(a)Or, 2062]**

48. At what angle do forces equal to (P+Q) and (P–Q) act so that the resultant may be $\sqrt{P^2+Q^2}$? **[Q.N. 5(a), 2060]**

$\left(\text{Ans: } \cos^{-1}\left\{\frac{(P^2+Q^2)}{2(P^2-Q^2)}\right\}\right)$

49. Find the resultant of n number of coplanar forces acting at a point. [Q.N. 13(a), 2060]
50. The resultant of two forces P and Q is R. If Q is doubled, the new resultant is perpendicular to P, prove that R = Q. [Q.N. 13(a)Or, 2060]
51. The resultant of two forces P and Q is R. If Q is doubled the new resultant is perpendicular to P. Prove that Q = R. [Q.N. 5(a), 2059]
52. State 'Triangle of forces'. [Q.N. 5(c), 2059]
53. A uniform sphere of weight 3N rests in contact with a smooth vertical wall. It is supported by a string whose length equals the radius of the sphere joining a point on the surface of the sphere to a point of the wall. Calculate the tension in the string and the reaction of the wall. [Q.N. 13(b), 2059]

(Ans: $T = 2\sqrt{3}$ N and $R = \sqrt{3}$ N)

54. Two forces equal to 2 P and P respectively act on a particle. If the first be doubled and the second is increased by 12 N, the direction of the resultant is unaltered. Find the value of P. [Q.N. 13(b), Or, 2059]

(Ans: P = 12 N)

55. At what angle do forces equal to (P + Q) N. and (P – Q) N. act so that the resultant may be $\sqrt{P^2 + Q^2}$? [Q.N. 5(a), 2058]

$\left(\text{Ans: } \cos^{-1}\left\{\frac{(P^2 + Q^2)}{2(P^2 - Q^2)}\right\}\right)$

56. State and prove "Triangle of forces". [Q.N. 13(a), 2058]
57. Two forces P and Q act at a point. Their resultant R is at right angles to P. Show that $Q^2 - P^2 = R^2$ and the angle between the forces is $\cos^{-1}\left(-\frac{P}{Q}\right)$. [Q.N.13(a)Or,2058]
58. Write the expression for the magnitude and the direction of the resultant of two forces acting at a given angle. [Q.N. 5(a), 2057]

$\left(\text{Ans: } R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \text{ and } \theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$

59. State and prove iLami's Theoremi. [Q.N. 13(a), 2057]
60. The resultant of two forces P and Qacting at an angle α is $(2m+1)\sqrt{P^2 + Q^2}$ when they act at an angle $(90° - \alpha)$ the resultant is $(2m-1)\sqrt{P^2 + Q^2}$. Prove that $\tan \alpha = \frac{m-1}{m+1}$. [Q.N. 13(a) Or, 2057]

Unit 13: Statics (Continued)

13.1 Like and unlike parallel forces

1. Two like parallel forces P and Q act at points 18m apart, if the resultant force is 9N and acts at a distance 12m from Q, find P. 2[Q.N.12(b), 2072'C']

(Ans: 6N)

2. Find the resultant of two like parallel forces. 6[Q.N.15, 2072'E']
3. Find the resultant of two unlike parallel forces. A man carries a bundle at the end of a stick 75cm long which is placed on his shoulder. What should be the distance between his hand and shoulder, in order that the pressure on the shoulder may be three times the weight of the bundle?

(Ans: Resultant = P – Q, P > Q, 25 cm) 6 [Q.N. 15(OR), Set 'C' 2071]

4. Two unlike parallel forces, the greater of which is 75N, have a resultant 25N. Find the ratio of the distances of the resultant from the component forces.

[Ans: 2 : 3] 2 [Q.N. 12(b), Set 'D' 2071]

5. P and Q are like parallel forces If P is moved parallel to itself through a distance x, show that the resultant of P and Q moves a distance $\frac{P.x}{P+Q}$. 6 [Q.N. 15(Or), 2070 'C']

6. Find two like parallel forces acting at a distance of 2.5m apart, which are equivalent to a given force of 30N. The line of action of one being at a distance of 50cm from the given force. 2 [Q.N. 12(b), 2070 'D']
(Ans: 24N, 6N)

7. Two like parallel forces P and Q act at points 18 m apart. If their resultant force be 9N and acts at a distance 12m from Q, find P. 2 [Q.N. 12(c), Supp. 2069]
[Ans: 3N]

8. Forces equal to 3, 4, 5, 6 N respectively act along the sides of a square ABCD taken in order. Find the magnitude, direction and line of action of their resultant. 4 [Q.N. 13(a), Supp. 2069]
(Ans: $2\sqrt{2}$ N, 45° with force 5N, through E such that DE = $\frac{7a}{2}$, a = side of square.)

9. Find the resultant of two unlike parallel forces. [Q.N. 14(OR), Supp. 2069]

10. Find the resultant of two like parallel forces. [Q.N.. 15(OR), Set 'A' 2069]

11. Replace a force of magnitude 48 kgwt by two unlike parallel forces. one at a distance of 2m and other at 8m from the given force. [Q.N. 12(b), Set 'B' 2069]
(Ans: 64 kg wt, 16 kg wt.)

12. A straight uniform rod is 3m long. When a load of 10N is placed at one end it balances about a point 25 cms from that end. Find the weight of the rod.
(Ans: 2N) [Q.N. 12(b), 2069]

13. Find two unlike parallel forces acting at a distance of 12 cm which are equivalent to a force of 20N, the greater of the two forces being at a distance of 6cm from the given force. [Q.N. 6(a), 2068]
(Ans: 10N, 30N)

14. Find the resultant of two parallel forces of 15 kg wt. and 10 kg wt. acting at a distance 20 cm apart in the same direction. [Q.N. 6(a), 2067]
(Ans: 25 kg wt. at dist. of 8 km from 15 kg wt.)

15. Define parallel forces. Deduce the resultant of two like parallel forces. [Q.N.14(a),2067]

16. A straight weightless rod, 48 cms in length, rests in a horizontal position between two pegs placed at a distance of 6 cm apart, one peg being at one end of the rod, and a weight of 2 kg is suspended from the other end. Find the pressures on the pegs.
(Ans: 14 kg wt; 16 kg wt) [Q.N. 6(b), 2066]

17. A uniform bar 4m. long and weighing 3N passes over a prop and is supported in a horizontal position by a force of 1N acting vertically upwards at the other end. Find the distance of the prop from the centre of the bar. [Q.N.6(b), 2065]
(Ans: 1 m)

18. A straight uniform rod is 3 m long. When a load of 5N is placed at one end, it balances about a point 25 cm from that end. Find the weight of the rod.
(Ans: 1 N) [Q.N. 6(b), 2064]

19. Find the two unlike parallel forces acting at a distance of 12 cm which are equivalent to a force of 20 N, the greater of the two forces being at a distance of 6 cm from the given force. [Q.N. 6(b), 2063]
(Ans.: 10 N and 30 N)

20. Find the resultant of like parallel forces. [Q.N. 13(a)Or, 2061]

21. A straight uniform rod is 3m long. When a load of 5N is placed at one end it balances about a point 25 cm from that end. Find the weight of the rod.
(Ans: 1N) [Q.N. 6(a), 2061]

22. Forces equal to 3, 5, 3 and 5 newtons respectively act along the sides of a square taken in order, find their resultant. [Q.N. 6(b), 2061]
(Ans: Couple of moment 8a Nm, where a is side of the square)

23. Find two like parallel forces, acting at a distance of 2.5m apart, which are equivalent to a given force of 30N, the line of action of one being at a distance of 50 cm from the given force. [Q.N. 6(a), 2060]
(Ans: 24N and 6N)

24. Find the resultant of two like parallel forces acting on a rigid body. [Q.N. 13(a), 2059]

25. A straight uniform rod is 3m long, when a load of 5 N is placed at one end it balances about a point 25 cm from that end. Find the weight of the road.
(Ans: 1N) [Q.N. 6(b), 2058]

26. Replace a force of magnitude 50 kg wt by two like parallel forces one at a distance of 2 m and other at 8 m from the given force. [Q.N. 6(b), 2057]
(Ans: P = 40 kg wt and Q = 10 kg wt)

13.2 Moment of a Force

1. Define moment. State and prove Varignon's theorem. [Q.N.14(Or), 2072'C']

2. Three forces P, 2P and 3P act along the sides AB, BC and CA of an equilateral triangle of side a, find the magnitude, direction and line of action of the resultant. 6[Q.N.14, 2072'C']
(Ans: $P\sqrt{3}$, Perpendicular to BC, passes through a distance $\frac{a}{2}$ from C.)

3. A uniform beam AB is 16m long and weighs 50 kg weights of 20kg and 50 kg are suspended from A and B respectively. At what point must the beam be supported so that it may rest horizontally. 2[Q.N.12(b), 2072'D']
(Ans: At a distance of 10 m from A)

4. Define coplanar forces. Forces equal to P, 2P, 3P and 4P act along the sides of a square ABCD taken in order; find the magnitude, direction and the line of action of the resultant. 6[Q.N.14, 2072'D']
(Ans: $2\sqrt{2}P$, Parallel to AC cuts CD produced at a distance of $\frac{3a}{2}$ where 'a' is side of ABCD)

5. ABCD is a square; along AB, CB, AD and DC equal forces P act; show that the magnitude of their resultant is equal to double of any components and acts along DC. [Q.N.15(Or), 2072'E']

6. Three forces p, 2p and 3p act along the sides AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant.
(Ans: $P\sqrt{3}$, line of action passes through D at a distance $\frac{a}{2}$ from C, where a is the length of side of triangle) 6 [Q.N. 15, Set 'C' 2071]

7. Define moment of a force about a point,. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. 6 [Q.N. 15, Set 'D' 2071]

8. Define moment of a force about a point. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. 6 [Q.N. 15, 2070 'C']

9. Define moment of a force about a point. Prove that the algebraic sum of the moments of two like parallel forces about any point in their plane is equal to the moment of their resultant about the same point. 6 [Q.N. 15, 2070 'D']

10. ABCD is a square, along AB, CB, AD and DC equal forces P act. Show that the magnitude of their resultant is equal to double of any components and acts along DC. 6 [Q.N. 14, Supp. 2069]

11. ABC is an isosceles tringle whose angle A is 120 ∫ and forces of magnitudes, 1, 1 and $\sqrt{3}$ N act along AB, AC and BC; show that the resultant bisects BC and is parallel to one of the other side of the triangle. 6 **[Q.N. 15, Set 'A' 2069]**

12. Forces equal to 3p, 4p and 5p act along the sides AB, BC and CA of an equilateral triangle ABC, find the magnitude, direction and the line of action of the resultant. **[Q.N. 15, Set 'B' 2069]**

(Ans: $P\sqrt{3}$ ⊥r to BC resultant passes through D such that $CD = \frac{3}{2}BC$)

13. A light rod of length 72cm. has equal weights attached to it, one at 18cm. from one end and the other at 30cm. from the other end; if it be supported by two vertical strings attached to its ends and if the strings cannot support a tension greater than the weight of 50 kg, what is the greatest magnitude of the equal weight ? **[Q.N. 14(a), 2068]**

(Ans: $42\frac{6}{7}$ kg)

14. Three forces P, 2P and 3P act along the sides AB, BC and CA of an equilateral triangle ABC of side a. Find the magnitude, direction and line of action of the resultant. ***(Ans: $P\sqrt{3}$, 90° to BC and $\frac{a}{2}$ from C)*** **[Q.N. 14(a), 2066]**

15. Forces equal to 3P, 4P, 5P act along the sides AB, BC and CA of an equilateral triangle ABC, find the magnitude, direction and line of action of the resultant. **[Q.N.14(a), 2065]**

(Ans: $p\sqrt{3}$, 90°, passes through at a distance of $\frac{3}{2}$ BC from c)

16. A light rod of length 72 cms has equal weights attached to it, one at 18 cms from one end and the other at 30 cms from the other end. If it is supported by two vertical strings attached to its ends and if the string can not support a tension greater than the weight of 50 kg, what is the greatest magnitude of the equal weights ? **[Q.N. 14(a), 2064]**

(Ans: $42\frac{6}{7}$ kg)

17. Prove that the algebraic sum of the moments of any two like parallel forces about a point in their plane is equal to the moment of their resultant force about the same point. **[Q.N. 14(a), 2063]**

18. Define moment of a force about a point. Give the geometrical meaning of the moment of a force about a point. **[Q.N. 5(a), 2062]**

19. The wire passing round a telegraph pole is horizontal and the two portion attached to the pole are inclined at an angle 60° to one another. The pole is supported by a wire attached to the middle point of the pole and inclined at 60° to the horizon; show that the tension of this wire is $4\sqrt{3}$ times that of the telegraph wire. **[Q.N. 14(a), 2062]**

20. ABCD is a square along AB, CB, AD and DC equal forces, P act. Show that the magnitude of their resultant is equal to double of any component and acts along DC. **[Q.N. 14(a), 2061]**

21. Prove that the algebric sum of moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. **[Q.N.14(a), 2060]**

22. A uniform bar 4 m long and weighing 3 N passes over a prop and is supported in horizontal position by a force of 1 N vertically upwards at the other end. Find the distance of the prop from the centre of the bar.
(Ans: 1m) **[Q.N. 6(b), 2059]**

23. Prove that the algebraic sum of moments of two like parallel forces, about any point in their plane is equal to the moment of their resultant about the same point. **[Q.N.14(a),2058]**

24. Define a couple and the moment of a couple. Express the moment of a couple mathematically. [Q.N. 6(a), 2057]

(Ans: Magnitude of one of the forces x arm of the couple)

25. Prove that the algebraic sum of the moments of any two forces, meeting at a point, about any point in their plane is equal to the moment of their resultant about the same point. [Q.N. 14(a), 2057]

13.3 Couple

1. Define arm of a couple and the moment of a couple. [Q.N. 6(b), 2060]
2. Define a couple. What do you mean by arm of a couple ? [Q.N. 6(a), 2058]

Unit 14: Dynamics

14.1 Motion with Uniform Acceleration

1. If a, b, c be the spaces described by a particle during the p^{th}, q^{th}, r^{th} seconds of its motion respectively, prove that : $a(q-r) + b(r-p) + c(p-q) = 0$.
[6[Q.N. 14, Set 'C' 2071]

2. A railway train goes from one station to another moving during the first part of the journey with uniform acceleration a; when steam is shut off and the brakes are applied, it moves with retardation a^1. If S be the distance between the stations, show that the time, the train takes is $\sqrt{\frac{2s(a+a^1)}{aa^1}}$. 6 [Q.N. 14, 2070 'C']

3. If a, b, c be the spaces described by a particle during the p^{th}, q^{th}, r^{th} seconds of its motion respectively, prove that:
$a(q-r) + b(r-p) + c(p-q) = 0$ 4 [Q.N. 13(b), 2070 'D']

4. If a, b, c be the spaces described by a particle during the p^{th}, q^{th} and r^{th} second of its motion respectively, prove that:
$a(q-r)+b(r-p)+c(p-q) = 0$ [Q.N. 13(b), 2068]

5. A bullet fired into a target loses half its velocity after penetrating 6 cms. How much further will it penetrate ? [Q.N. 5(b), 2067]

(Ans: 2cm)

6. Prove that for a particle moving with uniform acceleration a in a straight line
$a = \frac{2\left(\frac{s^1}{t^1} - \frac{s}{t}\right)}{t + t^1}$, where s is the space described in t seconds and s^1 during the next t^1 seconds. [Q.N.13(b), 2065]

7. If a, b, c, are the spaces described by the particles during the pth, qth, rth seconds of its motion respectively, prove that :
$a(q-r) + b(r-p) + c(p-q) = 0$ [Q.N. 13(b), 2064]

8. A train moving with a velocity of 360 km/hr has the uniform acceleration 40 m/s^2. Obtain the distance covered by the train in $\frac{1}{2}$ minute. [Q.N. 5(a), 2063]

(Ans.: 18.3 km)

9. A railway train goes from one station to another moving during the first part of the journey with uniform acceleration f, when steam is shut off and the brakes are applied, it moves with retardation f^1. If 'a' be the distance between the stations, show that the time the train takes is :
$\sqrt{\frac{2a(f+f^1)}{ff^1}}$ [Q.N. 13(b), 2062]

10. A body moves for 3 seconds with a constant acceleration during which it describes 24.30 metres, the acceleration then ceases and during the next 3 seconds, it describes 21.60 metres. Find the initial velocity and the acceleration. ***(Ans: 9 m/s and –0.6 m/s²)*** **[Q.N. 13(b), 2060]**

11. Prove that for a particle moving with uniform acceleration f in a straight line $f = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t + t'}$ Where s is the space described in t secs. and s' during the next t' secs. **[Q.N. 14(a), 2059]**

12. If a, b, c be the space described by a particle during the p^{th}, q^{th} and r^{th} seconds of its motion respectively, prove that $a(q - r) + b(r - p) + c(p - q) = 0$. **[Q.N. 13 (b), 2057]**

14.2 Motion Under Gravity

1. A ball is thrown vertically upwards at a rate of 40ms⁻². Find the time taken to attain the maximum height. ($g = 10ms^{-2}$) **2[Q.N.12(c), 2072'C']**
(Ans: 4 sec)

2. A body is projected vertically upward with velocity u and t seconds afterwards another body is projected similarly with the same velocity. Show that they meet a height $\frac{4u^2 - g^2t^2}{8g}$ from the point of projection after $\left(\frac{u}{g} - \frac{t}{2}\right)$ secs form the instant of projection of the second body. **6[Q.N.14, 2072'E']**

3. A ball is thrown vertically upwards with a velocity of 30m/s. Find the time taken by the ball to reach the ground again. ($g = 10m/s^2$).
[Ans: 6 sec] **2[Q.N. 12(c), Set 'D' 2071]**

4. A ball is projected vertically upwards with a velocity of 40m/s. Find its velocity and position at the end of 3s. ($g = 10m/s^2$) **2 [Q.N. 12(c), 2070 'D']**
(Ans: 10 m/s, 75 m)

5. If a ball is thrown vertically upwards at a rate of 20ms⁻¹, find the time taken to attain the maximum height. ($g = 10\ ms^{-2}$) **2 [Q.N. 12(b), Supp. 2069]**
[Ans: 2 sec]

6. A body is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4m. Find the velocity of the stone with which it is projected. ($g = 9.8m/s^2$) **[Q.N. 12(c), Set 'A' 2069]**
(Ans: 39.2 m/sec)

7. A ball thrown up vertically return to the thrower after 6 secs. Find the velocity with which it was thrown up. **[Q.N. 12(c), Set 'B' 2069]**
(Ans: 30 m/sec)

8. A stone is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4cm. Find the velocity of the stone with which it is projected. ($g = 9.8m/s^2$)
(Ans: 39.2 m/sec) **[Q.N. 6(b), 2068]**

9. A stone is dropped from the top of tower 200 m high and at the same time another is projected vertically upwards from the ground with a velocity of 50 ms⁻¹. Find where and when the will meet ($g = 9.8\ ms^{-2}$) **[Q.N. 13(b), 2067]**
(Ans: at 121.6 m from ground, 4 secs.)

10. A stone is dropped form a balloon at a height 116.4 m above the ground and it reaches the ground in 6 secs. Find the velocity with which the balloon was rising.
(Ans: 0.109 m/sec) **[Q.N. 5(b), 2066]**

11. A ball thrown up vertically returns to the thrower after 6 seconds. Find the velocity with which it was thrown up. ($g = 10\ m/s^2$) **[Q.N. 5(a), 2064]**
(Ans: 30 m/sec)

12. A stone is dropped from the top of a tower 200 m. high and at the same time another is projected vertically upwards from the ground with a velocity of 50 m/s. Find where and when the two will meet ? ($g = 9.8\ m/s^2$). [Q.N. 13(b), 2063]
(Ans.: Two stones meet at a height of 121.6 m from the ground after 4 sec.)

13. A stone is projected vertically upwards from the foot of the tower with a velocity just sufficient to carry it to 78.4 *m*. Find the velocity of the stone with which it is projected. ($g = 9.8 m/s^2$) [Q.N. 5(b), 2062]
(Ans.: 39.2 m/sec)

14. A body falls from rest from the top of a tower and during the last second it falls $\frac{16}{25}$ th of the whole height. Find the height of the tower. ($g = 10 m/sec^2$)
(Ans: 31.5 m) [Q.N. 13 (b), 2061]

15. A body is projected vertically with a velocity of 9.8 ms^{-1}, how long it takes to return to the point of projection ? ($g = 9.8\ ms^{-2}$) [Q.N. 5(b), 2058]
(Ans: 2secs)

16. A body falls from rest from the top of a tower and during the last second it falls $\frac{16^{th}}{25}$ of the whole height. Find the height of the tower ($g = 10 ms^{-2}$).
(Ans: 31.5 m) [Q.N. 13(b), 2058]

14.3 Motion Down a Smooth Inclined Plane

1. A particle slides down a smooth inclined plane 10m long and acquires a velocity $10\sqrt{2}\ ms^{-1}$. Find the inclination of the plane. ($g = 10 ms^{-2}$) 2[Q.N.12(c), 2072'D']
(Ans: 90°)

2. A ball is projected up a smooth plan with velocity 25m/s. If the inclination of the plane to the horizon be 30°, find the velocity of the ball when it travels a distance of 22.5m. ($g = 10 m/s^2$) 2[Q.N.12(b), 2072'E']
(Ans: 20 m/sec)

3. A ball is thrown up an inclined place with a velocity of 14.7 m/s. Where will the velocity of the ball be 4.9m/s? Assume that the inclination of the plane to the horizon is 30°. ($g = 9.8 m/s21^2$) 2 [Q.N. 12(c), Set 'C' 2071]
(Ans: t = 1 sec)

4. A ball is projected up a smooth inclined plane with velocity 25m/s. If the inclination of the plane to the horizon be 30°, find the velocity of the ball when it travels a distance of 22.5m. [$g = 10 m/s^2$] 2 [Q.N. 12(c), 2070 'C']
[Ans: 20 m/s]

5. A particle slides down from rest from the top of a smooth plane of height 1962 cms and inclination 30° with the horizon. Divide the plane into three parts so that a particle at the top of the plane may describe each part in equal times.
($g = 981 cm/sec^2$). [Q.N. 13(b), 2066]
(Ans: 436 cm, 1308 cm, 22180 cm)

6. A particle slides down an inclined plane 20m. long and acquires a velocity of $10\sqrt{2}$ m/sec. Find the inclination of the plane. ($g = 10 m/sec^2$). [Q.N.5(a), 2065]
(Ans: 30°)

7. A particle slides down an inclined plane 30 m. long and acquires a velocity of $\sqrt{300\sqrt{3}}\ ms^{-1}$. Find the inclination of the plane. ($g = 10 ms^{-2}$). [Q.N. 5(b), 2059]
(Ans: 60°)

Unit 15: Dynamics (Continued)

15.1 Newton's Laws of Motion

1. State laws of motion. A body of mass 50kg falling from a certain height is brought to rest after striking the ground with a speed of 5ms^{-1}. If the resistance force of ground is 500N, find the duration of the contact. **(Ans: 0.5 sec)** **4[Q.N.13(b), 2072'C']**
2. State laws of motion. Using Network's Law to define an absolute unit of force. **4[Q.N.13(b), 2072'D']**
3. State Newton's laws of motion. Prove that Newton's second law provides the measurement of the force. **4[Q.N.13(b), 2072'E']**
4. A body of mass 50kg is falling from a certain height is brought to rest after striking the ground with a speed of 5m/s. If the resistance force of the ground is 500N, find the duration of contact. **(Ans: t = 0.5 sec))** **2 [Q.N. 12(b), Set 'C' 2071]**
5. A gun of mass 400kg fires a shot of mass 3kg, with a velocity of 200m/s, find the constant force which acting on the gun would stop it after a recoil of 2.5 meters. **[Ans: 180 N]** **4[Q.N. 13(b), Set 'D' 2071]**
6. A cart is pushed on a frictionless smooth plane with an average force of 20N for 5 seconds. If the cart with mass 50kg is at rest in the beginning, find the velocity acquired by the cart. **2 [Q.N. 12(b), 2070 'C']**
[Ans: 2m/s]
7. A gun of mass 400kg fires a shot of mass 3kg with a velocity of 200m/sec, find the constant force which acting on the Gun would stop it after recoil of 2.5 m. **[Ans: 1600 N]** **[Q.N. 13(b) (OR), Supp. 2069]**
8. A mass of 5kg falls 300 cm from rest and is then brought to rest by penetrating 30 cm into some sand, find the average thrust of the sand. ***(Ans: 539 N upward)*** **[Q.N. 13(b), Set 'A' 2069]**
9. State Newton's laws of motion. Show that Newton's second law gives the measurement of a forces. **[Q.N. 13(b), Set 'B' 2069]**
10. A constant force of 10N acting on an object reduces its velocity from 15m/s to 5m/s in 2 seconds. Find the mass of the object. **[Q.N. 5(b), 2068]**
[Ans: 2 kg)
11. State Newton's second law of motion hence define a force. **[Q.N. 6(b), 2067]**
12. A body of mass 1 kg is falling under gravity at the rate of 28 ms^{-1}. What uniform force will stop it in 0.1 second? (g = 9.8 ms^{-2})
(Ans: 290N) **[Q.N. 6(a), 2066]**
13. The pull of the earth on a body is 49N. If the acceleration due to gravity is g = 9.8 m/sec^2. Find the mass of the body. **[Q.N.6(a), 2065]**
(Ans: 5 kg)
14. Show that Newton's second law of motion gives the measurement of a force.
[Q.N. 6(a), 2064]
15. State Newton's laws of motion. Show that Newton's second law of motion gives the measurement of a force. **[Q.N. 14(b)Or, 2063]**
16. A constant force of 10N acting on an object reduces its velocity from 15ms^{-1} to 5ms^{-1} in 2 seconds. Find the mass of the object. **[Q.N. 6(a), 2062]**
(Ans.: 2 kg)
17. A body of mass 1 kg is falling under gravity at the rate of 28 ms^{-1}. What is the uniform force that will stop it in (i) 0.1 sec (ii) 20 cm (g = 10 ms^{-2}).
Instead of falling under gravity if the body is moving at the rate of 28 ms^{-1} along a horizontal line, what will be the force required in above two cases ?
(Ans: (i) 290 N (ii) 80 N and (i) 280 N (ii) 70 N) **[Q.N. 14(b), 2059]**

15.2 Projectiles

1. If R be the horizontal range of a projectile and h its greatest height, prove that its initial velocity is $\sqrt{2g\left(h+\frac{R^2}{16h}\right)}$. 6[Q.N.15, 2072'C']

2. Describe motion of a projectile. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower and also find the striking velocity. [Q.N.15(Or), 2072'D']
(Ans: At a distance of 2h from the foot of tower, $2\sqrt{gh}$)

3. With what velocity must a body be projected at an angle of 45° from the top of a tower 65m high, if it is to reach a point on the ground 180m from the base of the tower. **(Ans: 25.71 m/sec)** [Q.N.13(b)(Or), 2072'E']

4. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity?
(Ans: $2\sqrt{gh}$) 4[Q.N. 13(b), Set 'C' 2071]

5. A cannon ball has the same range R on a horizontal plane for two different angles of projection. If H and H' are the greatest heights in two paths for which this is possible, prove that : $R^2=16HH'$. [Q.N. 14(OR), Set 'D' 2071]

6. Find the velocity and the direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g = 9.8m/s^2$) 4
[Ans: 70cm/s, 45° with horizon.] [Q.N. 13(b), 2070 'C']

7. A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60m in front of the point of projection. Find the velocity of projection. ($g = 10m/s^2$) 6 [Q.N. 14, 2070 'D']
(Ans: 25 m/s)

8. If R be the horizontal range of a projectile and h its greatest height, prove that its initial velocity is $\sqrt{2g(h+\frac{R^2}{16h})}$. 6 [Q.N. 15, Supp. 2069]

9. The horizontal and the vertical components of the initial velocity of a projectile are U and V. If R be the range and H, the greatest height attained, prove that:
(a) $\frac{4H}{R}=\frac{V}{U}$ (b) $\left(\frac{R}{U}\right)^2=\frac{8H}{g}$ [Q.N. 14, Set 'A' 2069]

10. If R be the horizontal range of a projectile and h is greatest height, prove that its initial velocity is $\sqrt{2g(h+\frac{R^2}{16h})}$. [Q.N. 14, Set 'B' 2069]

11. The horizontal and vertical components of the initial velocity of a projectile are U and V respectively. If R be the range and the H the greatest height attained, prove that:
$\frac{4H}{R}=\frac{V}{U}$ [Q.N. 14(b), 2068]

12. Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall which is 250 m off and 125 m high. ($g = 9.8\ ms^{-2}$) **(Ans: 70 m/sec, 45°)** [Q.N. 14(b), 2067]

13. Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall 250 m off and 125 m high ($g = 9.8\ ms^{-2}$).
(Ans: 70 cm/sec, 45°) [Q.N. 14(b), 2066]

14. If R be the horizontal range and T the time of flight of a projectile, show that $\tan\alpha=\frac{gT^2}{2R}$, where α is the angle of projection. [Q.N.14(b), 2065]

15. A stone is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. Find where it will strike the level ground through the foot of the tower. What will be its striking velocity ? [Q.N. 14(b), 2064]

$\left(Ans: 2h, 2\sqrt{gh}\right)$

16. A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60 m. in front of the point of projection : Find the velocity of projection. ($g = 10\ m/s^2$) [Q.N. 14(b), 2063]

(Ans.: 25 m/sec.)

17. If R be the horizontal range and T be the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$ where α is the angle of projection. [Q.N. 14(b)Or, 2062]

18. If R be the horizontal range of a projectile and h its greatest height. Prove that its initial velocity is $\sqrt{2g\left(h + \frac{R^2}{16h}\right)}$ [Q.N. 14(b), 2061]

19. If u and α be the velocity and angle of projection of a projectile, then find the time of flight. [Q.N. 6(a), 2059]

$\left(Ans: \frac{2usin\alpha}{g}\right)$

20. A particle is projected with a velocity u. If the greatest height attained by the particle be H, prove that the range R on the horizontal plane through the point of projection is $R = 4\sqrt{H\left(\frac{u^2}{2g} - H\right)}$. [Q.N. 14(b), Or, 2059]

21. If R be the horizontal range and T, the time of flight of a projectile, show that $\tan\alpha = \frac{gT^2}{2R}$, where α is the angle of projection. [Q.N. 14(b), 2058]

22. If R be the horizontal range of a projectile and h is greatest height, prove that its initial velocity is $\sqrt{2g\left(h + \frac{R^2}{16h}\right)}$ [Q.N. 14(b), 2057]

15.3 Work, Energy and Power

1. State the principle of conservation of energy. Also prove that the sum of the kinetic and potential energies of a moving body remains constant throughout the motion. [Q.N.13(b)(Or), 2072'C']

2. Define energy. State principle of conservation of energy. Also prove that the sum of the kinetic and potential energy of a moving body remains constant throughout the motion. 6[Q.N.15, 2072'D']

3. Calculate the power of a pump which can lift 300 kg of water through a vertical height of 4m in 10 sec. ($g = 10 m/s^2$) 2[Q.N.12(c), 2072'E']

(Ans: 1200 W)

4. Find the H.P. of an engine which can travel at the rate of 144km/hr up an incline of 1 in 200, the mass of the engine and load being 15 metric tons and the resistance due to friction etc. being 15kg weight per metric ton. ($g = 10 m/sec^2$)

(Ans: 160.86 HP) [Q.N. 13(b (Or)), Set 'C' 2071]

5. Define potential energy and kinetic energy of a body. Prove that the K.E. and P.E. of a freely falling body at any instant is constant. 6[Q.N. 14, Set 'D' 2071]

6. A bullet passes through two planks in succession whose initial velocity is 1200m/s and loses a velocity of 200m/s in penetrating each plank. Find the ratio of the thickness of the planks. Assuming that they offer the same average resistance.

[Ans: 11:9] [Q.N. 13(a)(OR), 2070 'C']

7. A bullet of mass 20g is fired horizontally into a suspended stationary wooden block of mass 380 g with a velocity of 200m/s. What is the common velocity of the bullet and the block if the bullet is embedded into the block? Find the loss of K.E. by the impact. ($g = 10m/s^2$) **[Q.N. 14(Or), 2070 'D']**
(Ans: 10 m/s, 380 J)

8. A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $\sqrt{\frac{2mE}{M(M+m)}}$. 4
[Q.N. 13(b), Supp. 2069]

9. Define work done by a force. Prove that the change in kinetic energy of a body is equal to the work done by the force. **[Q.N. 13(c), Set 'A' 2069]**

10. A car of mass 2000 kg moves up an inclined plane at an angle 30 ∫ to the horizon at a constant speed of 20m/s. If the frictional force is 2000N, calculate the power developed by the engine ($g = 10m/s^2$) **[Q.N. 14(Or), Set 'B' 2069]**
(Ans: 240 kW)

11. If a force be applied on the body, prove that the change in kinetic energy of a body is equal to the workdone by the force. **[Q.N. 14(b) (Or), 2068]**

12. A bullet of mass 200 gm is fired into a target with a velocity of 500 ms^{-1}. If the man of the target is 4.8 kg and in free to move, find the loss of kinetic energy by the impact.
(Ans: 24000 Joules) **[Q.N. 14(b((Or)), 2067]**

13. A bullet loses $\frac{1}{20}$ th of its velocity in passing through a plank. Find how many such uniform planks it would pass through before coming to rest assumning the retardation to be uniform. **[Q.N. 14(b)Or, 2066]**
(Ans: 11 approx.)

14. State and Prove the Principle of Conservation of Energy. **[Q.N.14(b)or, 2065]**

15. If a force acts on a body, prove that the change in kinetic energy of a body is equal to the work done by the force. **[Q.N. 14(b)Or, 2064]**

16. A pump having a power of 294 w pumps water at the rate of 90 liters per minute. Find the height to which the water is raised. ($g = 9.8 m/s^2$, 1 litre of water = 1 kg)
(Ans.: 20 m) **[Q.N. 6(a), 2063]**

17. An engine pumps 746 liters of water per minute from a well through an average height of 60m. Find the horse power of the engine if 50% of the power is wasted.
(1 liter of water = 1 kg., $g = 10 m/s^2$) **[Q.N. 14(b), 2062]**
(Ans.: 20 H.P.)

18. A car of mass 1000 kg. moves up an incline of 30° at a constant speed of 20 m/sec. If the frictional force is 2000N, calculate the power developed by the engine. ($g = 10m/sec^2$) **[Q.N. 5(b), 2061]**
(Ans: 140kw)

19. State the principle of conservation of energy. Illustrate it with the consideration of a body sliding down a smooth inclined plane. **[Q.N. 14(b) Or, 2061]**

20. A car covers a distance of 50m in 5 secs against a frictional force. If the power of the engine is 4000 watts, find the frictional force. **[Q.N. 5(b), 2060]**
(Ans: 400N)

21. From a point on the ground at a distance 'x' from the foot of a vertical wall, a ball is thrown at an angle of 45° which just clears the top of the wall and afterwards strikes the ground at a distance 'y' on the other side. Prove that the height of the wall is $\frac{xy}{x+y}$.
[Q.N. 14(b), 2060]

22. A shot whose mass is 40kg is discharged from a 700kg gun with a velocity of 140ms^{-1}. Find the constant force which acts on the gun would stop it after a recoil of 6.4m. [Q.N. 14(b)Or, 2060]
(Ans: 3500 N)

23. A car is moving at 36 kmh^{-1}. What velocity will double its kinetic energy? [Q.N. 6(c), 2059]
(Ans: $10\sqrt{2}$ ms^{-1})

24. "The change is kinetic energy of a body is equal to the work done by the acting force". Prove this statement. [Q.N. 14(b)Or, 2058]

25. Calculate the power of a pump which can lift 300 kgs of water through a vertical height of 4 m in 10 secs. [g = 10m s^{-2}] [Q.N. 5(b), 2057]
(Ans: 1200 watts)

26. Define work, power and energy. Prove that the sum of the kinetic and potential energies of a freely falling body remains constant throughout the motion. [Q.N. 14(b)Or, 2057]

Group 'C'

Unit 16: Linear Programming

1. Shade the feasible region for the constraints $x + 2y \leq 7$, $x, y \geq 0$. [Q.N.16(a), 2072'C']
2. Solve by Simplex method, the LP problem to maximize $z = 7x + 5y$ subject to $x + 2y \leq 6$, $4x + 3y \leq 12$, $x, y, \geq 0$. 6[Q.N.18, 2072'C']
(Ans: Max Z = 21 at (3, 0))
3. Draw the graph of the inequality : $3x - 3 \leq 5x - y$. 2 [Q.N.16(a), 2072'D']
4. Using Simplex method, find the optimal solution of $z = 7x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12, x_1, x_2 \geq 0$. 6[Q.N.18, 2072'D']
(Ans: Max. value of 21 at (3, 0))
5. Determine the feasible region bounded by the following system of inequalities: $x + y \leq 6$, $2x + y \geq 8$, $y \geq 0$. 2 [Q.N.16(a), 2072'E']
6. Using Simplex method, Maximize $F = 5x - 3y$, subject to $3x + 2y \leq 6$, $-x + 3y \geq -4$, $x, y \geq 0$ 6[Q.N.18, 2072'E']
(Ans: Max F = 10 at (2, 0))
7. Determine the feasible region of the following system of inequalities: $2x + y \leq 8$, $x + 2y \leq 10$, $x, y \geq 0$. 2 [Q.N. 16(a), Set 'C' 2071]
8. Using simplex method, maximize $f = 15x_1 + 10x_2$ Subject to $2x_1 + x_2 \leq 10$, $x_1 + 3x_2 \leq 10$, $x_1, x_2 \geq 0$. 6 [Q.N. 18, Set 'C' 2071]
[Ans: Max. f =80 at $x_1 = 4$ and $x_2 = 2$]
9. Determine graphically the feasible region determined by the following inequalities: $3x + 4y \leq 24$, $x \geq 2$, $x \geq 1$. 2[Q.N. 16(a), Set 'D' 2071]
10. Using simplex method, maximize $U = 25x + 45y$ Subject to $x + 3y \leq 21$, $2x + 3y \leq 24$, $x, y \geq 0$. 6 [Q.N. 18, Set 'D' 2071]
[Ans: Max U = 345, When x = 3, y = 6]
11. Draw the graph of the following inequalities: $3x + 4y \leq 24$, $0 \leq y \leq 4$, $0 \leq x \leq 7$. Shade the feasible region. 2 [Q.N. 16(a), 2070 'C']

12. Using simplex method, Max. $z = 5x_1 + 7x_2$ **6 [Q.N. 18, 2070 'C']**

13. Draw the graph of the following inequalities: **2 [Q.N. 16(a), 2070 'D']**
$x + y \leq 6,\ 2x + y \leq 8,\ y \leq 0.$
Shade the feasible region.

14. Using simplex method, **6 [Q.N. 18, 2070 'D']**
Max. $P = 50x_1 + 80x_2$
Subject to $x_1 + 2x_2 \leq 32,\ 3x_1 + 4x_2 \leq 84,\ x_1, x_2 \geq 0.$
(Ans: Max P = 1480 at (20, 6)

15. Shade the feasible region of the constraints. **2 [Q.N. 16(a), Supp. 2069]**
$2x + y \leq 40,\ x + 2y \leq 50,\ x \geq 0,\ y \geq 0$

16. Use Simplex method to maximize $Z = 7x_1 + 5x_2$ subject to
$x_1 + 2x_2 \leq 6, 4x_1 + 3x_2 \leq 12, x_1 \geq 0, x_2 \geq 0.$
[Ans: $\max_Z = 21$ at (3, 0)] **6 [Q.N. 18, Supp. 2069]**

17. Using simplex method, **[Q.N. 18, Set 'A' 2069]**
maximize $Z = 7x_1 + 5x_2$
Subject to
$x_1 + 2x_2 \leq 6$
$4x_1 + 3x_2 \leq 12$
$x_1, x_2 \geq 0$
(Ans: 21 when $x_1 = 3, x_2 = 0$))

18. Using simplex method, Maximize $Z = 7x_1 + 5x_2$ **6 [Q.N. 18, Set 'B' 2069]**
Subject to:
$x_1 + 2x_2 \leq 6, \quad 4x_1 + 3x_2 \leq 6, \quad x_1, x_2 \geq 0$
(Ans: $\text{Max}_Z = 10.5$, at $x_1 = 4.5, x_2 = 0$))

19. Shade the feasible region determined by the following inequalities:
$3x + 2y \leq 12,\ x + y \leq 5,\ x, y \geq 0$ **[Q.N. 16(a), Set 'A' 2069]**

20. Shade the feasible region determined by the inequalities: **[Q.N. 16(a), Set 'B' 2069]**
$x + 2y \leq 10,\ x + y \leq 6,\ x, y \geq 0.$

Unit 17: Computational Method

1. Convert the decimal number 3159 into hexadecimal form. **2[Q.N.16(b), 2072'C']**
(Ans: $C57_{16}$)

2. Apply the method of bisection to find the root of the equation $x^3 - 2x - 5 = 0$ in (2, 3) correct to three places of decimal. **6[Q.N.19, 2072'C']**
(Ans: 2.094)

3. Convert hexadecimal number $10A_{16}$ into binary form. **2[Q.N.16(b), 2072'D']**
(Ans: 011100001010_2)

4. Using the bisection method find the root of the equation $x^2 + x - 4 = 0$ in (1, 2) correct to two place of decimals. **4[Q.N.17(b), 2072'D']**
(Ans: 1.56)

5. Convert the decimal numeral 1503 into hexadecimal form.
(Ans: 5D F_{16}) **2[Q.N.16(b), 2072'E']**

6. Using bisection method, find the root of the equation: **6[Q.N.19, 2072'E']**
$2x^3 - 5x + 2 = 0, x \in (1, 2)$ with error less than 10^{-2}.
(Ans: 1.31641)

7. Find a root of the equation $2x^2 - 3x - 1 = 0, x \in (1, 2)$ using Newton Raphson method with error less than 10^{-4}. **[Q.N.19(Or), 2072'E']**
(Ans: 1.780776406)

8. Convert the decimal number 2567_{10} to octal form. 2 [Q.N. 16(b), Set 'C' 2071]
(Ans: 5007_8)

9. Find a root of an equation $x^3 + x - 4 = 0$ in the interval [1, 4] within an accuracy of 10^{-1}. [Q.N. 19, Set 'C' 2071]
[Ans: 1.375]

10. Find a root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to three places of decimal by Newton-Raphson method. [Q.N. 19(OR), Set 'C' 2071]
[Ans: 1.796]

11. Convert the hexadecimal number $AB5_{16}$ to the decimal number. 2 [Q.N. 16(b), Set 'D' 2071]
[Ans: 2741_{10}]

12. Using the bisection method, find a root of the equation: $f(x) = 2x^3 - 5x + 2 = 0$, between 1 and 2 with error less than 10^{-2}. 6 [Q.N. 19, Set 'D' 2071]
[Ans: 1.31641]

13. Derive the formula for Newton-Raphson method. Using Newton Raphson method, find a positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to three places of decimals. [Q.N. 19(OR), Set 'D' 2071]
[Ans: 1.154]

14. Using Newton–Raphson method, find the positive root of $x^3 - 18 = 0$ in (2, 3) [Q.N. 19(OR), 2070 'D']
(Ans: 2.62)

15. Convert the decimal number 3058 to hexadecimal form. 2 [Q.N. 16(b), 2070 'D']
(Ans: $BF2_{16}$)

16. Applying the method of successive bisection, find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to 2 places of decimals.
(Ans: 1.86) 6 [Q.N. 19, 2070 'D']

17. Solve $2x^2 - 3x - 1 = 0$ using Newton–Raphson method taking $x_0 = 1$ with error less than 10^{-4}. 6[Q.N. 19(OR), 2070 'C']
[Ans: 1.780776406]

18. Convert the decimal numeral 1503 to hexadecimal form. 2 [Q.N. 16(b), 2070 'C']
[Ans: $5DF_{16}$]

19. Find the root of the equation $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to three places of decimals by successive bisection method.
[Ans: 2.094] 6 [Q.N. 19, 2070 'C']

20. Convert decimal number 687 into binary system. 2 [Q.N. 16(b), Supp. 2069]
[Ans: 1010101111_2]

21. Show that the equation $f(x) = x^3 - x - 4 = 0$ has only one positive root and find the positive root correct to 3 decimal places using bisection method
(Ans: 1.796) 6 [Q.N. 19, Supp. 2069]

22. Convert the octal numeral 3733_8 into decimal form.
(Ans: 2011_8) [Q.N. 16(b), Set 'A' 2069]

23. Using method of bisecion, find the root of the equation $x^3 - x - 4 = 0$ lying between 1 and 2 correct to 3 places of decimals. [Q.N. 19, Set 'A' 2069]
(Ans: 1.796)

24. Using Newton–Raphson's method, find the square root of 153 correct to 3 places of decimals. [Q.N. 19(OR), Set 'A' 2069]
(Ans: 12.369)

25. Using Newton Raphson's method find the positive root of the equation $f(x) = x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to 3 places of decimals.
(Ans: 2.094) [Q.N. 19(OR), Set 'B' 2069]

26. Convert the hexadecimal numeral 2E4B into decimal form.
(Ans: 11851_{10}) [Q.N. 16(b), Set 'B' 2069]

27. Show that the equation $f(x) = x^3 - x - 4$ has one positive root and using the method of bisection, find the positive root correct to 3 places of decimals.
(Ans: 1.796) [Q.N. 19, Set 'B' 2069]

28. Using Newton–Raphson method, find the positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to 3 places of decimals. [Q.N. 19(OR), Supp. 2069]
(Ans: 1.154)

Unit 18: Computational Method (Continued)

1. Write the conditions for the system of equations
$a_{11}x + a_{12}y = b_1$, $a_{21}x + a_{22}y = b_2$, to be ill conditioned.
(Ans: $a_{11}a_{22} - a_{21}a_{12} = 0$) 2[Q.N.16(c), 2072'C']

2. Using Gauss Seidel method, solve the equations
$3x + 2y = -9$, $2x - 3y = -6$. 4[Q.N.17(a), 2072'C']
(Ans: – 3, 0)

3. Solve by Gauss elimination method:
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$ [Q.N.17(b)(Or), 2072'C']
(Ans: – 15, 8, 2)

4. Test whether the system of equations
$12x + 3y - 5z = 1$, $x + 5y + 3z = 28$ and $3x + 7y + 13z = 1$
is diagonally consistent? 2 [Q.N.16(c), 2072'D']
(Ans: diagonally consistent)

5. Using Gauss Seidel method, solve:
$3x + 4y + 8z = 7$, $x + 20y + z = -18$, $25x + y - 5z = 19$ 4[Q.N.17(a), 2072'D']
(Ans: 1, – 1, 1)

6. Use Gauss elimination method to solve:
$4x - y + z = 8$, $2x = 5y + 2z = 3$, $x + 2y + 4z = 11$ [Q.N.17(b)(Or), 2072'D']
(Ans: 1, – 1, 3)

7. Using Gauss-elimination method, Solve the following system of equations.
$2x_2 + 3x_3 = 7$, $3x_1 - 2x_2 + 2x_3 = 1$, $2x_1 + 3x_2 - 3x_3 = 5$. 4[Q.N.17(a), 2072'E']
(Ans: 1, 2, 1)

8. Solve the following equation using matrix inversion method:
$3x + y + z = 15$, $x + y + z = 3$, $y - z = -1$ [Q.N.17(a)(Or), 2072'E']
(Ans: 6, – 2, – 1)

9. Using Gauss-elimination method, solve the following system of equation:
$x + 3y - z = -2$, $3x + 2y - z = 3$, $-6x - 4y - 2z = 18$.
(Ans: x = 1, y = – 3, z = – 6) 4 [Q.N. 17(a), Set 'C' 2071]

10. Using inverse matrix method, solve the following system of equations:
$3x + y + z = 15$, $x + y + z = 3$, $y - z = -1$.
(Ans: x = 6, y = – 2, z = – 1) 4 [Q.N. 17(a) (Or), Set 'C' 2071]

11. Solve the following system of equations using inverse matrix method:
$x_1 - 2x_2 - x_3 = 1$, $x_1 - x_2 + 2x_3 = 9$, $2x_1 - 3x_2 - x_3 = 4$
[Ans: $x_1 = 2$, $x_2 = -1$, $x_3 = 3$] 4 [Q.N. 17(a) (Or), Set 'D' 2071]

12. Using Gauss-elimination method, solve the following system of equations:
$x - 2y + 3z = 2$, $2x - 3y + z = 1$, $3x - y + 2z = 9$.
[Ans: x = 3, y = 2, z = 1] 4 [Q.N. 17(a), Set 'D' 2071]

13. Using Simpson's $\frac{1}{3}$ rule, evaluate:
$\int_0^1 \sqrt{1+2x^2}\, dx$, h = 0.25. 4 [Q.N. 17(b), Set 'C' 2071]
[Ans: 1.2712]

14. Solve, using Gauss elimination method, the following equations.
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$ 4 [Q.N. 17(a), 2070 'C']
[Ans: –15, 8, 2]

15. Solve the following equation using Gauss Seidel method:
$3x_1 + x_2 = 5$, $x_1 + 2x_2 = 5$. [Q.N. 17(a) (Or), 2070 'C']
[Ans: 1, 2]

16. Solve the following system of equations by Gauss Seidel method
$3x + y - z = 2$, $2x - 5y + z = 20$, $x - 3y - 8z = 3$ [Q.N. 17(a) (Or), 2070 'D']
(Ans: 2, – 3 , 1)

17. Solve the following system of equations by Gaussian elimination method.
$x + 3y - 2z = 5$, $3x + 5y + 6z = 7$, $2x + 4y + 3z = 8$ 4 [Q.N. 17(a), 2070 'D']
(Ans: – 15, 8, 2)

18. Examine whether the following system of equations
$x_1 + 2x_2 + 5x_3 = 1$
$2x_1 - 3x_2 + 5x_2 = 2$,
$5x_1 + 3x_2 + 6x_3 = 3$; is diagonally dominant? 2 [Q.N. 16(c), Supp. 2069]
[Ans: not dominant]

19. Solve by Gauss elimination method, the system of equations
$3x + 2y - z = 1$, $x - y + 2z = -1$, $-x + \frac{1}{2}y - z = 0$. 4[Q.N. 17(a), Supp. 2069]
[Ans: 1, –2, –2]

20. Solve by Gauss–Seidel method:
$3x_1 + x_2 = 5$
$x_1 - 3x_2 = 5$ [Q.N. 17(a) (Or), Supp. 2069]
[Ans: 2, –1]

21. Examine whether the following system of equations are ill conditioned.
$2x_1 + x_2 = 25$
$2.001x_1 + x_2 = 25.01$ [Q.N. 16(c), Set 'A' 2069]
(Ans: Ill - conditioned)

22. Using Gauss elimination method, solve the following system of equations:
$x - 2y + 3z = 2$, $2x - 3y + z = 1$, $3x - y + 2z = 9$
(Ans: 3, 2, 1) [Q.N. 17(a), Set 'A' 2069]

23. Solve the following equations using Guess-Seidel method:
$2x_1 - x_2 = 8$
$3x_1 + 7x_2 = -5$ [Q.N. 17(a) (Or), Set 'A' 2069]
(Ans: 3, – 2)

24. Using Gauss elimination method, solve the following system of equations:
$x_1 - 2x_2 + 3x_3 = 10$,
$2x_1 + 3x_2 - 2x_3 = 1$,
and $-x_1 - 2x_2 + 4x_3 = 13$.
(Ans: 1, 3, 5) [Q.N. 17(a) Set 'B' 2069]

25. Solve the following equations using Guess-Seidel method:
$3x_1 + x_2 = 5$
$x_1 - 3x_2 = 5$ [Q.N. 17(a) (Or), Set 'B' 2069]
(Ans: 1, 2)

Unit 19: Numerical Integration

1. Using Simpson's $\frac{1}{3}$ rule, calculate $\int_1^5 x^4\,dx$ with n = 4. 4[Q.N.17(b), 2072'C']
(Ans: 625.33)

2. State and prove Trapezoidal rule of numerical approximation. 6[Q.N.19(Or), 2072'C']

3. Approximate the value using trapezoidal rule for $\int_{-1}^{1} e^x\,dx$, n = 2. 6[Q.N.19, 2072'D']
$\left(\text{Ans: } \frac{(e+1)^2}{2e}\right)$

4. Evaluate $\int_0^1 \sqrt{1+x^3}\,dx$ using Simpson's $\frac{1}{3}$ rule with n = 4. [Q.N.19(Or), 2072'D']
(Ans: 1.111)

5. Find the approximate value of $\int_0^{0.2} \sqrt{1-2x^2}\,dx$, n = 2, using Simpson's $\frac{1}{3}$ rule.
(Ans: 0.197298809) 2 [Q.N.16(c), 2072'E']

6. Evaluate using composite trapezoidial rule, the integral $\int_0^{\pi} \sin x\,dx$, n = 4.
(Ans: 1.896) [Q.N.17(b), 2072'E']

7. Using the trapezoidal rule, evaluate: $\int_0^1 \frac{dx}{1+x^2}$, n = 2.
(Ans: 0.775) 2 [Q.N. 16(c), Set 'C' 2071]

8. Using the trapezoidal rule, evaluate: $\int_0^2 (2x^2 - 1)dx$, n = 4.
[Ans: 3.5] 2 [Q.N. 16(c), Set 'D' 2071]

9. Estimate the following integral using Simpson's $\frac{1}{3}$ rule,

$$\int_0^{\pi} \sin x\,dx,\ n = 6$$

4[Q.N. 17(b), Set 'D' 2071]

10. Using trapezoidal rule, evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx,\ n = 2.$

[Ans: 1.052] 2 [Q.N. 16(c), 2070 'C']

11. Using Simpsons's $\frac{1}{3}$ rule, evaluate $\int_0^1 \frac{dx}{1+x^2},\ n = 4.$

[Ans: 1, 0.785] 4 [Q.N. 17(b), 2070 'C']

12. Using trapezoidal rule, evaluate $\int_0^3 (3x^2 - 4x)dx,\ n = 3.$

(Ans: 10.5) 2 [Q.N. 16(c), 2070 'D']

13. Using Simpons's $\frac{1}{3}$ rule, evaluate $\int_0^1 \frac{dx}{1+x},\ n = 4.$ 4 [Q.N. 17(b), 2070 'D']

(Ans: 0.693255)

14. Evaluate using Simpson's rule $\int_0^1 \frac{dx}{1+x}$. Estimate the error in using the approximation for n = 4. 4 [Q.N. 17(b), Supp. 2069]

[Ans: 0.693255, error ≤ 0.00052]

15. Estimate the following integral using Trape-Zoidal rule. [Q.N. 17(b), Set 'A' 2069]

$$\int_0^1 \frac{dx}{1+x},\ n = 4$$

Estimate the error with respect to the actual value.

(Ans: 0.69702, 0.00388)

16. Given $I = \int_0^4 x^3\,dx,\ n = 4$

Estimate the value of I using Trapezoidal rule.

(Ans: 68) [Q.N. 16(c), Set 'B' 2069]

17. Evaluate the following integral using Simpson's rule. 4 [Q.N. 17(b), Set 'B' 2069]

$\int_0^{\pi} \sin x\,dx,\ n=6$

(Ans: 2.0008)