30. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$.
    (Ans: 41 sq. units) [Q.N. 3(b), 2064]

31. Using vector method, prove in any triangle, that :
    $b^2 = c^2 + a^2 - 2ca \cos B$ [Q.N. 11(a), 2064]

32. If $\vec{i}$, $\vec{j}$, $\vec{k}$ are three mutually perpendicular unit vectors and
    $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - 3\vec{j} - \vec{k}$, find the cosine of the angle between the two vectors. [Q.N. 4(a), 2063]
    $$\left(Ans.: \cos^{-1} \sqrt{\frac{7}{12}}\right)$$

33. Using vector method, prove in any triangle that :
    $a = b \cos C + c \cos B$ [Q.N. 11(a), 2063]

34. Find the area of the parallelogram determined by the vectors
    $\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$ [Q.N. 3(b), 2062]
    (Ans.: $6\sqrt{5}$ sq. unit)

35. Prove vectorically that : $\cos(A-B) = \cos A \cos B + \sin A \sin B$ [Q.N. 11(a), 2062]

36. Find a unit vector perpendicular to $2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{i} + \vec{j} - 2\vec{k}$.
    $$\left(Ans: \frac{-5}{\sqrt{35}}\vec{i} + \frac{3}{\sqrt{35}}\vec{j} - \frac{1}{\sqrt{35}}\vec{k}\right)$$ [Q.N. 3(b), 2061]

37. If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them. Show that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$ [Q.N. 4(a), 2061]

38. Prove, in any triangle, by vector method that :
    $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ [Q.N. 11(a), 2061]

39. Find the cosine of the angle between the vectors
    $2\vec{i} + \vec{j} + \vec{k}$ and $4\vec{i} + 3\vec{j} + 5\vec{k}$ [Q.N. 3(b), 2060]
    $$\left(Ans: \frac{8}{5\sqrt{3}}\right)$$

40. Prove by vector method: $\sin(A-B) = \sin A \cos B - \cos A \sin B$. [Q.N. 11(a), 2060]

41. Find the angle between two vectors $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$.
    (Ans: 60°) [Q.N. 4(a), 2059]

42. Prove by vector method: $\sin(A+B) = \sin A \cos B + \cos A \sin B$. [Q.N. 11(a), 2059]

43. Show that the area of the parallelogram determined by:
    $\vec{i} + \vec{j} - 3\vec{k}$ and $-\vec{i} - 2\vec{j} - 3\vec{k}$ is $\sqrt{118}$ sq. units [Q.N. 3(b), 2058]

44. Prove by vector method.
    $\cos(A-B) = \cos A \cos B + \sin A. \sin B.$ [Q.N. 11(a), 2058]

45. Prove by vector method
    $\cos(A-B) = \cos A \cos B + \sin A \sin B$ [Q.N. 11(a), 2057]

Unit 7: Derivative and its Application

7.1   Continuity and differentiability

1. Show that $\lim\limits_{x \to 0} \dfrac{\log(1+x)}{x} = 1$ [Q.N. 2(b), 2057]

7.2   Differential Coefficients by definition (by first principle)

1. Find, from first principles, the derivative of xlnx.   [Q.N.11(Or), 2072'C']
    (Ans: $1 + \ell nx$)

2. Find from first principles the derivative of ln cos⁻¹x.   [Q.N.11(Or), 2072'D']
    $\left(\text{Ans: } \dfrac{-1}{\sqrt{1-x^2}\cos^{-1}x}\right)$

3. Solve: $x\dfrac{dy}{dx} + y - 1 = 0$.   2[Q.N.4(a), 2072'E']
    (Ans: x(y − 1) = e)

4. Find from first principles the derivative of sinx².   [Q.N.11(Or), 2072'E']
    (Ans: 2x cosx²)

5. Find from first principle, the derivative of tan⁻¹x.   6 [Q.N. 11(OR), Set 'C' 2071]
    $\left(\text{Ans: } \dfrac{dy}{dx} = \dfrac{1}{1+x^2}\right)$

6. Find from first principle, the derivative of: $\ln(\sin\frac{x}{a})$.
    $\left(\text{Ans: } \dfrac{d}{dx}\left(\log\sin\frac{x}{a}\right) = \frac{1}{a}\cot\frac{x}{a}\right)$ 6 [Q.N. 11(OR), Set 'D' 2071]

7. Find from first principles, the derivative of log (tanx).   [Q.N. 11(OR), 2070 'C']
    $\left(\text{Ans: } \dfrac{1}{\sin x.\cos x}\right)$

8. Find from first principles, that derivative of xˣ.   [Q.N. 11(OR), 2070 'D']
    (Ans: xˣ (1 + logx))

9. Find from first principles the derivative of logcos⁻¹x.   [Q.N. 11(OR), Supp. 2069]
    $\left(\text{Ans: } \dfrac{-1}{\cos^{-1}x\sqrt{1-x}}\right)$

10. Find from first principle, the derivative of sin(logx)   [Q.N. 11(OR), Set 'A' 2069]
    $\left(\text{Ans: } \frac{1}{x}\cos(logx)\right)$

11. Find from first principle, the derivative of eˢⁱⁿˣ.   [Q.N. 11(OR), Set 'B' 2069]
    (Ans: cosx. eˢⁱⁿˣ)

12. Find from first principle, the derivative of : eˢⁱⁿˣ   [Q.N. 10(b), 2068]
    (Ans: cosx eˢⁱⁿˣ)

13. Find from first principles, the derivative of : sin⁻¹x.   [Q.N. 10(b), 2067]
    $\left(\text{Ans: } \dfrac{1}{\sqrt{1-x^2}}\right)$

14. Find the derivative of Sin (log x) from first principles.   [Q.N. 10(b), 2066]
    $\left(\text{Ans: } \frac{1}{x}\cos(logx)\right)$

15. Find from first principles, the derivative of aˣ.   [Q.N.10(b), 2065]
    (Ans: aˣloga)

16. Find from first principles, the derivative of log sinx.   [Q.N. 10(b), 2064]
    (Ans: cotx)

17. Find from first principles, the derivative of *log tanx*   **[Q.N. 10(b), 2063]**

$$\left(Ans.: \frac{1}{sinx \cdot cosx} \right)$$

18. Find from first principles, the derivative of $tan^{-1}x$.   **[Q.N. 10(b), 2062]**

$$\left(Ans.: \frac{1}{1+x^2} \right)$$

19. Find from first principles, the derivative of $e^{\sqrt{x}}$.   **[Q.N. 10(b), 2061]**

$$\left(Ans.: \frac{1}{2\sqrt{x}} e^{\sqrt{x}} \right)$$

20. Find from first principles, the derivative of e^{sinx} **[Q.N. 10(b), 2060]**

$$\left(Ans.: cosx.e^{sinx} \right)$$

21. Find from first principles, the derivative of e^{tanx}.   **[Q.N. 10(b), 2058]**

$$\left(Ans: sec^2x \ e^{tanx} \right)$$

7.3 Derivative of hyperbolic function

1. Find the derivative of: $\left(Cosh \dfrac{x}{a} \right)^{logx}$ **[Q.N. 2(b), 2068]**

$$\left[Ans: \left(cosh \frac{x}{a} \right)^{logx} \left(\frac{1}{a^2} tanh \frac{x}{a} log x + \frac{1}{x} log cosh \frac{x}{a} \right) \right]$$

2. Find the derivative of $x^{cosh \frac{x}{a}}$.   **[Q.N. 2(b), 2067]**

$$\left[Ans: x^{cos h \frac{x}{a}} \left[\frac{cos h \frac{x}{a}}{x} + \frac{log x \ sin h \frac{x}{a}}{a} \right] \right]$$

3. Find the derivative of $2 \ tanh^{-1} \left(tan \frac{1}{2} x \right)$ **[Q.N. 2(b), 2066]**

(Ans: secx)

4. Find the derivative of Arc tan Sin hx.   **[Q.N.2(b), 2065]**

$$\left(Ans: \frac{coshx}{1 - sinh^2x} \right)$$

5. Find the derivative of : $\left(sin h \dfrac{x}{a} \right)^{x^2}$ **[Q.N. 2(b), 2064]**

$$\left(Ans: \left(sin \frac{x}{a} \right)^{x^2} \left[\frac{x^2}{a} cot \frac{x}{a} + 2x \ log \ sinh \frac{x}{a} \right] \right)$$

6. Find the derivative of $x^{Cosh^2 \frac{x}{a}}$.   **[Q.N. 2(b), 2063]**

$$\left(Ans.: x^{cosh^2 \frac{x}{a}} \left(\frac{1}{x} \cdot cosh^2 \frac{x}{a} + \frac{1}{a} \cdot logx. \ sin \frac{2x}{a} \right) \right)$$

7. Find the derivative of $log(sinh^x/a)$ **[Q.N. 2(b), 2062]**

$$\left(Ans.: \frac{1}{a} coth \frac{x}{a} \right)$$

8. Find the derivative of : $x^{\cos h\, x/a}$ [Q.N. 2(b), 2061]

$$\left(\text{Ans: } x^{\cosh\frac{x}{a}}\left[\frac{\cosh x/a}{x} + \frac{\log x \sinh x/a}{a}\right]\right)$$

9. Find the derivative of $2\tan^{-1}\left(\tanh\frac{x}{2}\right)$ [Q.N. 2(b), 2060]

    (Ans: Sechx)

10. Find the derivative of $e^{\cosh^{-1} x}$. [Q.N. 2(a), 2059]

$$\left(\text{Ans: } \frac{1}{\sqrt{x^2-1}}\, e^{\cosh^{-1}x}\right)$$

11. Find, from definition, the derivative of $e^{\tan x}$. [Q.N. 10(b), 2059]

$$\left(\text{Ans: } \sec^2 x e^{\tan x}\right)$$

12. Find the derivative of $x^{\cosh x}$ [Q.N. 2(b), 2058]

$$\left(\text{Ans: } x^{\cosh x}\left(\frac{\cosh x}{x} + \log x \sinh x\right)\right)$$

13. Find the derivative of $\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}$ [Q.N. 10(b), 2057]

$$\left(\text{Ans: } n\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)^{nx}\left[\frac{x}{a} + \log\left(\sinh\frac{x}{a} + \cosh\frac{x}{a}\right)\right]\right)$$

7.4 Tangent and Normal

1. Find the points on the circle $x^2 + y^2 = 16$ at which the tangents are parallel to X-axis.
    (Ans: (0, 4), (0, – 4))   2[Q.N.3(a), 2072'E']

2. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangent is parallel to the x-axis.   2
    (Ans: (0, 1), (2, – 3))   [Q.N. 3(a), Set 'C' 2071]

3. Find the equation of the tangent to the curve $y = 2x^3 - 5x^2 + 8$ at (2, 4)
    (Ans: 4x – y – 4 = 0)   2 [Q.N. 3(a), 2070 'D']

4. At what angle does the curve $y(1+x) = x$ cut the x-axis ?   [Q.N. 5(c), 2068]

$$\left(\text{Ans: } \frac{\pi}{4}\right)$$

5. Find the angle of intersection between the curves $y = x^2$ and
    $6y = 7 - x^3$ at (1, 1).   [Q.N. 2(c), 2067]

$$\left(\text{Ans: } \tan^{-1}\left(\pm\frac{3}{2}\right)\right)$$

6. Find the angle of intersection of the curves $y^2 = x^3$ and $y = 2x$ at the point (0, 0).

$$\left(\text{Ans: } \tan^{-1}(\pm 2) \text{ at } (0, 0) \tan^{-1}\left(\pm\frac{1}{7}\right) \text{ at } (4, 8)\right) \quad \text{[Q.N. 2(c), 2066]}$$

7. Find where the tangent is parallel to the x-axis for the curve
    $y = x^3 - 3x^2 - 9x + 15$.   [Q.N.2(c), 2065]
    (Ans: (–1, 20) and (3, –12))

8. Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to x-axis.
    (Ans: (0, 1), (2, –3))   [Q.N. 2(c), 2064]

9. Find the angle of intersection of the curves   [Q.N. 2(c), 2062]
    $4y = x^2 + 12$ and $y^2 = 8x$ at (2,4)
    (Ans.: 0°)

10. Find the slope and inclination with the x-axis of the tangent of $y = -3x - x^4$ at $x = -1$.

$$\left(\text{Ans: } 1 \text{ and } \frac{\pi}{4}\right) \quad \text{[Q.N. 2(c), 2061]}$$

11. Find the points on the curve $4y = x^4 - 8x^2$ where the tangents are paralleled to the x-axis. **[Q.N. 5(c), 2060]**
    (Ans: (0, 0), (-2, -4) and (2, -4))

12. Find the slope and inclination with x -axis of the tangent of: $x^2 + y^2 = 36$ at (0,6).
    (Ans: 0 and 0°) **[Q.N. 5(c), 2058]**

13. Find the slope and inclination with X -axis of the tangent of the curve
    $2y = 2 - x^2$ at x =1. **[Q.N. 5(c), 2057]**
    $\left(Ans: -1, \dfrac{3\pi}{4}\right)$

7.5 L Hospital's rule, Roll's Theorem & Mean Value Theorem

1. Evaluate, using L' Hospitals rule: $\underset{x \to 0}{Lt} \dfrac{\tan ax}{\tan bx}$. **2[Q.N.3(a), 2072'C']**
   $\left(Ans: \dfrac{a}{b}\right)$

2. State Rolle's theorem. Verify Rolle's theorem for the functions $f(x) = 2x^3 - 3x + 1$ in
   $\left[\dfrac{1}{2}, 1\right]$. **6[Q.N.11, 2072'C']**

3. Using L' Hospitals rule, evaluate: $\underset{x \to 0}{Lt} \dfrac{e^x - x - 1}{x^2}$. **2[Q.N.3(b), 2072'D']**
   $\left(Ans: \dfrac{1}{2}\right)$

4. State Mean Value theorem. Verify the mean value theorem for the function
   $f(x) = \sqrt{x^2 - 4}$, $x \in [2, 4]$. **6[Q.N.11, 2072'D']**
   (Ans: $c = \sqrt{6}$)

5. State Rolle's theorem. What is the geometrical interpretation of Rolle's theorem. Verify
   Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$, $x \in [-1, 1]$. **6[Q.N.11, 2072'E']**
   (Ans: $c = 0$)

6. State mean value theorem. Interpret it geometrically. Verify the mean value theorem for
   the function $f(x) = (x - 1)(x - 2)(x - 3)$ in [1, 4]. **6 [Q.N. 11, Set 'C' 2071]**

7. Using L Hospital's rule, evaluate:
   $\underset{x \to 0}{\lim} \dfrac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$. **2 [Q.N. 3(a), Set 'D' 2071]**
   [Ans: 1]

8. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function
   $f(x) = x(x-3)^2$ for $x \in [0, 3]$. **6 [Q.N. 11, Set 'D' 2071]**

9. Using L Hospital's rule, evaluate: $\underset{x \to 0}{\lim} \dfrac{\tan x - x}{x - \sin x}$ **2 [Q.N. 3(a), 2070 'C']**
   [Ans: 2]

10. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function
    $f(x) = x(x-3)^2$ for $x \in [0, 3]$. **6 [Q.N. 11, 2070 'C']**
    [Ans: C = 1]

11. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the
    function $f(x) = x(x - 1)^2$ in [0, 2]. **6 [Q.N. 11, 2070 'D']**
    $\left(Ans: c = \dfrac{4}{3}\right)$

12. Evaluate, using L'Hospital rule $\underset{x \to 0}{Lt} \dfrac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$ **[Q.N. 3(a), Supp. 2069]2**
    [Ans: 1]

13. Using mean value theorem, find a point on the parabola $y = (x - 3)^2$ where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

$\left(\text{Ans:} \dfrac{7}{2}, \dfrac{1}{4}\right)$ 6 [Q.N. 11, Supp. 2069]

14. Using L Hospital's rule, evaluate: $\displaystyle\lim_{x \to 0} \dfrac{e^x - x - 1}{x^2}$ [Q.N. 3(a), Set 'A' 2069]

$\left(\text{Ans:} \dfrac{1}{2}\right)$

15. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$.   6 [Q.N. 11, Set 'A' 2069]

16. Using L Hospital's rule, evaluate:

$\displaystyle\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$.   [Q.N. 3(a), Set 'B' 2069]

$\left(\text{Ans:} \dfrac{1}{6}\right)$

17. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1,4]$.   [Q.N. 11, Set 'B' 2069]

Unit 8: Antiderivatives

1. Evaluate : $\displaystyle\int \dfrac{dx}{1 - 2\cos x}$ 2[Q.N.3(b), 2072'C']

$\left(\text{Ans:} \dfrac{1}{\sqrt{3}} \log \dfrac{\sqrt{3} \tan \frac{x}{2} - 1}{\sqrt{3} \tan \frac{x}{2} + 1} + C\right)$

2. Evaluate : $\displaystyle\int \dfrac{dx}{(x-2)^2 (x-3)^3}$ 4[Q.N.7(a), 2072'C']

$\left(\text{Ans:} -\dfrac{1}{2}\left(\dfrac{x-2}{x-3}\right)^2 + 3\left(\dfrac{x-2}{x-3}\right) - \log \dfrac{x-2}{x-3} - \dfrac{x-3}{x-2} + C\right)$

3. Compute the integral $\displaystyle\int \dfrac{\coth x \, dx}{\sinh x - 9 \, \text{cosech} x}$ 2[Q.N.3(a), 2072'D']

$\left(\text{Ans:} \dfrac{1}{6} \log \dfrac{\sinh x - 3}{\cosh x + 3} + c\right)$

4. Evaluate : $\displaystyle\int \dfrac{dx}{(x-1)^2 (x-2)^3}$ 4[Q.N.7(a), 2072'D']

$\left(\text{Ans:} -\dfrac{1}{2}\left(\dfrac{x-1}{x-2}\right)^2 + 3\left(\dfrac{x-1}{x-2}\right) - 3 \log \left(\dfrac{x-1}{x-2}\right) - \dfrac{x-2}{x-1} + c\right)$

5. Evaluate: $\displaystyle\int \dfrac{dx}{\sqrt{(x-\alpha)(x-\beta)}} (\beta > \alpha)$ 2[Q.N.3(b), 2072'E']

$\left(\text{Ans:} 2\log \left(\sqrt{x-\alpha} + \sqrt{x-\beta}\right) + c\right)$

6. Evaluate : $\int \dfrac{dx}{2+3\cos x}$.   4[Q.N.7(a), 2072'E']

$$\left(\text{Ans: } \frac{1}{\sqrt{5}}\log\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}+C\right)$$

7. Evaluate : $\int \dfrac{6x+1}{x^2+9}\ dx$.   2 [Q.N. 3(b), Set 'C' 2071]

$$\left(\text{Ans: } 3\log(x^2+9)+\frac{1}{3}\tan^{-1}\frac{x}{3}+C\right)$$

8. Evaluate: $\int \dfrac{dx}{1-2\cos x}$.   4 [Q.N. 7(a), Set 'C' 2071]

$$\left(\text{Ans: } \frac{1}{\sqrt{3}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2}-1}{\sqrt{3}\tan\frac{x}{2}+1}\right)+C\right)$$

9. Evaluate : $\int \dfrac{dx}{\sqrt{2ax+x^2}}$ 2 [Q.N. 3(b), Set 'D' 2071]

[Ans: $\log(x+a+\sqrt{x^2+2ax})+C$

10. Evaluate: $\int \dfrac{dx}{1-3\sin x}$ 4 [Q.N. 7(a), Set 'D' 2071]

$$\left(\text{Ans:}\left(\frac{1}{2\sqrt{2}}\log\frac{\tan\frac{x}{2}-3-2\sqrt{2}}{\tan\frac{x}{2}-3+2\sqrt{2}}+C\right)\right)$$

11. Evaluate : $\int \dfrac{2x+3}{4x^2+1}\ dx$.   2 [Q.N. 3(b), 2070 'C']

$$\left(\text{Ans: } \frac{1}{4}\log(4x^2+1)+\frac{3}{2}\tan^{-1}2x+C\right)$$

12. Evaluate: $\int \dfrac{dx}{2+\cos x}$ 4 [Q.N. 7(a), 2070 'C']

$$\left(\text{Ans: } \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan^{-1}\frac{x}{2}}{\sqrt{3}}\right)+C\right)$$

13. Evaluate : $\int \dfrac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$, $(\beta>\alpha)$ 2 [Q.N. 3(b), 2070 'D']

(Ans: $2\log(\sqrt{x-\alpha}+\sqrt{x-\beta})+C$)

14. Evaluate: $\int \dfrac{dx}{1+2\sin x}$ 4 [Q.N. 7(a), 2070 'D']

$$\left(\text{Ans: } \frac{1}{\sqrt{3}}\log\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}+C\right)$$

15. Evaluate : $\int \dfrac{dx}{1 + \sin x + \cos x}$. 2 [Q.N. 3(b), Supp. 2069]

$\left(\text{Ans: } \log\left(1 + \tan\dfrac{x}{2}\right) + c\right)$

16. Evaluate: $\int \dfrac{x^2}{x^4 - 2x^2 - 15}\, dx$. 4 [Q.N. 7(a), Supp. 2069]

$\left(\text{Ans; } \dfrac{\sqrt{3}}{8} \tan^{-1}\dfrac{x}{3} + \dfrac{\sqrt{5}}{16} \log \dfrac{x - \sqrt{5}}{x + \sqrt{5}} + C\right)$

17. Evaluate : $\int \dfrac{dx}{\sqrt{2ax - x^2}}$. [Q.N. 3(b), Set 'A' 2069]

$\left(\text{Ans: } \sin^{-1}\dfrac{x - a}{a} + c\right)$

18. Evaluate: $\int \dfrac{dx}{3\sin x - 4\cos x}$. [Q.N. 7(a), Set 'A' 2069]

$\left(\text{Ans: } \dfrac{1}{5} \dfrac{\tan\dfrac{x}{2} - \dfrac{1}{2}}{\tan\dfrac{x}{2} + 2} + c\right)$

19. Evaluate : $\int \dfrac{dx}{e^x + e^{-x}}$ [Q.N. 3(b), Set 'B' 2069]

$(\text{Ans: } \tan^{-1} e^x + c)$

20. Evaluate: $\int \dfrac{dx}{2 + 3\cos x}$. [Q.N. 7(a), Set 'B' 2069]

$\left(\text{Ans: } \dfrac{1}{\sqrt{5}} \log \dfrac{\sqrt{5} + \tan\dfrac{x}{2}}{\sqrt{5} - \tan\dfrac{x}{2}} + C\right)$

21. Evaluate: $\int \dfrac{dx}{\sqrt{2ax - x^2}}$ [Q.N. 3(a), 2068]

$\left(\text{Ans: } \sin^{-1}\dfrac{x - a}{a} + C\right)$

22. Evaluate: $\int \dfrac{dx}{1 + \sin x + \cos x}$ [Q.N. 11(b), 2068]

$\left(\text{Ans: } \log\left(1 + \tan\dfrac{x}{2}\right) + C\right)$

23. Evaluate : $\int \dfrac{1}{x^2} e^{\frac{-1}{x}}\, dx$ [Q.N. 3(a), 2067]

$(\text{Ans: } e^{-\frac{1}{x}} + C)$

24. Evaluate : $\int \dfrac{dx}{1 + 2\sin x}$ [Q.N. 11(b), 2067]

$\left(\text{Ans: } \dfrac{1}{\sqrt{3}} \log \dfrac{\tan\dfrac{x}{2} + 2 - \sqrt{3}}{\tan\dfrac{x}{2} + 2 + \sqrt{3}} + C\right)$

25. Integrate: $\int \dfrac{dx}{\sqrt{2ax + x^2}}$ [Q.N. 3(a), 2066]

    (Ans: $log(x + a) + \sqrt{x^2 + 2ax} + c$)

26. Integrate: $\int \dfrac{x^2}{(x + 2)(x + 3)^2} \, dx$ [Q.N. 11(b), 2066]

    $\left(Ans: 4log(x + 2) - 3log(x + 3) + \dfrac{9}{x + 3} + c\right)$

27. Evaluate : $\int \dfrac{dx}{e^x + e^{-x}}$ [Q.N.3(a), 2065]

    (Ans: $tan^{-1}(e^x) + c$)

28. Integrate : $\int \dfrac{dx}{3 + 4 \, Cos \, hx}$ [Q.N.11(b), 2065]

    $\left(Ans: \dfrac{2}{\sqrt{7}} tan^{-1}\left(\dfrac{tan \, h\dfrac{x}{2}}{\sqrt{7}}\right) + c\right)$

29. Evaluate : $\int \dfrac{dx}{x^2 - 16}$ [Q.N. 3(a), 2064]

    $\left(Ans: \dfrac{1}{8} log \dfrac{x - 4}{x + 4} + C\right)$

30. Integrate : $\int \dfrac{cosx - sinx}{\sqrt{sin2x}} \, dx$ [Q.N. 11(b), 2064]

    $\left(Ans: log \left\{(sinx + cosx) + \sqrt{sin2x}\right\} + c\right)$

31. Integrate : $\int \dfrac{dx}{1 + sinx + cosx}$ [Q.N. 11(b), 2063]

    $\left(Ans.: log \left(1 + tan\dfrac{x}{2}\right) + C\right)$

32. Integrate : $\int \dfrac{dx}{\sqrt{2ax - x^2}}$ [Q.N. 3(a), 2062]

    $\left(Ans.: Sin^{-1}\dfrac{x - a}{a} + C\right)$

33. Integrate : $\int \dfrac{dx}{Sinx + Cosx}$ [Q.N. 11(b), 2062]

    $\left(Ans.: \dfrac{1}{\sqrt{2}} log\left[tan\left(\dfrac{x}{2} + \dfrac{\pi}{8}\right)\right] + C\right)$

34. Evaluate : $\int \sqrt{\dfrac{1 + x}{1 - x}} \, dx$ [Q.N. 3(a), 2061]

    $\left(Ans: -\sqrt{1 - x^2} + sin^{-1} x + c\right)$

35. Find the value of : $\int \frac{dx}{3\,Sinx - 4\,Cosx}$ **[Q.N. 11(b), 2061]**

$$\left(Ans: \frac{1}{5} \log \frac{Tan\frac{x}{2} - \frac{1}{2}}{Tan\frac{x}{2} + 2} + C\right)$$

36. Evaluate : $\int \frac{dx}{\sqrt{a^2 - x^2}}$ **[Q.N. 3(a), 2060]**

$$\left(Ans: sin^{-1}\frac{x}{a} + C\right)$$

37. Find the value of :

$$\int \frac{sinx \, . \, cos\,x}{(sin\,x + cos\,x)^2} dx$$ **[Q.N. 11(b), 2060]**

$$\left(Ans: -\frac{1}{2}\frac{1}{(Tanx + 1)} + C\right)$$

38. $\int \frac{dx}{a + b\,cos\,x}$ a<b .   **[Q.N. 11(b), 2059]**

$$\left(Ans: \frac{1}{\sqrt{b^2 - a^2}} \log \left(\frac{\sqrt{b+a} + \sqrt{b-a}\,tan\frac{x}{2}}{\sqrt{b+a} - \sqrt{b-a}\,tan\frac{x}{2}}\right) + C\right)$$

39. Prove that: $\int \frac{dx}{\sqrt{a^2 - x^2}} = sin^{-1}\frac{x}{a} + c.$ **[Q.N. 3(a), 2058]**

40. Integrate: $\int \sqrt{\frac{1+x}{1-x}} dx.$ **[Q.N. 11(b), 2058]**

$$\left(Ans: -\sqrt{1-x^2} + sin^{-1}x + c\right)$$

41. Prove: $\int Cosecx\,dx = \log\left|tan\frac{x}{2}\right| + c$ **[Q.N. 3(a), 2057]**

42. Integrate: $\int \frac{dx}{a + b\,cosx}$ when a> b.   **[Q.N. 11(b), 2057]**

$$\left(Ans: \frac{2}{\sqrt{a^2 - b^2}} Tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\,tan\frac{x}{2}\right) + C\right)$$

Unit 9: Differential Equations and their Applications

1. Solve : $\frac{dy}{dx} + \frac{y}{x} = 1.$ 2**[Q.N.4(a), 2072'C']**

$$\left(Ans: xy = \frac{x^2}{2} + C\right)$$

2. Solve : $\dfrac{dy}{dx} = y \tan x - 2 \sin x$ 4[Q.N.7(b), 2072'C']

$\left(\text{Ans: } y\cos x = \dfrac{\cos 2x}{2} + C \right)$

3. Solve : $xy \dfrac{dy}{dx} - y^2 = x^2$ 4[Q.N.7(b)(Or), 2072'C']

(Ans: $y^2 = 2x^2 (\log x + C)$)

4. Solve : $\dfrac{dy}{dx} + \dfrac{1 + \cos 2y}{1 - \cos 2y} = 0$.   2[Q.N.4(a), 2072'D']

(Ans: $\tan y - y + x = c$)

5. Reduce the equation $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2$ in liner form hence solve it.   4[Q.N.7(b), 2072'D']

$\left(\text{Ans: } \dfrac{1}{xy} = - \log x + c \right)$

6. Solve : $\dfrac{dy}{dx} = \dfrac{y + 1}{x + y + 1}$ [Q.N.7(b)(Or), 2072'D']

(Ans: $y + 1 = ce^{\frac{x}{y + 1}}$)

7. Solve: $\dfrac{dy}{dx} = \dfrac{y}{x} - \sin^2 \dfrac{y}{x}$.   4[Q.N.7(b), 2072'E']

$\left(\text{Ans: } \cot \left(\dfrac{y}{x} \right) = \log x + c \right)$

8. Solve : $\sin x \dfrac{dy}{dx} + \cos xy = x \sin x$.   [Q.N.7(b)(Or), 2072'E']

(Ans: $y \sin x = - x\cos x + \sin x + c$)

9. Solve: $e^{x-y}. dx + e^{y-x} dy = 0$.   2 [Q.N. 4(a), Set 'C' 2071]

(Ans: $e^{2x} + e^{2y} = C$)

10. Solve: $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{2x^2}$.   4 [Q.N. 7(b), Set 'C' 2071]

(Ans: $2x = (x - y) \log (cx)$)

11. Solve: $\sin x \dfrac{dy}{dx} + (\cos x). y = \sin x.\cos x$.   4 [Q.N. 7(b)(OR), Set 'C' 2071]

$\left(\text{Ans: } y \sin x + \dfrac{1}{4} \cos 2x = c \right)$

12. Solve: $\dfrac{dy}{dx} + 4x = 2e^{2x}$.   2 [Q.N. 4(a), Set 'D' 2071]

[Ans: $y = e^{2x} - 2x^2 + C$]

13. Solve: $xy \dfrac{dy}{dx} = x^2 + y^2$.   4 [Q.N. 7(b), Set 'D' 2071]

[Ans: $y^2 = 2x^2 (\log x + C)$]

14. Solve: $\dfrac{dy}{dx} + \dfrac{2x}{1 + x^2} . y = \dfrac{1}{(1 + x^2)^2}$.   4 [Q.N. 7(b)(OR), Set 'D' 2071]

[Ans: $y(1 + x^2) = \tan^{-1} x + C$]

15. Solve: $\dfrac{dy}{dx} = \dfrac{e^x + 1}{y}$ 2 [Q.N. 4(a), 2070 'C']

[Ans: $y^2 = 2e^x + 2x + C$]

16. Solve: $\cos^2 x \dfrac{dy}{dx} + y = 1$.   [Q.N. 7(b), 2070 'C']

[Ans: $y = 1 + Ce^{-\tan x}$]

17. Solve: $\dfrac{dy}{dx} = \dfrac{y}{x} - \sin^2\dfrac{y}{x}$. [Q.N. 7(b)(OR), 2070 'C']

$\left(\text{Ans: } \cot\dfrac{y}{x} = \log x + C\right)$

18. Solve: $x^2 dy - y^2 dx = 0$ 2 [Q.N. 4(a), 2070 'D']
(Ans: $y - x = Axy$)

19. Solve: $(1+x^2)\dfrac{dy}{dx} + 2xy = 4x^2$. 4 [Q.N. 7(b), 2070 'D']

$\left(\text{Ans: } (1 + x^2)\, y = \dfrac{4}{3}x^3 + C\right)$

20. Solve: $(x^2 + y^2)dy = xy\,dx$ [Q.N. 7(b)(OR), 2070 'D']
(Ans: $x^2 = 2y^2 \log(yc)$)

21. Solve: $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$. 2 [Q.N. 4(a), Supp. 2069]

[Ans: $y - x = C(1 + xy)$]

22. Solve: $\tan x\,\dfrac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Supp. 2069]

[Ans: $y \sin x = x + C$]

23. Solve: $\dfrac{dy}{dx} = \dfrac{y - x + 1}{y - x + 5}$ [Q.N. 7(b)(OR), Supp. 2069]

[Ans: $x^2 + y^2 - 2xy + 10y - 2x = C$]

24. Solve: $\dfrac{dy}{dx} = e^{x-y} + x^3 \cdot e^{-y}$ [Q.N. 4(a), Set 'A' 2069]

$\left(\text{Ans: } e^y = e^x + \dfrac{x^4}{4} + c\right)$

25. Solve: $\tan x\,\dfrac{dy}{dx} + y = \sec x$ 4 [Q.N. 7(b), Set 'A' 2069]

(Ans: $y\sin x = x + c$)

26. Solve: $xy\,\dfrac{dy}{dx} = x^2 + y^2$ [Q.N. 7(b)(OR), Set 'A' 2069]

(Ans: $y^2 = 2x^2(\log x + c)$)

27. Solve: $e^{x-y}\,dx + e^{y-x}\cdot dy = 0$ [Q.N. 4(a), Set 'B' 2069]
(Ans: $e^{2x} + e^{2y} = C$)

28. Solve: $(1 + x^2)\dfrac{dy}{dx} + 2xy = 4x^2$ [Q.N. 7(b), Set 'B' 2069]

$\left(\text{Ans: } (1 + x^2)\, y = \dfrac{4}{3}x^3 + C\right)$

29. Solve: $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$ [Q.N. 7(b)(OR), Set 'B' 2069]

$\left(\text{Ans: } \sin\left(\dfrac{y}{x}\right) = Cx\right)$

30. Solve: $x^2 dy - y^2 dx = 0$ [Q.N. 6(c), 2068]
(Ans: $y - x = c\,y$, c is a constant.)

31. Solve: $\sin x\,\dfrac{dy}{dx}\cos x.y = x\sin x$ [Q.N. 11(b)(Or), 2068]

(Ans: $y + x\cot x - c\,\text{cosec}\ x = 1$; c is constant.)

32. Solve: $x\,dy + (x+y)\,dx = 0$ [Q.N. 6(c), 2067]
(Ans: $2xy + x^2 = c^2$)

33. Solve : $(1 - x^2)\dfrac{dy}{dx} = 1 + xy$ [Q.N. 11(b) (Or)), 2067]

(Ans: $\sqrt{1 - x^2}\ y = sin^{-1} x + C$)

34. Solve the differential equation $(x + 2y -3)\ dy - (2x - y + 1)\ dx = 0.$ [Q.N. 6(c), 2066]
(Ans: $xy + y^2 - x^2 - 3y - x = c$)

35. Solve the differential equation: $(1 + x^2)\dfrac{dy}{dx} + y = e^{tan^{-1}x}$ [Q.N. 11(b)Or, 2066]

$\left(Ans: y = \dfrac{1}{2} e^{tan^{-1}x} + ce^{-tan^{-1}x}\right)$

36. Solve: $(xy^2 + x)\ dx + (yx^2 + y)\ dy = 0.$ [Q.N.6(c), 2065]
(Ans: $2x^3 + 2y^3 + 3x^2y^2 = c$)

37. Solve: $\dfrac{dy}{dx} + y\ Cot\ x = x.$ [Q.N.11(b, or), 2065]

(Ans: $y\ sinx = - x\ cosx + sinx + c$)

38. Solve : $xdy + (x + y)\ dx = 0$ [Q.N. 6(c), 2064]
(Ans: $y = x\ log\ (cx)$)

39. Solve : $2\dfrac{dy}{dx} = \dfrac{y}{x} + \dfrac{y^2}{x^2}$ [Q.N. 11(b)Or, 2064]

$\left(Ans: (y - x)^2 = cxy^2\right)$

40. Solve : $e^{x-y}\ dx + e^{y-x}\ dy = o$ [Q.N. 6(c), 2063]
(Ans.: $e^{2x} + e^{2y} = C$)

41. Solve : $\dfrac{dy}{dx} = \dfrac{y}{x} + tan\ \dfrac{y}{x}$.   [Q.N. 11(b) Or, 2063]

$\left(Ans.: sin\ \dfrac{y}{x} = cx\right)$

42. Solve : $x^2dy - y^2dx = 0$ [Q.N. 6(c), 2062]
(Ans.: $y - x = Axy$)

43. Solve : $\dfrac{dy}{dx} + \dfrac{1}{x} \cdot\ y = x^2$ [Q.N. 11(b)Or, 2062]

$\left(Ans.: xy = \dfrac{x^4}{4} + C\right)$

44. Solve : $x^2\ dy - y^2\ dx = 0$ [Q.N. 6(c), 2061]
(Ans: $y - x = Axy$)

45. Solve: $(x^2 - y^2)\dfrac{dy}{dx} = xy$ [Q.N. 11(b)Or, 2061]

(Ans: $2y^2\ logyC + x^2 = 0$)

46. Solve : $\sqrt{1 - x^2}\ dy + \sqrt{1 - y^2}\ dx = 0$ [Q.N. 6(c), 2060]
(Ans: $y = tan^{-1} x + C$)

47. Solve: $tanx\dfrac{dy}{dx} + y = sec\ x$ [Q.N. 11(b)Or, 2060]

(Ans: $ysinx = x + C$)

48. Solve: $x\ dy - y\ dx = 0.$ [Q.N. 2(b), 2059]
(Ans: $y = cx$)

49. Solve: $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy}$ [Q.N. 11(b) Or, 2059]

$\left(Ans: y^2 + x^2 = cx\right)$

50. Solve: $\dfrac{dy}{dx} = \dfrac{x^3 + 1}{y^3 + 1}$ [Q.N. 6(c), 2058]

$\left(Ans: \dfrac{y^4}{4} + y = \dfrac{x^4}{4} + x + c\right)$

51. Solve : $\cos^2 x \dfrac{dy}{dx} + y = 1$.   [Q.N. 11(b)Or, 2058]

$\left(Ans: y = 1 + ce^{-tanx}\right)$

52. Solve: $\dfrac{dy}{dx} = \dfrac{x^2 + x + 1}{y^2 + y + 1}$ [Q.N. 6(c), 2057]

$\left(Ans: \dfrac{y^3}{3} + \dfrac{y^2}{2} + y = \dfrac{x^3}{3} + \dfrac{x^2}{2} + x + A\right)$,

53. Solve: $\tan x \dfrac{dy}{dx} + y = \sec x$.   [Q.N. 11(b)Or, 2057]

  (Ans: ysinx = x + c)

Unit 10: Dispersion, Correlation and Regression

10.1 Measures of dispersions

1. The information about the daily temperature of two cities X and y are as follows:

	X	Y
Average temp. (°F)	84	92
Variance of temp.	16	25

    Determine which city has greater consistency in climate.   2[Q.N.4(b), 2072'C']
  (Ans: City X)

2. In the distribution of data 20, 25, 30, 36, 32, 43 ; find standard deviation.
  (Ans: 7.39)   2[Q.N.4(b), 2072'D']

3. For a group of 50 items; circle $\Sigma x^2 = 600$, $\Sigma x = 150$ and $m_0 = 1.75$, find the Pearsonian coefficient of skewness.   2[Q.N.4(b), 2072'E']
  (Ans: 0.72)

4. Calculate the coefficient of Skewness based on mean, mode and standard deviation from the following data: 4 [Q.N. 8(a), Set 'C' 2071]

Wages (in Rs.)	100	110	120	130	140
No. of persons	2	6	10	8	4

  (Ans: 0.18)

5. If n = 10, $\Sigma x = 120$, $\Sigma x^2 = 1530$, find the standard deviation and the coefficients of variation.   2 [Q.N. 4(b), Set 'D' 2071]
  [Ans: r = 3, C.V. = 25%]

6. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, N = 10 and $M_0 = 12.45$ find the skewness based on mean, mode and standard deviation.   4[Q.N. 8(a), 2070 'C']
  [Ans: –0.22]

7. Consider the following distribution.

	Distribution A	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation:	10	10

  Is the distribution A same as the distribution B regarding the degree of variation and skewness?   4[Q.N. 8(a), 2070 'D']
  (Ans: Yes, same)

8. The information about the daily temperature of two cities A and B are as follows:

	City A	City B
Average Temperature (F)	84	92
Variance of Temperature	16	25.

Determine which city has greater consistency in climate.

[Ans: A]   2 [Q.N. 4(b), Supp. 2069]

9. Determine the standard deviation and the coefficient of variation from the following distribution.   4 [Q.N. 8(a), Set 'A' 2069]

Profit (in Rs.)	0-10	10-20	20-30	30-40	40-50
No. of shops	8	13	16	8	5

(Ans: Rs. 11.88, 52.11%)

10. If $\Sigma fx = 110$, $\Sigma fx^2 = 1650$, $N = 10$ and $M_0 = 12.45$, find the skewness based on mean, mode and standard deviation.   [Q.N. 8(a), Set 'B' 2069]
(Ans: – 0.22)

11. Weights of a group of individuals are given below. Find out the mean and the standard deviation.   [Q.N. 12(a), 2068]

Weight (in kg.)	0-10	10-20	20-30	30-40	40-50
Frquency	12	33	30	15	10

(Ans: 27, 11.45)

12. Consider the following distribution:

	distribution A	distribution B
Arithmetic mean	100	90
Medium	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the degree of variation and skewness ?   [Q.N. 12(b), 2068]
(Ans: Yes, same)

13. In a frequency distribution of a set of data C.V. = 5%, $\sigma = 2$ and Karl Pearson coefficient of skewness = 0.5; find the mean of the data.   [Q.N. 4(b), 2067]
(Ans: 40)

14. Define standard deviation. Also prove that the root mean square deviation is not less than the standard deviation.   [Q.N. 12(b), 2067]

15. Find the standard deviation from the following data:

x: 10 11 12 13 14
f: 3 12 18 12 2

(Ans: 0.967)   [Q.N. 4(b), 2066]

16. Calculate the coefficient of skewness from the following frequency distribution: [Q.N.12(b), 2066]

Investment	10-20	20-30	30-40	40-50	50-60	60-70	70-80
	12	18	20	15	10	3	2

(Ans: 0.227)

17. Calculate Karl Pearson's coefficient of skewness of the data: [Q.N.12(a), 2065]

Marks	above 0	above 10	above 20	above 30	above 40
Frequency	150	140	100	80	80

(Ans: 0.89)

18. The information about the wages distribution of the firms A and B are given below :

	Firm A	Firm B
No. of workers	500	600
Average monthly wages	Rs. 586	Rs. 575
Variable of wages distribution	81	100

In which firm is the wages distribution uniform ?   [Q.N. 4(b), 2064]
(Ans: firm A)

19. For a group of 10 items, $\Sigma x = 452$, $\Sigma x^2 = 24,270$ and mode = 43.7, find the Pearson's coefficient of skewness. [Q.N. 12(a), 2064]
    (Ans: 0.076)

20. Consider the following distribution.

	Distribution a	Distribution B
Arithmetic mean:	100	90
Median:	90	80
Standard deviation	10	10

Is the distribution A same as the distribution B regarding the skewness ?
(Ans.: Same) [Q.N. 4(b), 2063]

21. Find the standard deviation of the following data : [Q.N. 4(b), 2062]
    10, 15, 20, 25, 30, 35, 40
    (Ans.: 10)

22. Following are the marks obtained by two students A and B in 10 tests of 100 marks each.

Test:	1	2	3	4	5	6	7	8	9	10
Marks of A :	44	80	76	48	52	72	68	56	60	54
Marks of B :	48	75	54	60	63	69	72	51	57	66

Who is intelligent ? [Q.N. 12(a), 2062]
(Ans.: B)

23. Find the mean deviation from mean of the following data :
    6, 8, 10, 13 and 5. [Q.N. 4(b), 2061]
    (Ans. 2.72)

24. Find out the mean and Standard Deviation from the following data :

Variable:	5-10	10-15	15-20	20-25	25-30	30-35
Frequency:	2	9	29	54	11	5

(Ans: 23.95 and 4.87) [Q.N. 12(a), 2061]

25. The median, mode and coefficient of skewness for a certain distribution are respectively 17.4, 15.3 and 0.35. Calculate mean and C.V. [Q.N. 12(b), 2061]
    (Ans: 18.45 and 48.78%)

26. Find the mean deviation from median of the numbers 5, 7, 10, 12 and 6.
    (Ans: 1.57) [Q.N. 4(b), 2060]

27. Find the mean and S.D. from the following table wages (Rs.) :

Wages:	10–20	10–30	10–40	10–50	10–60
No. of workers :	15	33	63	83	100

(Ans: Mean = 35.06 and S.d. = 12.45) [Q.N. 12(a), 2060]

28. Prove that in a discrete distribution the standard deviation is not less than the mean deviation from the mean. [Q.N. 12(b), 2060]

29. The coefficient of variation and mean of a certain frequency distribution are 50.2% and 22.8 respectively. Find the s.d. [Q.N. 4(c), 2059]
    (Ans: 11.4456)

30. Find the standard deviation of the following data:
    100, 150, 200, 250, 300. [Q.N. 4(b), 2058]
    (Ans: 70.7)

31. Write the expressions for the first four central moment and for β_2. Hence show that for a discrete distribution, $\beta_2 \geq 1$. [Q.N. 12(b), 2057]

$$\left(\text{Ans: } \mu_1 = 0, \mu_2 = \frac{\Sigma x^2}{N}, \mu_3 = \frac{\Sigma x^3}{N}, \mu_4 = \frac{\Sigma x^4}{N}, \beta_2 = \frac{\mu_4}{\mu_2} \right)$$

10.2 Correlation

1. Define correlation. Find Karl Pearson's coefficient of correlation of the marks of the following distribution. **4[Q.N.8(a), 2072'D']**

X	20	30	40	50	60
Y	50	46	30	24	8

(Ans: – 0.937)

2. If n = 15, σ_X = 3.2, σ_y = 3.4 and $\Sigma(X - \bar{X})(Y - \bar{Y})$ = 122, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), Set 'C' 2071]**
(Ans: 0.75)

3. Calculate Karl Pearson's correlation coefficient between the two variables height (in cms) and weight (in kg) from the data gives below: **4 [Q.N. 8(a), Set 'D' 2071]**
[Ans: 0.51]

Height	160	162	165	161	163
Weight	63	62	64	60	61

4. If $\Sigma(X - \bar{X})^2$ = 40, $\Sigma(Y-\bar{Y})^2$ = 63 and $\Sigma(X-\bar{X})(Y-\bar{Y})$= 35, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), 2070 'C']**
[Ans: 0.697]

5. If n = 10, ΣX = 60, ΣY = 60, ΣX^2 = 400, ΣY^2 = 580 and ΣXY = 415, find the correlation coefficient between the two variables. **2 [Q.N. 4(b), 2070 'D']**
(Ans: 0.59)

6. Find Karl Pearsons' coefficient of correlation of the marks of the following two group of students **4 [Q.N. 8(a), Supp. 2069]**
[Ans: –0.937]

X	20	30	40	50	60
Y	50	46	30	24	8

7. Calculate the correlation coefficient between two variables from the following data: Σx^2 = 114, Σy^2 = 422, and Σxy = 174 **[Q.N. 4(b), 2068]**
[Ans: 0.793]

8. If the covariance between the variable x and y is 18 and the variances of x and y are 16 and 81 respectively, find the coefficient of correlation between them.
(Ans: 0.5) **[Q.N.4(b), 2065]**

9. From the following table, calculate the coefficient of correlation by Karl Pearson's method. **[Q.N. 12(b), 2063]**

X : 6, 2, 10, 4, 8
Y : 9, 11, –, 8, 7

Arithmetic means of X and Y series are 6 and 8 respectively.
(Ans.: – 0.92)

10. Prove that the correlation coefficient between two variables lies between −1 and +1. **[Q.N. 12(b), 2062]**

11. Calculate Karl Pearson's coefficient of correlation from the following data:

x:	12	9	8	10	11	13	7
y:	14	8	6	9	11	12	3

(Ans: 0.95) **[Q.N. 12(b), 2059]**

12. Prove that the coefficient of correlation lies between-1 and 1. **[Q.N. 12(b), 2058]**

13. Calculate r_{xy} if Σx^2= 114; Σy^2 = 442 ; Σxy = 174. **[Q.N. 4(b), 2057]**
(Ans: 0.793)

10.3 Regression

1. Define regression and lines of regression. Find the correlation coefficients between the two variables when $b_{xy} = 1.8$ and $b_{yx} = 0.35$. 4[Q.N.8(a), 2072'C']
   (Ans: 0.79)

2. The regression coefficients of x on y and y on x are 0.84 and 0.32 respectively. if the arithmetic means of x and y series are 42 and 26 respectively, find two equations of lines of regression. 4[Q.N.8(a), 2072'E']
   (Ans: y = 0.32x + 12.56, x + 0.84y + 20.16)

3. Find the regression equation of y on x when:
   $\Sigma x = 15$, $\Sigma y = 25$, $\Sigma x^2 = 55$, $\Sigma y^2 = 140$, $\Sigma xy = 78$, n = 5.
   (Ans: y = 0.3x + 4.1) [Q.N. 4(b), Set 'A' 2069]

4. The regression coefficient of y on x is 0.32. If the arithmetic means of x and y series are 42 and 36 respectively, find the regression equation of y on x.
   (Ans: 0.32x – y – 6 = 0) [Q.N. 4(b), Set 'B' 2069]

Unit 11: Probability

11.1 Probability

1. In rolling a pair of dice, determine the probability of obtaining a sum of 10.
   $\left(\text{Ans: } \dfrac{1}{12}\right)$ 2[Q.N.4(c), 2072'C']

2. In a draw of a card from well shuffled deck of 52 cards what is the probability that it is a king or a queen?
   $\left(\text{Ans: } \dfrac{2}{13}\right)$ 2[Q.N.4(c), 2072'D']

3. Two dice are rolled once. What is the probability of getting a total of 8 or 7 ?
   $\left(\text{Ans: } \dfrac{11}{36}\right)$ 2[Q.N.4(c), 2072'E']

4. The chance that A can solve the problem is $\dfrac{3}{5}$ and the chance that B can solve the problem is $\dfrac{2}{3}$. Find the probability that the problem is solved. 2[Q.N. 4(c), Set 'C' 2071]
   $\left(\text{Ans: } \dfrac{13}{15}\right)$

5. Two coins are tossed simultaneously. Find the sample space. Find the probability that both are heads. 2 [Q.N. 4(c), Set 'D' 2071]
   [Ans: S ={HH, HT, TH, TT}, $P(HH) = \dfrac{1}{4}$]

6. A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that one is boy and one girl? 2 [Q.N. 4(c), 2070 'C']
   $\left(\text{Ans: } \dfrac{16}{33}\right)$

7. A card is drawn from a well–shuffled back of 52 cards. What is the probability that it is a king or a Diamond? 2 [Q.N. 4(c), 2070 'D']
   $\left(\text{Ans: } \dfrac{4}{13}\right)$

8. Two dice are rolled simultaneously. What is the probability of turning the same digit in both dice? 2 [Q.N. 4(c), Supp. 2069]

$\left(\text{Ans:} \dfrac{1}{6}\right)$

9. From 20 tickets marked from 1 to 20, one is drawn at random. Find the probability that it is a multiple of 4 or 5. [Q.N. 4(c), Set 'A' 2069]

$\left(\text{Ans:} \dfrac{2}{5}\right)$

10. A bag contains 5 red and 6 white balls. Two balls are drown at random. Find the probability that (i) both are red (ii) both are of the same colour. [Q.N.8(b), Set 'B' 2069]

$\left(\text{Ans: (i)} \dfrac{2}{11} \text{(ii)} \dfrac{5}{11}\right)$

11. A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability that is a red 8, a red 9 or a red 10? [Q.N. 4(c), 2068]

$\left(\text{Ans:} \dfrac{3}{26}\right)$

12. The chance that A can solve a certain problem is $\dfrac{3}{4}$. The chance that B can solve it is $\dfrac{2}{3}$, find the chance that the problem will be solved if they both try. [Q.N.8(b),2068]

$\left(\text{Ans:} \dfrac{3}{4}\right)$

13. Define mutually exclusive events and dependent cases with example while performing an experiment. [Q.N. 4(c), 2067]

14. A class consists of 40 boys and 60 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl? [Q.N. 8(a), 2067]

$\left(\text{Ans: (a)} \dfrac{^{60}C_2}{^{100}C_2}, \text{(b)} \dfrac{^{40}C_2}{^{100}C_2}, \text{(c)} \dfrac{^{60}C_1 \infty^{40}C_1}{^{100}C_2}\right)$

15. The chance that A can solve a problem is $\dfrac{1}{4}$, the chance that B can solve it is $\dfrac{2}{3}$. Find the probability that the problem will be solved if both of them try. [Q.N. 4(c), 2066]

$\left(\text{Ans:} \dfrac{3}{4}\right)$

16. Five men in a group of 20 are graduates. If three men are chosen out of 20 at random, what is the probability of at least one being graduates? [Q.N. 8(a)Or, 2066]

$\left(\text{Ans:} \dfrac{137}{22}\right)$

17. Given P(A) = 0.4, P(A ∪ B) = 0.56, P(B) = 0.3.
Are A & B independent ? [Q.N.4(c), 2065]
(Ans: not independent)

18. State and prove the theorem of compound probability [Q.N.8(a), 2065]

19. A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability of drawing (i) a white ball (ii) not a black ball. [Q.N. 4(c), 2064]

$\left(\text{Ans: (i)} \dfrac{7}{20}, \text{(ii)} \dfrac{4}{5}\right)$

20. If P(A) and P(B) are the probabilities of the happening of the events A and B respectively, prove that : P(A∪B) = P(A) + P(B) − P(A∩B)
where P(A∪B) and P(A∩B) have the usual meanings. What will be the form of the above formula if A and B are independent events ? [Q.N. 8(a), 2064]

21. The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that the problem will be solved if they both try.

$\left(Ans.: \frac{3}{4}\right)$ **[Q.N. 4(c), 2063]**

22. If P(A) and P(B) be the probabilities of the independent events A and B respectively, prove that :

$$P(A \cap B) = P(A). P(B)$$

Where $P(A \cap B)$ has the usual meaning.   **[Q.N. 8(a), 2063]**

23. What is the probability that an English alphabet selected at random is (i) a vowel (ii) a consonant ?   **[Q.N. 4(c), 2062]**

$\left(Ans.: (i) \frac{2}{7} \ (ii) \frac{5}{7}\right)$

24. State and prove "The Theorem of Total Probability".   **[Q.N. 8(a), 2062]**

25. A lot contains 10 items of which 3 are defective. Three items are chosen from the lot at random one after another without replacement. Find the probability that :

    (i)   All three are defective.

    (ii)   None of them are defective.   **[Q.N. 8(a)Or, 2062]**

$\left(Ans.: (i) \frac{1}{120} \ (ii) \frac{119}{120}\right)$

26. State and prove the "Theorem of Compound Probability".   **[Q.N. 8(a), 2061]**

27. A class consists of 60 boys and 40 girls. If two students are chosen at random, what will be the probability that (a) both are boys (b) both are girls (c) one boy and one girl ?   **[Q.N. 8(a) Or, 2061]**

$\left(Ans: (i) \frac{^{60}C_2}{^{100}C_2} \ (ii) \frac{^{40}C_2}{^{100}C_2} \ (iii) \frac{^{60}C_1 \infty \ ^{40}C_1}{^{100}C_2}\right)$

28. Two dice are thrown. Determine the probability of getting a sum ≤ 5.

$\left(Ans.: \frac{5}{18}\right)$ **[Q.N. 4(c), 2060]**

29. State and prove the "Theorem of total probability".   **[Q.N. 8(a), 2060]**

30. Two letters are selected at random from the word "examination". Find the probability that both of them are same letters.   **[Q.N. 3(c), 2059]**

$\left(Ans.: \frac{3}{55}\right)$

31. If A, B, C are three mutually exclusive events with $\frac{1}{3}$ P(A) = $\frac{2}{3}$ P(B) = $\frac{1}{6}$ P(C), find P(A); P(B); and P(C).   **[Q.N. 8(a), 2059]**

$\left(Ans.: \frac{2}{7}, \frac{1}{7}, \frac{4}{7}\right)$

32. If A and B are two independent events with P(A) = $\frac{2}{3}$ and P(B) = $\frac{3}{5}$, find P(A∪B). $\left(Ans.: \frac{13}{15}\right)$ **[Q.N. 4(c), 2058]**

33. State and prove the "Theorem of total probability".   **[Q.N. 8(a), 2058]**

34. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability of being it (i) a red card (ii) a heart. **[Q.N. 4(c), 2057]**

$$\left(Ans: (i)\frac{1}{2} \text{ and } (ii)\frac{1}{4}\right)$$

35. State and prove the iTheorem of Total probabilityî **[Q.N. 8(b), 2057]**

11.2 Binomial Distribution

1. Define Binomial distribution. Find the probability of getting (i) two heads (ii) at least two heads in 5 tosser of a coin. **4[Q.N.8(b), 2072'C']**

$$\left(Ans: (i)\frac{5}{16} (ii)\frac{13}{16}\right)$$

2. The probability of hitting a target is found to be 0.25. If eight hits are made, find the probability that (i) none will hit the target (ii) exactly two will hit the target. **4[Q.N.8(b), 2072'D']**

$$\left(Ans: \frac{6561}{65536}, \frac{5103}{16384}\right)$$

3. Four coins are tossed simultaneously. What is the probability of getting
   i) 2 heads ii) 4 heads. **4[Q.N.8(b), 2072'E']**

$$\left(Ans: (i)\frac{3}{8} (ii)\frac{1}{16}\right)$$

4. A dice is rolled 3 times. Getting 5 or 6 is numbered as success. Find the probability of getting (i) 2 successes (iii) 3 successes. **4[Q.N. 8(b), Set 'C' 2071]**

$$\left(Ans: \frac{2}{9}, \frac{1}{27}\right)$$

5. Suppose that in a certain city 60% of all recorded births are males, suppose we select 5 birth records from the population. What is the probability that : i) three of them are males ii) more than 4 are males. **4[Q.N. 8(b), Set 'D' 2071]**

$$[Ans: (i)\frac{216}{625}, (ii)\frac{243}{3125}]$$

6. A certain manufacturing process produces electrical fuses of which 15% are defective. Find the probability that in a sample of 10 fuses selected at random there will be i) no defective ii) not more than one defective. **4[Q.N. 8(b), 2070 'C']**

   [Ans: (i) 0.1969, (ii) 0.5443]

7. A coin is tossed 5 times. Find the probability of getting
   i) two heads   ii) atleast two hands. **4[Q.N. 8(b), 2070 'D']**

$$\left(Ans: (i)\frac{5}{16} (ii)\frac{13}{16}\right)$$

8. Four coins are tossed simultaneously. What is the probability of getting (i) 2 heads and 2 tails (ii) at least 2 heads. **4[Q.N. 8(b), Supp. 2069]**

$$\left(Ans: (i)\frac{3}{8}, (ii)\frac{11}{16}\right)$$

9. If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random
   (i) l   (ii) at most 2 bulbs will be defective. **4[Q.N. 8(b), Set 'A' 2069]**

$$\left(Ans: (i)\frac{256}{625} (ii)\frac{608}{625}\right)$$

10. A dice is thrown 3 times. Getting a 2 or 3 is numbered as a success. Find the probabilities of getting two successes. **[Q.N. 4(c), Set 'B' 2069]**

$$\left(Ans: \frac{2}{9}\right)$$

11. A dice is thrown 3 times. Getting a '5' or '6' is numbered a success. Find the probability of getting (i) 3 success (ii) exactly 2 successes. **[Q.N. 8(b)(Or), 2068]**

$\left(Ans: (i) \dfrac{1}{27} (ii) \dfrac{2}{9} \right)$

12. Define Binomial distribution, and its mean and variance, hence find the probability of getting three heads in five tosses of two coins. **[Q.N. 8(Or), 2067]**

$\left(Ans: \dfrac{45}{512} \right)$

13. The probability of hitting a target is $\dfrac{1}{6}$, if eight hittings are made, find the probability that (i) none will strike the target, (ii) exactly two will strike the target. **[Q.N. 8(a), 2066]**

$\left(Ans: (i) \dfrac{390625}{1679616} (ii) \dfrac{109375}{419904} \right)$

14. If three dices are thrown what is the probability of getting
     (i)   exactly 3 sixes
     (ii) exactly 2 sixes. **[Q.N.8. (a, or), 2065]**

$\left(Ans: \dfrac{5}{72} \right)$

15. Suppose that in a certain city 60% of all recorded births are males. If we select 5 births from the population, what will be the probability that :
     (i)   none of them is male
     (ii) exactly three of them are male **[Q.N. 8(a)Or, 2064]**

$\left(Ans: (i) \dfrac{243}{3125} (ii) \dfrac{216}{625} \right)$

16. The incidence of occupation disease in an industry is such that the workmen have a 20% chance of suffering from it. What is the probability that out of six workmen four or more will contact the disease ?

$\left(Ans.: \dfrac{53}{3125} \right)$ **[Q.N. 8(a) Or, 2063]**

17. Find the binomial distribution having mean = 12 and variance = 8.

$\left(Ans: \left(\dfrac{2}{3} + \dfrac{1}{3} \right)^{36} \right)$ **[Q.N 4(c), 2061]**

18. A sample of 100 fuses is known to have an average 5 defective fuses. Three fuses of sample are tested. What is the probability that (i) none of them is defective (ii) exactly one of them is defective ? **[Q.N. 8(a) Or, 2060]**

$\left(Ans: (i) \dfrac{6859}{8000} (ii) \dfrac{1141}{8000} \right)$

19. If 4 dice are thrown, what is probability of getting (i) exactly 3 sixes (ii) exactly 2 sixes and (iii) no sixes. **[Q.N. 8(a) Or, 2059]**

$\left(Ans: (i) \dfrac{5}{324} (ii) \dfrac{25}{216} (iii) \dfrac{625}{1296} \right)$

20. The probability of hitting a target is $\dfrac{1}{5}$. If six hittings are made, find the probability that:
     (i) none will strike the target (ii) exactly 2 will strike the target. **[Q.N. 8(a) Or, 2058]**

$\left(Ans: (i) \dfrac{4096}{15625} (ii) \dfrac{512}{3125} \right)$

21. A dice is thrown 3 times. Getting a '5' or '6' is numbered a success. Find the probability of getting (a) 3 successes and (b) exactly 2 successes. **[Q.N. 8(b) Or, 2057]**

$\left(Ans: (a) \dfrac{1}{27} and (b) \dfrac{2}{9} \right)$