5. Mathematics-II

Course Content

Group 'A'

Unit 1: Permutation and Combination. **10 hrs**

Basic principle of counting, Permutation of (a) set of objects all different (b) set of objects not all different (c) circular arrangement (d) repeated use of the same object. Combination of things all different, Properties of combination.

Unit 2: Binomial Theorem **10 hrs**

Binomial theorem for a positive integral index, general term. Binomial coefficients, Binomial theorem for any index (Without proof), Application to approximation, Euler's number. Expansion of ex, ax and log (1 + x) (without proof).

Unit 3: Elementary Group Theory **8 hrs.**

Binary operation, Binary operation on sets of integers and their properties, Definition of a Group, Groups whose element are not numbers, Finite and infinite groups, Uniqueness of identity, Uniqueness of inverse, Cancellation law, Abelian Group.

Unit 4: Conic Sections **12 hrs**

Standard equation of parabola, Ellipse and Hyperbola, Equations of tangent and normal to a parabola at a given point.

Unit 5: Co-ordinates in Space **12 hrs**

Co-ordinate axes, Co-ordinate planes, The octants,, Distance between two points, External and internal point of division, Direction cosines and ratios , fundamental relation between direction cosines, Projections, Angle between two lines. General equation of a plane, Equation of a plane in intercept and normal form, Plane through three given points, Plane through the intersection of two given planes, Parallel and perpendicular planes, angle between two planes distance of a point from a plane.

Unit 6: Vectors and its Applications **14 hrs**

Cartesian representation of vectors, Collinear and non-collinear vectors, Coplanar and non-Coplanar vectors, Linear combination of vectors. Scalar product of two vectors, Angle between two vectors, Geometric interpretation of scalar product, Properties of Scalar Product, Condition of perpendicularity. Vector product of two vectors, Geometric interpretation of vector product, Properties of Vector Product, Application of product of vectors in plane trigonometry.

Unit 7: Derivative and its Application **14 hrs**

Derivative of inverse trigonometric, exponential and logarithmic functions by definition, Relationship between continuity and differentiability, Rules for differentiating hyperbolic function and inverse hyperbolic function, Composite function and function of the type $f(x)g(x)$. L'Hospital's rule (for 0/0, %), Differentials, Tangent and Normal, Geometric interpretation and application of Rolle's theorem and Mean value theorem.

Unit 8: Antiderivatives **7 hrs**

Antiderivatives, Standard integrals, Integrals reducible to standard forms, Integrals of rational functions.

Unit 9:Differential Equations and their Applications **7 hrs**

Differential equation and its order and degree, Differential equations of first order and first degree: Differential equations with separable variables, homogeneous and exact differential equations.

Unit 10: Dispersion, Correlation and Regression **12 hrs**

Dispersion, Measures of dispersion (Range, Semi interpqrtile range, Mean deviation, Standard deviation) variance, Coefficient of variation, Skewness, Karl Pearson's and Bowley's Coefficient of Skewness, Bivariate distribution, Correlation, Nature of correlation, Correlation coefficient by Karl Pearson's method. Interpretation of correlation coefficient, Properties of correlation coefficient (Without proof) Regression equation, Regression line of y on x and x on y.

Unit 11: Probability **8 hrs**

Random experiment, sample space, Event, Equally likely cases, Mutually exclusive events, Exhaustive cases, Favourable cases, Independent and dependent cases, Mathematical and empirical definition of probability, Two basic laws of probability, Conditional probability (without proof), Binomial distribution, Mean and Standard deviation of binomial distribution (without proof).

Group 'B'

Unit 12: Statics 9 hrs.

Forces and resultant forces, Parallelogram of forces, Composition and resolution of forces, Resultant of coplanar forces acting at a point, Triangle of forces and Lami's theorem.

Unit 13: Statics (Continued) 9 hrs

Resultant of like and unlike parallel forces, Moment of a force, Varignon's theorem, Couple and its properties (without proof).

Unit 14: Dynamics 9hrs

Motion of particle in a straight line, Motion with uniform acceleration, Motion under gravity, Motion down a smooth inclined plane. The concepts and theorems be restated and formulated as application of calculus.

Unit 15: Dynamics (Continued) 9 hrs

Newton's laws of motion, Impulse, Work, Energy and Power, Projectiles.

Group 'C'

Unit 16: Linear Programming 11 hrs

Introduction of a linear programming problem (LPP), Graphical solution of LPP in two variables, Solution of LPP by simplex method (two variables).

Unit 17: Computational Method 9 hrs

Introduction to Numerical computing (Characteristics of Numerical computing Accuracy, Rate of Convergence, Numerical Stability, Efficiency); Number systems (Decimal, Binary, Octal & Hexadecimal system conversion of one system into another), Approximations and error in computing Roots of nonlinear equation, Algebraic, polynomial & transcendental equations and their solution by bisection and Newton – Raphson Methods,

Unit 18: Computational Method (Continued) 8 hrs

Solution of system of linear equations by Gauss elimination method, Gauss-Seidel method, Ill conditioned systems, Matrix inversion method.

Unit 19: Numerical Integration 8 hrs

Trapezoidal and Simpson's rules, estimation of errors.

Model Question 2068

Time: 3 hrs. **Full Marks:- 100**

Pass Marks:- 35

Attempt all questions of group A and group B or C.

Group A

1. (a) In an examination paper containing 10 questions, a candidate has to answer 7 questions. If two questions are made compulsory, in how many ways can be choose 7 questions in all ? (2)

(Ans: 56 ways) **(From Unit: 1.2)**

(b) Find the middle term in the expansion of $\left(2x + \frac{1}{3x^2}\right)^9$. (2)

$\left(\text{Ans: } \left(\frac{448}{9}x^{-3}, \frac{224}{27}x^{-6}\right)\right)$ **(From Unit: 2.1)**

(c) Let S = {–1, 1} and * denote the usual operation of multiplication. Represent it by Cayley is table. Show that * is a binary operation on S. (2)

(From Unit: 3.1)

2. (a) Find the eccentricity and the foci of the ellipse : $x^2 + 4y^2 - 4x + 24y + 24 = 0$. (2)

$\left(\text{Ans: } \left[\frac{\sqrt{3}}{2}, (2 \pm 2\sqrt{3}, -3)\right]\right)$ **(From Unit: 4.2)**

(b) Find the point where the line through the points (1, 2, 3) and (4, –4, 9) meets the zx – plane. (2)

(Ans: 2, 0, 5) **(From Unit: 5.2)**

(c) Are the three points with position vectors $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$ collinear ? Justify your answer. (2)
(Ans: yes) **(From Unit: 6.1)**

3. (a) Using L' Hospitals rule, evaluate

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{\sin^2 x}$$ (2)

(Ans: 2) **(From Unit: 7.5)**

(b) Evaluate : (2)

$$\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}} \ (\beta > \alpha).$$ **(From Unit: 8]**

$$\left(\textbf{Ans: } 2\log\left(\sqrt{x-\alpha} + \sqrt{x-\beta} + C\right)\right)$$

(c) If $\vec{a} = 6\vec{i} + 3\vec{j} - 5\vec{k}$ and $\vec{b} = \vec{i} - 4\vec{j} + 2\vec{k}$ show that $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. (2) **(From Unit: 6.2)**

4. (a) Solve : $x\frac{dy}{dx} + y - 1 = 0$. (2)
(Ans: x(y – 1) = C) **(From Unit: 9)**

(b) If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. (2)
(Ans: 0.59) **(From Unit: 10.2)**

(c) Two dice are rolled once. What is the probability of getting a total of 9 or 6 ?(2)
$\left(\textbf{Ans: } \frac{1}{4}\right)$ **(From Unit : 11.2)**

5. (a) In how many ways can the letters of the word "COMPUTER" be arranged so that (4)
(i) all the vowels are always together ?
(Ans: 4320 ways)
(ii) the vowels may occupy only odd positions ?
(Ans: 2880 ways) **(From Unit: 1.1)**

(b) Given the algebraic structure (G,*) with G = {1, ω, ω²} where ω represents an imaginary cube root of unity and * stands for the binary operation of multiplication, show that (G,*) is a group. (4)
(From Unit: 3.2)

6. (a) Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1). Express it in the slope form.
(Ans: $yy_1 = 2a(x + x_1)$, $y = mx + \frac{a}{m}$**)**

OR What is a conic section ? Find the equation of the parabola in the standard form. (4)
(Ans: $y^2 = 4ax$) **(From Unit: 4.1)**

(b) Find the equation of the plane through the point (2, 1, 4) and perpendicular to each of the planes
$9x - 7y + 6z + 48 = 0$ and $x + y + z = 0$ (4)
(Ans: 13x + 3y – 16z + 35 = 0) **(From: 5.2)**

7. (a) Evaluate : (4)

$$\int \frac{dx}{a + b\cos x} (a > b > 0).$$ (4)

$\left(\textbf{Ans: } \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \tan\frac{x}{2}\right) + C\right)$ **(From Unit: 8)**

(b) Solve : $x^2\frac{dy}{dx} + y^2 = xy$

(Ans: $x + y \log x = y C$) **(From Unit: 9)**

OR

Solve : $(1 - x^2)\frac{dy}{dx} - xy = 1$ (4)

(Ans: $y\sqrt{1 - x^2} = \sin^{-1}x + C$) **(From Unit : 9).**

8. (a) Find Karl Pearson's coefficient of skewness from the following distribution. (4)

(Ans: –0.41) **(From Unit : 10.1)**

Marks	Above 20	Above 30	Above 40	Above 50	Above 60
No. of students	50	46	30	24	8

(b) The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that (i) the problem will be solved if they both try (ii) A solves but B cannot.

$\left(\textbf{Ans: (i) } \frac{3}{4} \textbf{ (ii) } \frac{1}{12}\right)$ **(From Unit : 11.1)**

OR

Suppose that in a certain city 60% of all the recorded births are male. Suppose we select 5 birth records from population. What is the probability that (4)

(i) exactly three of them are male ?

$\left(\textbf{Ans: } \frac{216}{625}\right)$

(ii) 4 or more are male ?

$\left(\textbf{Ans: } \frac{343}{3125}\right)$ **(From Unit : 11.2)**

9. Show that :

$$\sum_{n=1}^{\infty} \frac{n^2}{(n+1)^2} = e - 1.$$ (6) **(From Unit: 2.2)**

10. Define scalar product of two vectors. Find the geometrical interpretation of scalar product of two vectors. Prove vectorically that
$\cos(A + B) = \cos A \cos B - \sin A \sin B$. (6)
(From Unit : 6.2)

11. State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = x(x - 1)^2$ in [0, 1]
Also, find the point on the curve where the tangent is parallel to the x–axis. (6)

$\left(\textbf{Ans: } (1, 0), \left(\frac{1}{3}, \frac{4}{27}\right)\right)$ **(From Unit: 7.5)**

OR

Find from first principle the derivative of $\log\cos^{-1}x$. (6)

$\left(\text{Ans: } \dfrac{-1}{\sqrt{1-x^2}\cos^{-1}x}\right)$ **(From Unit: 7.2)**

Group B

12. (a) Forces equal to 7P, 5P and 8P acting on a particle are in equilibrium. Find the angle between the latter pair of forces. (2)
(Ans: 120°) **(From Unit: 12)**

(b) A body is projected vertically upwards with a velocity of 19.6 m/s. How long will it take to reach a point 294m below the point of projection ? (g = 0.8 m/s) (2)
(Ans: 10 sec.) **(From Unit: 14.2)**

(c) A body of mass 50 kg falling from a certain height is brought to rest after striking the ground with a speed of 5m/s. If the resistance force of the ground is 500N, find the duration of the contact. (2)
(Ans: 0.5 sec.) **(From Unit: 15.1)**

13. (a) P and Q are two like parallel forces acting at A and B. Show that if they interchange positions, the point of application of the resultant is displaced by a distance $\dfrac{P-Q}{P+Q}$. AB. **(From Unit: 13.1)**

OR

Forces 1P, 2P and 3P act at a point in direction parallel to the sides of an equilateral triangle taken in order. Find their resultant. (4)
(Ans: $\sqrt{3}$ P, ⊥^{r} to force 2P) **(From Unit : 13.2)**

(b) Prove that the sum of the kinetic and potential energies of a freely falling body remains constant throughout the motion. (4)
(From Unit: 15.3)

14. The horizontal and the vertical components of the initial velocity of a projectile are U and V respectively. If R be the horizontal range and H, the greatest height attained, prove that

(i) $\dfrac{4H}{R} = \dfrac{V}{U}$ (ii) $\left(\dfrac{R}{U}\right)^2 = \dfrac{8H}{g}$. **(From Unit : 15.2)**

OR

A cat seeing a mouse at a distance of 15m before it, starts from rest with an acceleration of 2 m/s^2 and pursues it. If the mouse be moving uniformly with a velocity of 14 m/s, find when and where the cat will catch the mouse. (6)
(Ans: 15 sec, running 210m i.e 225m from the starting point of cat.)
(From Unit: 14.2)

15. Define the moment of a force about a point and interpret its geometrical meaning. Prove that the algebraic sum of the moments of two intersecting forces about any point in their plane is equal to the moment of their resultant about the same point. (6)
(From Unit: 13.2)

Group C

16. (a) If a man rides his car at 25 km/hr, he has to spend Rs. 2 per km on petrol. If he rides it at a faster speed of 40 km/hr, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Formulate the above problem as a linear programming problem. (2)

(Ans: Max F(x, y) = x + y subject to

$2x + 5y \le 100$

$8x + 5y \le 200$

$x \ge 0, y \ge 0$) **(From Unit :16)**

(b) Convert the decimal number 2011 into octal form. (2)

(Ans: 3733_8) **(From Unit :17)**

(c) Is the following equations diagonally dominant :

$12x + 3y - 5z = 1, x + 5y + 3z = 28, 3x + 7y + 13z = 1$? (2)

(Ans: Yes, diagonally dominated) **(From Unit :18)**

17. (a) Using Gauss elimination method, solve the following system of equations. (4)

$x + 3y - z = -2$

$3x + 2y - z = 3$

$-6x - 4y - 2z = 18$

(Ans: –24, 8, 2) **(From Unit: 18)**

Or

Solve the following equations using Gauss–Seidal method :

$2x_1 - x_2 = 8$

$3x_1 + 7x_2 = -5$

(Ans: 3, –2) **(From Unit: 18)**

(b) Evaluate the following integral using Simpson's rule :

$\int_0^1 \frac{dx}{1} + x^2$ taking 4 equal intervals (i.e. n = 4) (4)

(Ans: 0.785) **(From Unit 19)**

18. Using Simplex method, maximize $Z = 5x_1 + 7x_2$ subject to :

$2x_1 + 3x_2 \le 13$

$3x_1 + 2x_2 \le 12$

$x_1, x_2 \varepsilon 0$. (6)

(Ans: Max Z = 31, at (2, 3)) **(From Unit 16)**

19. Show that the equation $f(x) = x^3 - 18 = 0$ has only one positive root. Using bisection method, find the positive root correct to 3 places of decimal in the interval (2, 3). (6)

(Ans: 2.620) **(From Unit 17)**

OR

Use Netwon–Raphson method to find the positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to three places of decimals. (6)

(Ans: 1.154) **(From Unit 17)**

Exam Questions

Group 'A'

Unit 1: Permutation and Combination

1.1 Permutation

1. In how many ways can 7 students be seated in a circle? 2[Q.N.1(a), 2072'C']

(Ans: 720)

2. In how many ways the letters of the works ELEMENT can be arranged so that vowels are always together? 2[Q.N.1(a), 2072'D']

(Ans: 480)

3. Find the number of ways in which 4 men and 3 women can be seated in a row having seven seats so that the men and the women must alternate.
(Ans: 144) 2[Q.N.1(a), 2072'E']

4. In how many ways can the letters of the word "TUESDAY" be arranged ? How many of these arrangements do not begin with T? How many begin with T and do not end with Y? 4 [Q.N. 5(a), Set 'C' 2071]
(Ans: 5040, 4320, 600)

5. In how many ways can the letter of the word "COMPUTER" be arranged so that i) all vowels are always together? ii) the relative positions of the vowels and consonants are not changed? 4 [Q.N. 5(a), Set 'D' 2071]
[Ans: (i) 4320 (ii) 720

6. In how many ways can the letters of the word, 'CALCULUS" be arranged so that the two L's do not come together? 4 [Q.N. 5(a), 2070 'C']
[Ans: 3780]

7. In how many ways can the letters of the word "ELEMENT" be arranged?
(Ans: 840) 2 [Q.N. 1(a), 2070 'D']

8. In how many ways the letters of the word MILLENIUM can be arranged?
(Ans: 45360) 2 [Q.N. 1(a), Supp. 2069]

9. In how many ways can four boys and three girls be seated in a row containing seven seats if they may sit any where? [Q.N. 1(a), Set 'A' 2069]
(Ans: 5040)

10. In how many ways can the letters of the word "ARRANGE" be arranged so that no two R is come together? [Q.N. 5(a), Set 'A' 2069]
(Ans: 900)

11. In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begin with M and do not end with N? [Q.N. 5(a), Set 'B' 2069]
(Ans: 600, 96)

12. How many license plates consisting of 3 different digits can be made out of given integers 3,4,5,6,7 ? [Q.N. 2(a), 2068]
[Ans: 60)

13. In how many ways letters of the word PRECARIOUS can be arranged so that all the vowels are always together ? [Q.N. 2(a), 2067]
(Ans: 43,200)

14. How many numbers are there between 100 and 1000 such that every digit is either 2 or 9 ? [Q.N.2(a), 2065]
(Ans: 8)

15. Prove that the total no. of permutations of a set of n objects taken r at a time is given by $P(n, r) = n(n-1)(n-2) \ldots\ldots (n-r+1), n \in r$. [Q.N.7(b), 2065]

16. Show that the number of ways in which the letters of the word "arrange" can be arranged so that two r's do not come together is 900. [Q.N. 7(b), 2064]

17. How many numbers of three different digits less than 500 can be formed from the integers 1, 2, 3, 4, 5, 6 ? [Q.N. 2(a), 2063]
(Ans.: 80)

18. In how many ways can 4 Art students and 4 Science students be arranged alternately at a round table ? [Q.N. 7(b), 2062]
(Ans.: 144)

19. In how many ways can 6 different beads be strung on a necklace ?
(Ans: 60) [Q.N. 2(a), 2061]

20. In how many ways can the letters of the word 'MONDAY' be arranged ? How many of these arrangements do not begin with M ? How many begin with M and don't end with Y ? [Q.N. 7(b), 2061]
(Ans: 720, 600, 96)
21. Find the numbers of permutation of the letters of the word 'MATHEMATICS'.
(Ans: 4989600) [Q.N. 2(a), 2060]
22. How many permutations are there of the letters of the word 'mathematics' taken all together? [Q.N. 1(b), 2059]
(Ans: 4989600)
23. Prove that the total number of permutations of a set of n objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$. [Q.N. 7(b), 2059]
24. In a certain election, there are three candidates for president, five for secretary and only two for the treasurer. Find in how many ways the election may turn out.
(Ans: 30 ways) [Q.N. 2(a), 2058]
25. In how many ways can the letters of the word ARRANGE be arranged so that no two R's come together. [Q.N. 7(b), 2057]
(Ans: 900)

1.2 Combination

1. A committee of five persons is to be selected from 5 men and 4 ladies. In how many ways can this be done so that at least two ladies are always included.
(Ans: 120) 4[Q.N.5(a), 2072'C']
2. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives? 4[Q.N.5(a), 2072'D']
(Ans: 336)
3. In a group of 10 students, 6 are boys. In how many ways can 4 students be selected for mathematical competition so as to include at most two girls?
(Ans: 185) 4[Q.N.5(a), 2072'E']
4. A man has 5 friends. In how many ways can he invite one or more of them to a dinner? 2[Q.N. 1(a), Set 'D' 2071]
[Ans: 31]
5. Find the number of ways in which 5 courses out of 8 can be selected when 3 courses are compulsory.
(Ans: 10 ways) 2 [Q.N. 1(a), Set 'C' 2071]
6. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least two gentlemen?
(Ans: 240) 4 [Q.N. 5(a), 2070 'D']
7. From 10 persons, in how many ways can a selection of 4 be made when two particular persons are always included?. 2 [Q.N. 1(a), 2070 'C']
(Ans: 84)
8. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives. 4 [Q.N. 5(a), Supp. 2069]
(Ans: 336)
9. From 10 persons, in how many ways can a selection of 4 be made if two particular persons are always excluded. [Q.N. 1(a), Set 'B' 2069]
(Ans: 70)
10. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include atleast 2 ladies?
(Ans: 186) [Q.N. 5(a)(OR), Set 'B' 2069]

11. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives ? [Q.N. 7(b), 2068]
(Ans: 336)

12. A committee of five is to be constituted from six boys and five girls. In how many ways can this be done so as to include at least one boy and one girl ?
(Ans: 455) [Q.N. 7(b), 2067]

13. From 10 persons in how many ways can a selection of 4 be made if two particular persons are always excluded? [Q.N. 2(a), 2066]
(Ans: 70)

14. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite seven guests so that 5 of them may be relatives?
(Ans: 336) [Q.N. 7(b), 2066]

15. A person has got 12 acquaintances of whom 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives ? [Q.N. 2(a), 2064]
(Ans: 336)

16. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is not permitted to attempt more than 4 from any group. In how many different ways can he make up his choice ?
(Ans.: 200) [Q.N. 7(b), 2063]

17. From 10 persons, in how many ways can a committee of 4 be made when one particular person is always included. [Q.N. 2(a), 2062]
(Ans.: 84)

18. From 6 gentlemen and 4 ladies a committe of 5 is to be formed. In how many ways can this be done so as to include at least one lady ? [Q.N. 7(b), 2060]
(Ans: 246)

19. From 10 football players in how many ways can a selection of 4 be made
(i) when one particular player is always included (ii) when two particular players are always excluded. [Q.N. 7(b), 2058]
(Ans: (i) 84 (ii) 70)

20. A committee is to be chosen from 12 men and 8 women and is to consist of 3 men and 2 women. How many such committee can be formed ?
(Ans: 6160) [Q.N. 2(a), 2057]

Unit 2: Binomial Theorem

2.1 Binomial Theorem

1. Find the coefficient of the term containing x^2 in the expansion of $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$. 2
$\left(\text{Ans: } \frac{20}{3}\right)$ 2[Q.N.1(b), 2072'C']

2. State Binomial theorem. In the expansion of $(1 + x)^n$ prove that the sum of the coefficients of the odd terms is equal to the sum of coefficients of the even terms and each equals to 2^{n-1}. 6[Q.N.9, 2072'D']

3. If $(1 + x)^n = c_0 + c_1x + c_2x^2 + ... + c_nx^n$, prove that:
$c_0c_n + c_1c_{n-1} + c_2c_{n-2} + ... + c_nc_0 = \frac{2n!}{n!n!}$. 6[Q.N.9, 2072'E']

4. Find the term independent of x in the expansion of $\left(x^2-\frac{1}{3x^2}\right)^{12}$.
$\left(\text{Ans: } t_7 = \frac{308}{243}\right)$ 2 [Q.N. 1(b), Set 'C' 2071]

5. Find the coefficient of x in the expansion of $\left(x^2+\frac{a^2}{x}\right)^5$. 2 [Q.N. 1(b), Set 'D' 2071]

(Ans: $10a^6$)

6. Show that the middle term in the expansion of

$(x-\frac{1}{x})^{2n}$ is $\frac{1.3.5. \cdots (2n-1)}{n!}(-2)^n$ 6 [Q.N. 9, 2070 'C']

7. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_n X^n$, prove that:

$C_0C_n + C_1C_{n-1} + ... + C_n C_0 = \frac{2n!}{(n!)^2}$. 6 [Q.N. 9, 2070 'D']

8. Find the term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^9$.

(Ans: 84) 2 [Q.N. 1(b), Supp. 2069]

9. Which term is free from x in the expansion of $\left((x^2+\frac{1}{x}\right)^{15}$?

(Ans: 3003) [Q.N. 1(b), Set 'A' 2069]

10. If $(1+x)^n = c_0 + c_1x + c_2x^2 + ... + c_nx^n$, prove that:

$c_0c_n + c_1c_{n-1} + ... + c_nc_0 = \frac{2n!}{n!n!}$ [Q.N. 9, Set 'B' 2069]

11. Find the coefficient of x^5 in the expansion of $\left(x+\frac{1}{2x}\right)^7$. [Q.N. 1(b), 2068]

$\left(\textit{Ans: } \frac{7}{2}\right)$

12. Find the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{18}$. [Q.N. 1(b), 2067]

$\left(\textit{Ans: } \frac{18!}{(9!)^2}\right)$

13. If three consecutive coefficients in the expansion of $(1+x)^n$ be 165, 330 and 462, find n. [Q.N. 8(b), 2066]

(Ans: 11)

14. Find the term free from x in the expansion of $\left(\frac{3x^2}{2}+\frac{1}{3x}\right)^9$. [Q.N.1(b), 2065]

$\left(\textit{Ans: } \frac{7}{18}\right)$

15. Find the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{18}$ [Q.N. 1(b), 2064]

$\left(\textit{Ans: } \frac{18!}{(9!)^2}\right)$

16. Find the term independent of *x* in the expansions of $\left(x^2+\frac{1}{x}\right)^{12}$.

(Ans.: 9th term = 495) [Q.N. 1(b), 2063]

17. If $C_0, C_1, C_2, ... C_n$ are the binomial coefficients in the expansion of $(1+x)^n$, show that:

$C_0 + C_2 + C_4 + ... = 2^{n-1}$ [Q.N. 1(b), 2062]

18. Find the term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^{12}$

(Ans: 9th term = 495) [Q.N. 1(b), 2061]

19. If the three consecutive coefficients in the expansion of $(1 + x)^n$ be 165, 330, 462; find n. [Q.N. 8(b), 2061]
(Ans: 11)

20. Find the coefficient of x^5 in $\left(x + \frac{1}{2x}\right)^7$ [Q.N. 1(b), 2060]
$\left(Ans: \frac{7}{2}\right)$

21. If C_0, C_1, C_2 .... C_n are the binomial coefficients in the expansion of $(1+x)^n$, then prove that $C_0\ C_n + C_1\ C_{n-1} + \text{..........} + C_n\ C_0 = \frac{2n!}{n!\,n!}$. [Q.N. 8(b), 2059]

22. Find the seventh term of $(2x + y)^{12}$ [Q.N. 1(b), 2058]
$(Ans: 59136x^6y^6)$

23. Find the middle term in the expansion of $(1+x)^{2n}$, where n is a positive integer.
$\left(Ans: \frac{1.3.5 \text{........} (2n-1)\,(2x)^n}{n!}\right)$ [Q.N. 8(b), 2058]

Write the middle terms in the expansion of $(a + x)^n$ when n is odd. [Q.N. 1 (b), 2057]
$\left(Ans: C\left(n, \frac{n+1}{2}\right) a^{\frac{n-1}{2}} . x^{\frac{n+1}{2}}\right)$

2.2 Exponential and Logarithmic Series

1. Prove that : $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \text{.....} = 2e.$ 6[Q.N.9, 2072'C']

2. Prove that $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \text{.....} = e.$ 2[Q.N.1(b), 2072'D']

3. Show that $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \text{..... to } \infty = e.$ 2[Q.N.1(b), 2072'E']

4. Show that: $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1.$ 6 [Q.N. 9, Set 'C' 2071]

5. Show that: $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \text{...} = \frac{3e}{2}$ 6 [Q.N. 9, Set 'D' 2071]

6. Show that $\frac{1}{2}\left(e + \frac{1}{e}\right) = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \text{.........}$ 2 [Q.N. 1(b), 2070 'C']

7. Show that $\log_e 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \text{.........}$ 2 [Q.N. 1(b), 2070 'D']

8. Sum to infinity the series. $\left(1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \text{.....}\right)\left(1 - \frac{1}{1.2} + \frac{1}{1.2.3} - \text{.....}\right)$
[Ans: $e + e^{-1} - 2$] 6 [Q.N. 9, Supp. 2069]

9. Show that: $\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \text{.......}}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \text{.......}} = \frac{e-1}{e+1}$ [Q.N. 9, Set 'A' 2069]

10. Prove that: $\log_e 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \text{.........}$ [Q.N. 1(b), Set 'B' 2069]

11. Prove that: $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+.......\infty=\frac{3e}{2}$ [Q.N. 8(a), 2068]

12. Prove that $\left(\frac{1}{3}-\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{3^2}+\frac{1}{2^2}\right)+\frac{1}{3}\left(\frac{1}{3^3}-\frac{1}{2^3}\right)+.........\infty=0$ [Q.N. 8(b), 2067]

13. Prove that $\log_e 2=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+......$ [Q.N. 1(b), 2066]

14. Prove that : $\frac{1}{1.2}+\frac{1}{2.2^2}+\frac{1}{3.2^3}+.......=\log_e 2$ [Q.N.8(b), 2065]

15. If $y=x-\frac{1}{2}x^2+\frac{1}{3}x^3-....$, show that $x=y+\frac{1}{2!}y^2+\frac{1}{3!}y^3+.....$ [Q.N. 8(b), 2064]

16. If $y=\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........$ to ∞.
Prove that : $x=y-\frac{1}{2}y^2+\frac{1}{3}y^3-\frac{1}{4}y^4+........$ to ∞. [Q.N. 8(b), 2063]

17. Prove that : $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+.........$ to $\infty=\frac{1}{e}$ [Q.N. 8(b), 2062]

18. Prove that : $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+........=\frac{e}{2}$ [Q.N. 8(b), 2060]

19. Prove that: $\log_e 2=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+........$ [Q.N. 3(a), 2059]

20. Show that: $\dfrac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+..............}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+..............}=\frac{e-1}{e+1}$ [Q.N. 8(a), 2057]

Unit 3: Elementary Group Theory

3.1 Binary Operation and Algebraic Structure

1. If $a * b = 3a + 2b$ for $a, b, \in z$, the set of integers, show that * is a binary operation on z. 2[Q.N.1(c), 2072'C']
2. Test the commutative property for the operation * defined by $m*n = n$, $m, n \in z$. 2[Q.N.1(c), 2072'E']
3. Show that the multiplication is a binary operation on the set $S = \{-1, 0, 1\}$. 2[Q.N. 1(c), Set 'D' 2071]
4. Let $G = \{0, 1, 2\}$. Form a composition table of G under the multiplication modulo 3. Find the identify and inverse element of 2.
(Ans: identity element is 1, inverse element of 2 is 2) 2 [Q.N. 1(c), Set 'C' 2071]
5. A binary operation * defined on the set $S = \{a, b, c\}$ is presented in the following Cayley's table

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Show that: (S, *) forms a group. 4[Q.N. 5(b), Set 'D' 2071]
6. Let $G = \{0, 1, 2\}$. Form a composition table for G under addition modulo 3. Find the identity element of 1. 2 [Q.N. 1(c), 2070 'C']
[Ans: 0]
7. Show that the multiplication is a binary operation on the set $S = \{-1, 0, 1\}$. 2 [Q.N. 1(c), 2070 'D']

8. Using Cayley's table of the algebric structure (G,X) where G = {–1,1} and X stands for the usual operation of multiplication, find inverse elements of G.
(Ans: –1, 1) 2 **[Q.N. 1(c), Supp. 2069]**
9. Let S= {–1, 1} and * denote the usual operation of multiplication. Represent it by Cayley's table. Show that the multiplication is a bionary operation on S. **[Q.N. 1(c), Set 'A' 2069]**
10. If the binary operation * on Q, the set of rational numbers is defined by a*b = a+b+ab for every a,b$\in$ Q show that * satisfies associative property. **[Q.N. 1(c), Set 'B' 2069]**

3.2 Group

1. Show that the set of all vectors in space under addition is a group. 4**[Q.N.5(b), 2072'C']**
2. If a, b $\in$ (G, 0) Where G is a group. Prove
(i) $(a * b)^{-1} = b^{-1} * a^{-1}$ (ii) $(a^{-1})^{-1} = a$ **[Q.N.5(b)(Or), 2072'C']**
3. In a Caley's table for a finite group, why does each element occur exactly once in each row and exactly once in each column? 2**[Q.N.1(c), 2072'D']**
4. Let (G, *) be a group. If a, b $\in$ G, then prove that
(i) $(a * b)^{-1} = b^{-1} * a^{-1}$ (ii) $(a^{-1})^{-1} = a$ **[Q.N.5(b), 2072'D']**
5. Define a group. Let a, b, c and x be elements of a group G. Solve the following for x:
$x^2 = a^2$ and $x^5 = e$ 4**[Q.N.5(b) (Or), 2072'D']**
6. Show that the set T = {–1, 1} forms a group under multiplication operation.
4**[Q.N.5(b), 2072'E']**
7. Prove that every element in a group (G, o) has unique inverse.
4**[Q.N.5(b) (Or), 2072'E']**
8. Given the algebraic structure (G, *) with G = {1, ω, ω²} where ω represents the cube root of unity and * stands for the binary operation of ordinary multiplication of complex numbers, show that (G, *) is a group. 4**[Q.N. 5(b), Set 'C' 2071]**
9. If a, b, c are the elements of a group (G, *), prove that:
A * b = A * c $\Rightarrow$ b = c and boa = coa $\Rightarrow$ b = c. 4 **[Q.N. 5(b)(OR), Set 'C' 2071]**
10. Let a, b, c are the elements of a group (G, *).
i) if a * b = b, prove that : a = e
ii) if a * b = e, prove that : $b = a^{-1}$. 4 **[Q.N. 5(b)(OR), Set 'D' 2071]**
11. Show that the set T = {– 1, 1} forms a group under multiplication operation.
4 **[Q.N. 5(b), 2070 'C']**
12. a, b, c are the elements of a group (G, o)
i) if a o b = a o c prove that b = c
ii) if b o a = c o a prove that b = c. **[Q.N. 5(b)(OR), 2070 'C']**
13. Show that the set of integers Z forms a group under the operation of addition.
4 **[Q.N. 5(b), 2070 'D']**
14. If a and b are the elements of a group (G, O) prove that the equation aox = b has a unique solution in (G, O). **[Q.N. 5(b) (OR), 2070 'D']**
15. Define abelian group. If (G,*) is an abelian group, show that
$(a * b)^{-1} = a^{-1} * b^{-1} \forall$ a, b $\in$ G. 4 **[Q.N. 5(b), Supp. 2069]**
16. Let G = { 1, –1, i, –i} and the operation be of multiplication show that G forms a group under the operation of multiplication. 4 **[Q.N. 5(b)(OR), Supp. 2069]**
17. Define group. Let G = {1, –1, i, –i} where i is an imaginary unit and * stands for the binary operation of multiplications. Show that (G, *) forms a group.
[Q.N. 5(b), Set 'A' 2069]
18. If a and b are the elements of a group (G, o) prove that:
$(a \text{ o } b)^{-1} = b^{-1} \text{ o } a^{-1}$ **[Q.N. 5(b)(OR), Set 'A' 2069]**
19. Given the algebraic structure (G,*) with G = {1, w, w²} where w represents the imiaginary cube root of unity and * stands for the binary operation of multiplication, show that (G, *} is a group. **[Q.N. 5(b), Set 'B' 2069]**

Unit 4: Conic Section

4.1 Parabola

1. Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. 4 [Q.N.6(a), 2072'C']
2. If the tangent to the parabola $y^2 = 12x$ makes an angle 45° with the straight line x – 2y + 3 = 0, find its equation and the point of contact. 4 [Q.N.6(a), 2072'D']

   (Ans: $3x - y + 1 = 0$, $x + 3y + 27 = 0$, $\left(\frac{1}{3}, 2\right)$, (27, – 18)
3. Find the condition under which the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Find the equation of the tangent in slope form. 4[Q.N.6(b), 2072'E']

   $\left(\textbf{Ans: } c = \frac{a}{m}, y = mx + \frac{a}{m}\right)$
4. Find the equation of the normal to the parabola $y^2 = 4ax$ in the slope form.
   (Ans: $y = mx - 2am - am^3$) 4 [Q.N. 6(a), Set 'C' 2071]
5. Find the equation of the parabola in the standard form $y^2 = 4ax$.
   4 [Q.N. 6(a), Set 'D' 2071]
6. Prove that the line $3x + 4y + 6 = 0$ is tangent to the parabola $2y^2 = 9x$. Find its point of contact 4 [Q.N. 6(a), 2070 'D']
   (Ans: 2, – 3)
7. Find the equation of the tangent to the parabola $y^2 = 4ax$ at the point (x_1, y_1).
   [Ans: $yy_1 = 2a(x + x_1)$] 4 [Q.N. 6(a), 2070 'C']
8. Deduce the equation of a hyperbola with a focus to (6, 0) and a vertex at (4, 0).
   $\left(\textbf{Ans: } \frac{x^2}{16} - \frac{y^2}{20} = 1\right)$ [Q.N. 6(a) (OR), 2070 'D']
9. If the tangent to the parabola $y^2 = 12x$ makes an angle 45° with the straight line $x - 2y + 3 = 0$, find its equation and the point of contact. 4 [Q.N. 6(a), Supp. 2069]

   [Ans: $3x - y + 1 = 0$, $x + 3y + 27 = 0$, $(\frac{1}{3}, 2)$, (27, –18)]
10. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1) and express this in slope form. [Q.N. 6(a), Set 'A' 2069]
    (Ans: $y = mx - 2am - am^3$)
11. Find the condition under which the line $y = mx + c$ is tangent to the parabola $y^2 = 4ax$. Find the equation of tangent in slope form. Also, find the point of contact.
    $\left(\textit{Ans: } y = mx + \frac{a}{m}, \left(\frac{a}{m_2}, \frac{2a}{m}\right)\right)$ [Q.N. 6(a), Set 'B' 2069]
12. Prove that the line $\ell x+my+n=0$ touches the parabola $y^2=4ax$ if $\ell n=am^2$. [Q.N.2(c), 2068]
13. Find the coordinates of the focus, the vertex, theequation of thedirectrix and the length of the latus rectum of the parabola $y^2 = 6y - 12x + 45$. [Q.N. 9(b), 2068]

    $\left(\textit{Ans: vertex} = \left(\frac{9}{2}, 3\right), \textit{focus} = \left(\frac{3}{2}, 3\right) \textit{ directrix: } x = \frac{15}{2}, \textit{latus rectum} = 12\right)$
14. Show that the pair of tangents from the point (–2, 3) to the parabola $y^2 = 8x$ are at right angle. [Q.N. 9(b), 2067]
15. Find the equations of the tangents from the point (–6, 9) to the parabola $y^2 = 24x$.
    (Ans: $2x + y + 3 = 0$ and $x - 2y + 24 = 0$) [Q.N. 5(c), 2066]
16. Find the equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) on the parabola. [Q.N.9(b), 2065]

    (Ans: $yy_1 = 2a(x + x_1)$)

17. Find the equation of the parabola in which the ends of the latus rectum have the co-ordinates (–1, 5) and (–1, –11) and the vertex is (–5, –3). [Q.N. 5(c), 2064]
18. Deduce the equation of the parabola in the standard form $y^2 = 4ax$. [Q.N. 9(b), 2064]
(Ans: $y^2 + 6y - 16x - 71 = 0$)
19. Find the coordinates of the vertex and the focus of the parabola whose equation is $y^2 = 6y - 12x + 45$. [Q.N. 5(c), 2063]
$\left(\text{Ans.: vertex} = \left(\frac{9}{2}, 3\right) \text{ and Focus} = \left(\frac{3}{2}, 3\right)\right)$
20. Prove that the line $\ell x + my + n = 0$ touches the parabola $y^2 = 4ax$ if $\ell n = am^2$. [Q.N. 9(b), 2063]
21. Determine the equation of the chord joining the points t_1 and t_2 on the parabola $y^2 = 4ax$. [Q.N. 5(c), 2062]
$\left(\text{Ans.: } (t_1 + t_2)x - 2y = at_1(t_1^2 + t_1t_2 - 4)\right)$
22. Find the equation of the normal to the parabola $y^2 = 4ax$ in the slope form.
(Ans.: $y = mx - 2am - am^3$) [Q.N. 9(b), 2062]
23. Prove that the latus rectum of a parabola bisects the angle between the tangent and the normal at either extremity of the latus rectum. [Q.N. 9(b), 2061]
24. Find the equation of the normal to the parabola $y^2 = 5x$ perpendicular to the line $x + 2y = 7$ [Q.N. 2(c), 2060]
(Ans: $y = 2x - 15$)
25. Deduce the equation of the tangent to the parabola $y^2 = 4ax$ at (x_1, y_1) on the parabola.
(Ans: $yy_1 = 2a(x + x_1)$) [Q.N. 9(b), 2060]
26. Find the focus and directrix of the parabola $x^2 = 12y$. [Q.N. 2(c), 2059]
(Ans: (0, 3) and $y + 3 = 0$)
27. Find the condition that the line $y = mx + c$ may be a tangent to the parabola $y^2 = 4ax$.
$\left(\text{Ans: } c = \frac{a}{m}\right)$ [Q.N. 9(b), 2059]
28. Find the equation of the tangent to the parabola $y^2 = 16x$ at the point (4, 8).
(Ans: $x - y + 4 = 0$) [Q.N. 2(c), 2058]
29. Show that the normal to the parabola $y^2 = 8x$ at (2,4) meets the parabola again in (18,–12). [Q.N. 9(b), 2058]
30. Find the focus and directrix of the parabola $y^2 - 4y - 8x - 20 = 0$. [Q.N. 2(c), 2057]
(Ans: (–1, 2) and $x + 5 = 0$)
31. Find the equation of the parabola in the standard form $y^2 = 4ax$.
(Ans: $y^2 = 4ax$) [Q.N. 9(b), 2057]

4.2 Ellipse

1. Show that $x^2 + 4y^2 - 2x - 16y + 1 = 0$ represents the equation of the ellipse. Find its vertices and foci. [Q.N.6(a)(Or), 2072'C']
(Ans: (5, 2) and (– 3, 2); $(1 \pm 2\sqrt{3}, 2)$)
2. Find the eccentricity and coordinates of the foci of the curve $\frac{(x+6)^2}{4} + \frac{y^2}{36} = 1$
$\left(\text{Ans: } \frac{2\sqrt{2}}{3}, (-6, \pm 4\sqrt{2})\right)$ [Q.N.6(a)(Or), 2072'D']
3. Find the eccentricity and the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$.
$\left(\text{Ans: } \frac{4}{5}, (0, \pm 4)\right)$ 2[Q.N.2(a), 2072'E']

4. Find the eccentricity and the foci of the ellipse $25x^2 + 4y^2 = 100$.
$\left(\text{Ans: } e = \frac{\sqrt{21}}{5}, \text{ foci} = (0, \pm\sqrt{21})\right)$ 2 [Q.N. 2(a), Set 'C' 2071]

5. Find the equation of the ellipse whose distance between two foci is 8 and the semi-latus rectum is 6. 4 [Q.N. 6(a) (OR), Set 'D' 2071]
[Ans: $3x^2 + 4y^2 = 192$]

6. Find the eccentricity and the foci of the ellipse. $\frac{x^2}{9} + \frac{y^2}{16} = 1$ 2 [Q.N. 2(a), 2070 'C']
$\left(\text{Ans: } \frac{\sqrt{7}}{4}, (0, \pm\sqrt{7})\right)$

7. Find the eccentricity and the foci of the ellipse $3x^2 + 4y^2 = 36$
$\left(\text{Ans: } \frac{1}{2}, (\pm\sqrt{3}, 0)\right)$ 2 [Q.N. 2(a), 2070 'D']

8. Show that $x^2 + 4y^2 - 2x - 16y + 1 = 0$ represents the equation of an ellipse. Find it centre, vertex and foci. [Q.N. 6(a) (OR), Supp. 2069]
[Ans: Centre = (1, 2), vertices = (5, 2) & (–3, 2), foci = $(1 + 2\sqrt{3}, 2)$]

9. Find the eccentricity, the coordinates of the vertices and the foci of ellipse $9x^2 + 5y^2 - 30y = 0$ [Q.N. 6(a) (OR), Set 'A' 2069]
$\left(\textit{Ans: } \frac{2}{3}, (0, 3 \pm 3)\ (0, 3 \pm 2)\right)$

10. Find the eccentricity and the foci of the ellipse $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$
$\left(\textit{Ans: } \frac{\sqrt{7}}{4}, (-2 \pm \sqrt{7}, 5)\right)$ [Q.N. 2(a), Set 'B' 2069]

11. Find the coordinates of the vertices, the eccentricity and the coordinates of the foci the ellipse $25x^2 + 4y^2 = 100$ [Q.N. 9(b) (or), 2068]
$\left(\textit{Ans: } (0, 0), \frac{\sqrt{21}}{5}, (0, \pm\sqrt{21})\right)$

12. Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find its centre, vertex and focus. [Q.N. 9(b) (or), 2067]
[Ans: (2, – 3); (–2, – 3) & (6, – 3), $(2 \pm 2\sqrt{3}, -3)$]

13. Find the equation of the ellipse in the standard position whose latus rectum is equal to half its major axis and which passes through the point $(\sqrt{6}, 1)$.
$\left(\textit{Ans: } \frac{x^2}{8} + \frac{y^2}{4} = 1\right)$ [Q.N. 9(b), 2066]

14. Show that : $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represents the equation of an ellipse. Find its centre, vertex & focus. [Q.N.9(b, or), 2065]
$\left(\textit{Ans: } (1,2); (1,5) \text{ and } (1, -1); (1, 2\pm\sqrt{5})\right)$

15. Find the eccentricity and the foci of the ellipse :
$9x^2 + 5y^2 - 30y = 0$ [Q.N. 9(b)Or, 2064]
$\left(\textit{Ans: } \frac{2\sqrt{2}}{3}, (0, 3 \pm 2\sqrt{3})\right)$

16. Find the eccentricity and the coordinates of the foci of the ellipse :
$\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$ [Q.N. 9(b) Or, 2062]
$\left(\textit{Ans.: } \frac{1}{\sqrt{3}}, (0, 0), (0, 4)\right)$

17. Find the equation of the ellipse in the standard position with a focus at (0, – 5) and eccentricity $\frac{1}{3}$. [Q.N. 5(c), 2061]

(Ans: $9x^2 + 8y^2 = 180$)

18. Find the eccentricity and the foci of the ellipse : $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$

$\left(\text{Ans: } \frac{1}{\sqrt{3}}, (0, 4) \text{ and } (0, 0)\right)$ [Q.N. 9(b)Or, 2060]

19. Deduce the equation of the ellipse in the standard position if a focus is at (0, -5) and eccentricity is $\frac{1}{3}$. [Q.N. 9(b) Or, 2059]

(Ans: $9x^2 + 8y^2 = 180$)

20. Find the eccentricity and the foci of the ellipse $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$. [Q.N. 9(b)Or, 2058]

$\left(\text{Ans: } \frac{\sqrt{7}}{4}, (-2 \pm \sqrt{7}, 5)\right)$

21. Find the eccentricity, length of the latus rectum and coordinates of the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$. [Q.N. 9(b)Or, 2057]

$\left(\text{Ans: } \frac{\sqrt{3}}{2}, 2, (\pm 2\sqrt{3}, 0)\right)$

4.3 Hyperbola

1. Find eccentricity and foci of the hyperbola $\frac{x^2}{36} - \frac{y^2}{64} = 1$. 2[Q.N.2(a), 2072'C']

$\left(\text{Ans: } \frac{5}{3}, \pm 10, 0\right)$

2. Find the equation of the hyperbola with vertex (8, 0) and passing the through the point $(8\sqrt{2}, 4)$. 2[Q.N.2(a), 2072'D']

$\left(\text{Ans: } \frac{x^2}{64} - \frac{y^2}{16} = 1\right)$

3. Find the equation of the hyperbola with vertex at (0, 8) and passing through the point $(4, 8\sqrt{2})$. [Q.N.6(b)(Or), 2072'E']

$\left(\text{Ans: } \frac{y^2}{64} - \frac{x^2}{16} = 1\right)$

4. Find the equation of the hyperbola with a focus at (0, 5) and a vertex at (0, –3)

$\left(\text{Ans: } \frac{y^2}{9} - \frac{x^2}{16} = 1\right)$ [Q.N. 6(a) (OR), Set 'C' 2071]

5. Find the eccentricity and the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

$\left(\text{Ans: } e = \frac{5}{3}, \text{ foci} = (\pm 5, 0)\right)$ 2 [Q.N. 2(a), Set 'D' 2071]

6. Find the coordinates of the vertices, the eccentricity and the coordinates of the foci of the hyperbola $5x^2 - 20y^2 - 20x = 0$ [Q.N. 6(a) (OR), 2070 'C']

[Ans: Vertices = (0, 0) & (4, 0), $e = \frac{\sqrt{5}}{2}$, foci = $(2 \pm \sqrt{5}, 0)$]

7. Determine the equation of the hyperbola with a focus at (–5,0) and a vertex at (3, 0). 2
$\left(\text{Ans: } \frac{x^2}{9} - \frac{y^2}{16} = 1\right)$ [Q.N. 2(a), Supp. 2069]

8. Find the eccentricity and the foci of the hyperbola $3x^2 - 4y^2 = 36$.
$\left(\text{Ans: } \frac{\sqrt{7}}{2}, (\pm\sqrt{21}, 0)\right)$ [Q.N. 2(a), Set 'A' 2069]

9. Find the equation of the hyperbola with focus at (–5, 0) and vertex at (2, 0).
$\left(\text{Ans: } \frac{x^2}{4} - \frac{y^2}{21}\right) = 1$ [Q.N. 6(a) (OR), Set 'B' 2069]

10. Find the equation of hyperbola in the standard form with a focus at (0, 5) and a vertex at (0, –3). [Q.N. 5(c), 2067]
(Ans: $16y^2 - 9x^2 = 144$)

11. Determine the equation of the hyperbola in the standard position with focus at (– 7, 0) and eccentricity $\frac{7}{4}$. [Q.N. 9(b)Or, 2066]
$\left(\text{Ans: } \frac{x^2}{16} - \frac{y^2}{33} = 1\right)$

12. Find the eccentricity and foci of the hyperbola $3x^2 - 4y^2 = 36$. [Q.N.5(c), 2065]
$\left(\text{Ans: } \frac{\sqrt{7}}{2}, (\pm\sqrt{21}, 0)\right)$

13. Find the eccentricity and the coordinates or the foci of the hyperbola
$\frac{x^2}{16} - \frac{y^2}{4} = 1$. [Q.N. 9(b) Or, 2063]
$\left(\text{Ans.: Eccentricity} = \frac{\sqrt{5}}{2} \text{ and Foci} = (\pm 2\sqrt{5}, 0)\right)$

14. Find the eccentricity and the foci of the hyperbola : $3x^2 - 4y^2 = 36$. [Q.N. 9(b) Or, 2061]
$\left(\text{Ans: } \frac{\sqrt{7}}{2}, (\pm\sqrt{21}, 0)\right)$

Unit 5: Co-ordinates in Space

5.1 Coordinates in Space

1. Find the ratio in which the line joining the points P(– 2, 4, 7) and Q(3, – 5, – 1) is divided by the yz-plane. 2[Q.N.2(b), 2072'C']
(Ans: 3 : 2)

2. If P and Q denote the coordinates (2, 6, 2) and (4, 5, 0) respectively, find the direction cosines of the line PQ. 2[Q.N.2(b), 2072'D']
$\left(\text{Ans: } \frac{2}{3}, \frac{5}{3}, -\frac{2}{3}\right)$

3. Find the angle between the two lines whose direction ratios are 2,3,4 and 1, – 2, 1. 2[Q.N.2(b), 2072'E']
(Ans: 90°)

4. Show that the lin^ AB is perpendicular to CD if A, B, C, D are the points (2, 3, 4), (5, 4, –1), (3, 6, 2) and (1, 2, 0) respectively. 4 [Q.N. 6(b), Set 'C' 2071]

5. Find the direction cosines of a line passing through the points M(–2, 4, 3) and N(–1, 2, 5). 2 [Q.N. 2(b), Set 'D' 2071]
$\left(\text{Ans: } -\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right)$

6. Show that the direction cosines of a line equally inclined to the axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$. 2 [Q.N. 2(b), 2070 'C']

7. Find the angle between two straight lines whose direction cosines are ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2. 4[Q.N. 6(b), 2070 'D']
(Ans: $\cos^{-1}(\ell_1\ell_2 + m_1m_2 + n_1n_2)$)

8. Find the direction cosines of the line joining the points (1,2,3) and (4,5,7).
$\left(\text{Ans: } \frac{3}{\sqrt{34}}, \frac{3}{\sqrt{34}}, \frac{4}{\sqrt{34}}\right)$ 2 [Q.N. 2(b), Supp. 2069]

9. Find the angle between the two lines whose direction ratios are a_1, b_1, c_1, and a_2, b_2, c_2. Also, find the condition underwhich the two lines are perpendicular. [Q.N. 6(b), Set 'A' 2069]
$\left(\text{Ans: } \theta = \cos^{-1}\left\{\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{\Sigma a_1^2}\sqrt{\Sigma a_2^2}}, a_1a_2 + b_1b_2 + c_1c_2 = 0\right\}\right)$

10. Find the direction cosines of a line which are equally inclined to the axes.
$\left(\text{Ans: } \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \text{ or } \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)\right)$ [Q.N. 2(b), Set 'B' 2069]

5.2 Plane

1. Find the equation of the plane through the intersection of the planes $2x + 3y + 10z = 8$ and $2x - 3y + 7z = 2$, and perpendicular to the plane $3x - 2y + 4z = 5$. 4[Q.N.6(b), 2072'C']
(Ans: $2y + z - 2 = 0$)

2. Show that the plane $2x + 3y - 4z = 3$ is parallel to the plane $10x + 15y - 20z = 12$ and is perpendicular to the plane $3x + 2y + 3z = 5$. 4[Q.N.6(b), 2072'D']

3. Find the equation of the plane passing through the points (1, 1, 0), (–2, 2, –1) and (1, 2, 1). 4 [Q.N.6(a), 2072'E']
(Ans: $2x + 3y - 3z = 5$)

4. Find the equation of the plane through (1, 2, 3) and parallel to the plane $3x - 4y + 5z = 0$.
(Ans: $3x - 4y + 5z = 10$) 2 [Q.N. 2(b), Set 'C' 2071]

5. Find the equation of the plane through the points (–1,1,1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.
[Ans: $2x + 2y - 3z + 3 = 0$] 4[Q.N. 6(b), Set 'D' 2071]

6. Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6), and normal to the plane $2x + 6y + 6z = 9$. 4 [Q.N. 6(b), 2070 'C']
[Ans: $3x + 4y - 52 = 9$]

7. Find the equation of the plane which makes equal intercepts on the axes and passes through the point (2, 3, 4). 2 [Q.N. 2(b), 2070 'D']
(Ans: $x + y + z = 9$)

8. Show that the plane $2x + 3y - 4z = 3$ is parallel to the plane $10x + 15y - 20z = 12$ and is perpendicular to the plane $3x + 2y + 3z = 5$. 4 [Q.N. 6(b), Supp. 2069]

9. Find the equation of the plane through the point (3, –4, 5) and parallel to the plane $3x - 4y + 5z = 7$. 2 [Q.N. 2(b), Set 'A' 2069]
(Ans: $3x - 4y + 5z - 50 = 0$)

10. Find the equation of the plane through the points (1, 1, 0), (– 2, 2, – 1) and (1, 2, 1).
(Ans: $2x + 3y - 3z + 1 = 0$) [Q.N. 6(b), Set 'B' 2069]

Unit 6: Vectors and its Applications

6.1 Elements of Vectors

1. Prove that vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$ and $-7\vec{i} + 10\vec{k}$ are collinear ? 2[Q.N.2(c), 2072'C']

2. If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ find unit vector along $\vec{a} - 2\vec{b}$. 2
   $\left(\text{Ans: } \frac{7}{\sqrt{134}}, \frac{-9}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right)$ [Q.N.2(c), 2072'D']

3. If $3\vec{i} + \vec{j} + \vec{k}$, and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are colliner vectors. Find the value of λ.
   (Ans: $\lambda = -12$) 2 [Q.N. 2(c), Set 'C' 2071]

4. Show that the three points whose position vectors are
   $7\vec{j} + 10\vec{k}$, $-\vec{i} + 6\vec{j} + 6\vec{k}$ and $-4\vec{i} + 9\vec{j} + 6\vec{k}$ form an isosceles triangle.
   2 [Q.N. 2(c), Set 'D' 2071]

5. ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point show that: $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$. 2 [Q.N. 2(c), 2070 'C']

6. The vertices A, B, C of a triangle are (2, −1, −3), (4, 2, 3) and (6, 3, 4) respectively. Show that $\vec{AB} = (2, 3, 6)$ and AC = 9. 2 [Q.N. 2(c), 2070 'D']

7. If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-5, 7, -1)$, find $|\vec{a} - 2\vec{b} + \vec{c}|$.
   [Ans: 3] 2[Q.N. 2(c), Supp. 2069]

8. Show that the three points with position vectors.
   $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$ and $3\vec{i} + 8\vec{j} - 6\vec{k}$ are collinear.
   [Q.N. 2(c), Set 'A' 2069]

9. If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and a unit vector along the direction of $\vec{PQ}$. [Q.N. 2(c), Set 'B' 2069]
   $\left(\textit{Ans: } 4\vec{i} - 5\vec{j} + 11\vec{k}, \frac{4}{9\sqrt{2}}\vec{i} - \frac{5}{9\sqrt{2}}\vec{j} + \frac{11}{9\sqrt{2}}\vec{k}\right)$

10. ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point, show that $\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} = 4\vec{OG}$. [Q.N. 4(a), 2068]

11. Prove that the three vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $\vec{b} + 2\vec{c}$ are coplanar. [Q.N. 10(a), 2068]

12. Determine the unit vector of $2\vec{a} - 3\vec{b}$ where $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = 2\vec{i} + 3\vec{j}$
    $\left(\text{Ans: } \frac{14}{\sqrt{421}}\vec{i} + \frac{15}{\sqrt{421}}\vec{j}\right)$ [Q.N. 4(a), 2067]

13. Show that the three points whose position vectors are $2\vec{i} - \vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} - 5\vec{k}$ and $3\vec{i} - 5\vec{j} - 4\vec{k}$ form the sides of a right angled triangle.
    [Q.N. 10(a), 2067]

14. Prove that the points A, B, C are collinear, if
    $\vec{OA} = \vec{i} + 2\vec{j} + 4\vec{k}$, $\vec{OB} = 2\vec{i} + 5\vec{j} - \vec{k}$ and $\vec{OC} = 3\vec{i} + 8\vec{j} - 6\vec{k}$. [Q.N.4(a),2066]

15. Prove that the vectors:
$5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are coplaner. [Q.N. 10(a), 2066]

16. If $3\vec{i}+\vec{j}-\vec{k}$ and $\lambda\vec{i}-4\vec{j}+4\vec{k}$ are collinear vector. Find λ.
(Ans: –12) [Q.N.3(b), 2065]

17. Show that the following vectors are linearly dependent :
$5\vec{i}+6\vec{j}+7\vec{k}$, $7\vec{i}-8\vec{j}+9\vec{k}$ and $3\vec{i}+20\vec{j}+5\vec{k}$. [Q.N.10(a), 2065]

18. If $\vec{a}=(3, 4)$ and $3\vec{a}+2\vec{b}=(5, 6)$, find $\vec{b}$. [Q.N. 4(a), 2064]
(Ans: –2, –3)

19. Show that the three points whose position vectors are
$7\vec{j}+10\vec{k}$, $-\vec{i}+6\vec{j}+6\vec{k}$ and $-4\vec{i}+9\vec{j}+6\vec{k}$ form an isosceles right angled triangle. [Q.N. 10(a), 2064]

20. If $\vec{a}=(2,-3)$ and $\vec{b}=(4,-2)$, find unit vector along $4\vec{a}-3\vec{b}$.
$\left(Ans.:\left(\frac{-4}{2\sqrt{13}},\frac{-6}{2\sqrt{13}}\right)\right)$ [Q.N. 3(b), 2063]

21. ABCD is a parallelogram. G is the point of intersection of the diagonals and if O is any point, show that : $\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}=4\vec{OG}$ [Q.N. 10(a), 2063]

22. If $\vec{OP}=\vec{i}+3\vec{j}-7\vec{k}$ and $\vec{OQ}=5\vec{i}+2\vec{j}-4\vec{k}$ find $\vec{PQ}$ and determine its direction cosines. [Q.N. 4(a), 2062]
$\left(Ans.: 4\vec{i}-5\vec{j}+11\vec{k},\frac{4}{9\sqrt{2}},\frac{-5}{9\sqrt{2}},\frac{11}{9\sqrt{2}}\right)$

23. OB and OC are two straight lines and D is a point on BC such that
BD:DC = m:n, show that : $\vec{OD}=\frac{n\vec{OB}+m\vec{OC}}{m+n}$ [Q.N. 10(a), 2062]

24. Prove that the following vectors are coplanar:
$\vec{a}-3\vec{b}+5\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-4\vec{c}$ [Q.N. 10(a), 2061]

25. If $\vec{a}=(2,-3)$ and $\vec{b}=(4,-2)$. Find the unit vector along $4\vec{a}-3\vec{b}$.
$\left(Ans:\frac{-4}{2\sqrt{13}},\frac{-6}{2\sqrt{13}}\right)$ [Q.N. 4(a), 2060]

26. Show that the three points whose positions vectors are $7\vec{j}+10\vec{k}$, $-\vec{i}+6\vec{j}+6\vec{k}$ and $-4\vec{i}+9\vec{j}+6\vec{k}$ form an isosceles right angled triangle.
[Q.N. 10(a), 2060]

27. If $\vec{a}+\vec{b}=(5, 6)$ and $\vec{a}-\vec{b}=(3, 2)$, find $\vec{a}$ and $\vec{b}$. [Q.N. 3(b), 2059]
(Ans: (4, 4) and (1, 2))

28. Prove that the vectors $-\vec{a}+4\vec{b}+3\vec{c}$; $2\vec{a}-3\vec{b}-5\vec{c}$ and $2\vec{a}+7\vec{b}-3\vec{c}$ are coplanars, where $\vec{a}$, $\vec{b}$, $\vec{c}$ are any vectors. [Q.N. 10(a), 2059]

29. If $\vec{a}=(3,-1,-4)$, $\vec{b}=(-2,4,-3)$ and $\vec{c}=(-5,7,-1)$ find $\left|\vec{a}-2\vec{b}+\vec{c}\right|$
(Ans: 3) [Q.N. 4(a), 2058]

30. Show that the points A, B and C with position vectors $\vec{i} - 2\vec{j} + 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$, $-7\vec{j} + 10\vec{k}$ respectively are collinear. [Q.N. 10(a), 2058]

31. Show that the vectors $2\vec{i} + 3\vec{j} - 8\vec{k}$ and $2\vec{i} + 4\vec{j} + 2\vec{k}$ are orthogonal. [Q.N. 3(b), 2057]

32. If the position vector of M and N are $3\vec{i} + \vec{j} - 3\vec{k}$ and $4\vec{i} - 2\vec{j} + \vec{k}$ respectively, find $\vec{MN}$ and determine its direction cosines. [Q.N. 10(a), 2057]
$\left(Ans: \vec{MN} = \vec{i} - 3\vec{j} + 4\vec{k}, \left(\frac{1}{\sqrt{26}}, \frac{-3}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)\right)$

33. A B CD E F is a regular hexagon. Express $\vec{AC}$ and $\vec{AD}$ in terms of $\vec{AB}$ and $\vec{BC}$.
$\left(Ans: \vec{AC} = \vec{AB} + \vec{BC}, \vec{AD} = 2\vec{BC}\right)$ [Q.N. 4(a), 2057]

6.2 Product of Vectors

1. Find the angle between the vectors $2\vec{i} - \vec{j} + \vec{k}$ and $\vec{i} - 3\vec{j} - 5\vec{k}$.
(Ans: 90°) 2[Q.N.3(c), 2072'C']
2. Define Vector product of two Vectors. Prove by Vector method:
$\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6[Q.N.10, 2072'C']
3. If $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 0$, prove that $|\vec{a}| = |\vec{b}|$. 2[Q.N.3(c), 2072'D']
4. Define Vector product of two Vectors. Prove by Vector method that in any triangle ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 6[Q.N.10, 2072'D']
5. If $\vec{OP} = \vec{i} + 3\vec{j} - 7\vec{k}$ and $\vec{OQ} = 5\vec{i} - 2\vec{j} + 4\vec{k}$, find $\vec{PQ}$ and its direction cosines. 2[Q.N.2(c), 2072'E']
$\left(Ans: 4\vec{i} - 5\vec{j} + 11\vec{k}, \left(\frac{4}{9\sqrt{2}}, \frac{11}{9\sqrt{2}}\right)\right)$
6. Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$
(Ans: 20.5 sq. unit) 2[Q.N.3(c), 2072'E']
7. Define scalar product of two vectors. Give the geometrical interpretation of the scalar product of two vectors. In any triangle prove vectorically that $a^2 = b^2 + c^2 - 2bc \cos A$. [Q.N.10, 2072'E']
8. For what value of m is the pair of vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} - 7\vec{j} + m\vec{k}$ orthogonal ?
(Ans: m = 3) 2 [Q.N. 3(c), Set 'C' 2071]
9. Define vector product of two vectors. Prove by vector method that
$\sin(A + B) = \sin A\cos B + \cos A\sin B$. 6 [Q.N. 10, Set 'C' 2071]
10. Find a unit vector perpendicular to each of the vectors
$3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$. 2 [Q.N. 3(c), Set 'D' 2071]
$\left(Ans: \frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} - \frac{1}{\sqrt{3}}\vec{k}\right)$
11. Define scalar product of two vectors. Prove by vector method that:
$\cos(A + B) = \cos A \cos B - \sin A \sin B$. 6 [Q.N. 10, Set 'D' 2071]

12. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, prove that $\vec{a}$ is perpendicular to $\vec{b}$ 2 [Q.N. 3(c), 2070 'C']

13. Define vector product of two vectors. Using vector method, prove that:
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 6[Q.N. 10, 2070 'C']

14. Find the since of the angle between the two vectors
$2\vec{i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$. 2 [Q.N. 3(c), 2070 'D']
$\left(\text{Ans: } \sqrt{\frac{155}{156}}\right)$

15. Define scalar product of two vectors. 6[Q.N. 10, 2070 'D']
Prove by vector method that : cos (A – B) = cosA cosB + sinA sinB

16. If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$, prove that $|\vec{a}| = |\vec{b}|$. 2 [Q.N. 3(c), Supp. 2069]

17. Define vector product of two vectors and geometrically interpret it. Also show that $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ where $\vec{a}$ and $\vec{b}$ are any two non zero vectors.
6 [Q.N. 10, Supp. 2069]

18. Find the area of the parallelogram determined by the vectors $\vec{i} + 2\vec{j} + 3\vec{k}$ and $-3\vec{i} - 2\vec{j} + \vec{k}$ [Q.N. 3(c), Set 'A' 2069]
(Ans: $6\sqrt{5}$ sq uints)

19. Define scalar product of two vectors. Prove by the method of vectors that:
cos (A–B) = cosA cosB + sinA sinB. 6 [Q.N. 10, Set 'A' 2069]

20. If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ are any two vectors, find the cosine of the angle between the two vectors. [Q.N. 3(c), Set 'B' 2069]
$\left(\text{Ans: } \frac{1}{2}\right)$

21. Define vector product of two vectors. Interpret the vector product of two vectors geometrically. Prove by vector method that:
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ [Q.N. 10, Set 'B' 2069]

22. Find the area of the triangle determined by the vectors
$3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$ [Q.N. 3(b), 2068]
[Ans: 20.5 sq. units]

23. Using vector method prove that: $c^2 = a^2+b^2 - 2ab \cos C$. [Q.N. 11(b), 2068]

24. Given $\vec{a} = (3, 1, 2)$ and $\vec{b} = (2, -2, 4)$, find the projection of $\vec{a}$ on $\vec{b}$.
(Ans: $\sqrt{6}$) [Q.N. 3(b), 2067]

25. Prove by vector method : Cos (A+B) = Cos A Cos B – Sin A Sin B. [Q.N. 11(a), 2067]

26. For what value of m are the vectors $\vec{i} - 2\vec{j} + 4\vec{k}$ and $2\vec{i} + 7\vec{j} + m\vec{k}$ orthogonal?
(Ans: 3) [Q.N. 3(b), 2066]

27. Use vector method to prove that, in any triangle ABC, a = b cos C + c cos B.
[Q.N. 11(a), 2066]

28. Find the value of r if the vectors $3\vec{i} - \vec{j} - 2\vec{k}$ and $2\vec{i} - 2\vec{j} + r\vec{k}$ are orthogonal.
(Ans: 4) [Q.N.4(a), 2065]

29. By using vectors, prove that in any ΔABC,
$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$ [Q.N.11(a), 2065]